The Organization as a Rule of Intra-firm Bargaining

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February 26, 2007

Abstract

This paper studies the choice of organizational forms under the incomplete contract. We identify an organizational form with a rule of the ex post bargaining and compare four types of organization: horizontal organization (partnership), common agency, pyramidal hierarchy and vertical hierarchy. We show that if human capital investments of all members are complementary and essential to the production, the horizontal organization is chosen. If investments of two players including an owner are essential, then the common agency can be optimal. If the pyramidal hierarchy can motivate subordinates to invest, the pyramidal hierarchy is chosen. The vertical hierarchy may be realized if the owner can motivate a player who engages in the firm-specific investment by assigning him to the middle rank. We also examine who should be assign to the middle tier in the vertical hierarchy.

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1 Introduction

The multi-level pyramidal hierarchy is widespread in large firms and many papers show the rationale for hierarchical form. However, there are other organizational forms in the real world. For example, it is well known that law firms adopt the partnership (horizontal organization). There are often two bosses (common agency) in new establishing firms. For example, Yahoo! and Google were founded by two person. Soichiro Honda, who was a founder of one of the biggest auto companies, Honda, concentrated on the technology sector and his business partner, Takeo Fujisawa, engaged in management. Furthermore there are hierarchical organizations with the steep structure as well as hierarchies with the flat structure.

Why there are various forms of organization in the real world? We focus on incentives to invest the human capital under the incomplete contract and show that characteristics of investment exerted by members determine which organizational form is optimal for the owner of the firm.

The return is divided among the members of the organization after the human capital investment because of the incompleteness of the contract. The bargaining position of each member over the return from human capital investments is different across organizational forms. The bargaining power of the owner depends on whether the only person owns the firm, two people own together or all of the member own together. Workers in the higher hierarchical rank may have stronger bargaining positions than workers in the lower rank. The bargaining power of a worker depends on whether he has subordinates or not. Therefore, an organizational structure is regarded as an allocation of the bargaining power and we identify an organizational structure with a rule of intra-firm bargaining.

We model this idea by considering an organization of three players and comparing four types of organizational form: *horizontal organization* where all members have equal authority, *common agency* where there are two bosses, *pyramidal hierarchy* where there are one boss and two subordinates in a same rank and *vertical hierarchy* where there are one boss, one supervisor and one subordinate. This paper studies how the choice of organizational forms determines the ex-post bargaining rule and affects the incentive of human capital investments. The bargaining procedure runs as follows; a player in the higher tier is able to make a proposal prior to a player in the lower tier. If there is more than one player in the top rank, they are selected as a proposer with equal probability. A player makes a take-it-or-leave-it offer to her subordinates and so on. We call a player who chooses the organizational form 'player 1' and she is assumed to be in a top rank position. Therefore, player 1 is regarded as a principal in our model.

We obtain the following results in this paper. If investments of all members are (perfectly) complementary and essential for the production, the horizontal organization is chosen. If investments of two players including player 1 are essential and an investment of another player is marketable, then the common agency emerges in the case where two players engage in essential investments. If two subordinates intend to invest their human capital in the pyramidal hierarchy, player 1 chooses this form. We examine a tier-assignment problem in the vertical hierarchy when player 2 and player 3 are asymmetric. A player assigned to the bottom rank would invests only if his investment is marketable in our model. Since a player in the middle rank has a bargaining power due to the position, he has stronger incentive than the player in the bottom rank. We show if only one-player's investments is marketable, a player who engage in the firm-specific investment is assigned to the middle rank. If the investments for both players are firm-specific, a player whose investment increases firm's value more should be assigned to the middle rank. Finally, we compare the vertical hierarchy with the pyramidal hierarchy. The vertical hierarchy can be feasible only when the owner can motivate a player who engages in the firm-specific investment by assigning him to the middle rank. However the increase in wage is large relative to the benefit when the degree of firm-specificity is small. Then the owner chooses the pyramidal hierarchy even though it can not implement the player to invest.

Most closely related to our work is Hart and Moore (1990) who examine how the ownership of assets affects human capital investments and consider the boundary of the firm. To focus on the design of organization, our model does not consider the control structure of assets or the boundary of the firm. There are two other different points from Hart and Moore (1990).

First, the ex-post bargaining in our model is based on a noncooperative approach, not on a cooperative approach. Hart and Moore adopted the Shapley value as a solution concept to the bargaining problem. Since they adopted a noncooperative approach to the action decision problem, the solution concept is inconsistent. The cooperative game approach itself does not matter. Actually, some noncooperative bargaining game model has been provided to generate the Shapley value as a Nash equilibrium outcome (see, Gul (1989), Hart and Mas-Colell (1996), Hart and Moore (1988), Stole and Zwiebel (1996)). The noncooperative bargaining games to implement the Shapley value commonly contain the feature which every player has an equal position and equal treatment in the bargaining procedure[1]. The bargain-

[1] For example, every player is selected as a proposer with equal probability among all the players (in Gul (1989), Hart and Mas-Colell (1986)), or all players are line up in random

ing procedure in organizations may depend on the organization structure, but the cooperative bargaining approach cannot reflect it. We consider the intra-firm bargaining given the organization structure. The organizational structure affects the bargaining procedure and the possible coalitional deviations in renegotiations. Therefore the barganing power is expressed by the bargaining procedure not by the outside option in our model.

Second, Hart and Moore (1990) studies that the ownership and control structure of assets to maximize the social surplus. We consider a situation where lump-sum transfer is not feasible in ex-ante and Coase theorem can not be applied to our model. Therefore the organizational form is chosen to maximize the principal's payoff in our paper because she has a right to select a organizational form.

Four remarkable papers about the studies of the organizational form are Rajan and Zingales (2001), Demange (2005), Hart and Moore (2005) and Choe and Ishiguro (2005). Rajan and Zingales (2001) attempted a comparison between a vertical hierarchy and a horizontal hierarchy, focusing on the effects of specialization and competition. Competition in their model means the coalitional deviation and they showed that flat hierarchies promote the level of their human capital investments in the organization. Demange (2005) has been investigated organizational structures from the viewpoint of group stability and shown that the hierarchical structure achieves efficient coordination and is not blocked by any subgroup consisting of a superior and his or her subordinates. However she adopted a cooperative solution concept like core and the incentive problem of the human capital investments is absent.

This paper applies the concept of coalitional deviations according to Rajan and Zingales (2001) and Demange (2005) to a noncooperative bargaining game for allocating the return of the organization. The possibility of coalitional deviations depends on the organizational form.

Hart and Moore (2005) and Choe and Ishiguro (2005) are related papers to the allocation of authority in the organization. Hart and Moore (2005) regards the design of hierarchies as determination of the decision-making authority and suppose that a hierarchy of authority over decisions can be contractual specified ex ante. They explain why coordinator should be senior to specialist and why pyramidal hierarchies may be optimal. The delegation of authority is absent in our model, focusing instead on the relationship between organizational forms and incentives for human capital investment. Choe and Ishiguro (2005) considered an organization that consists of a principal and

order with each ordering being equally likely and each player makes a take-it-or-leave-it offer in the order (in Hart and Moore (1988)), or the ability of renegotiations is ensured for all players equally (in Stole and Zwiebel (1996)).

two agents and has two project. They compare three type of organizational structures; centralization where the principal have all decision-making authority, decentralization where the principal delegate the authority to each agent and the (vertical) hierarchy where the principal decides the project of the direct subordinate and the subordinate has the authority to the project of his subordinate. They showed that the optimal authority structure depends on externalities (or coordination benefit) between two projects and the incentive in human capital. The organization of three players is similar to our model and they also consider who should be at middle tier in the hierarchy when two agents are asymmetric in their ability. But the ex post bargaining is supposed to be bilaterally and applies a symmetric Nash bargaining solution.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 studies the bargaining procedure in each organization. Section 4 examines the incentive problem and the choice of organizational forms.Section 5 concludes. Appendix includes all proofs of the theorems and the propositions.

2 The Model

We shall consider a organization consisting of three risk-neutral players. The set of players is denoted by $N = \{1, 2, 3\}$, and a coalition S of players is the subset of N. There is an essential asset $\{a\}$ for production. We assume that player 1 owns the asset. In other words, player 1 has an ownership of the organization. The model is composed of three periods, date 0, date 1 and date 2.

At date 0, an organizational form is selected by player 1. At date 1, each player $i \in N$ chooses the level of human capital investment $e_i \in \{0, 1\}$. The human capital investment e_i affects the player's productivity or value at date 2. At date 2, players negotiate over the allocations of the return and production occurs.

We follow the incomplete contracting approach of Hart and Moore (1990). We suppose that the production and the allocation of the return at date 2 cannot be included in a date 0 contract because of the complication of investment and the transaction costs. Hence the initial contract specifies the organizational form only. The level of human capital investment is chosen noncooperatively by the players at date 1.

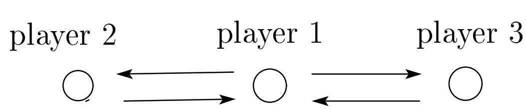
2.1 Organzational Forms

We study four kinds of organizational form; (i) horizontal organization, (ii) common agency, (iii) pyramidal hierarcy and (iv) vertical hierarchy. There are three tiers in the organization. It is assumed that player 1, who is the owner of the firm, belongs to the first tier. At date 0, player 1 arranges the assignment of tier to player 2 and player 3. Each organizational form is characterized by the tier-assignment to players. Let us introduce the tier-assignment fuction $t : N \to \{1, 2, 3\}$. That is, $t(i) = k$ means that player i belongs to tier k. By assumption, $t(1) = 1$. We say that a player in tier k is subordinate to a player in tier $k-1$ and a player in tier $k-1$ is superior to a player in tier k. Thus, a number of tier represents the rank in the organization. The triplet $(t(1), t(2), t(3))$ detemines a form of organization uniquely. We assume that if $t(i) = k\ (\geq 2)$ for some $i \in N$, then, for any tier $m < k-1$, there exists $j \in N$ such that $t(j) = m$. This implies that every player except for that in tier 1 has a direct superior in the organization.

(i) Horizontal organization

We call the *horizontal organization* as $(t(1), t(2), t(3)) = (1, 1, 1)$. All players belong to tier 1 and are on the level with each other. In this organization, all players have a same bargaining power in the bargaining stage of payoff allocation. Figure 1 represents the horizontal organization of $(t(1), t(2), t(3)) = (1, 1, 1)$. Each circle indicates the player in the organization. The symbol '$\leftrightarrows$' between two players represents that both players are in the same tier and they are on the equal footing. We write $\{i \leftrightarrows j\}$ as player i and j belong to the same tier.

(Figure 1)



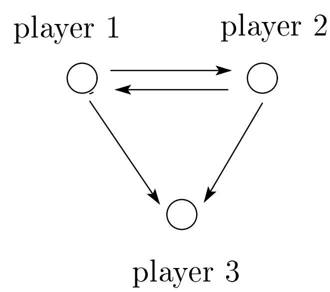
The horizontal organization of $(t(1), t(2), t(3)) = (1, 1, 1)$ is denoted by g^1.

(ii) Common agency

We call the organizational forms of $(t(1), t(2), t(3)) = (1, 1, 2)$ and of $(t(1), t(2), t(3)) = (1, 2, 1)$ the *common agency*. Tier 1 consists of two players and tier 2 contains the rest of players. A player in tier 2 becomes a common subordinate to two players in tier 1. The common agency $(t(1), t(2), t(3)) = (1, 1, 2)$ is shown in Figure 2. The symbol '$\to$' between two players represents the relationship between the superior and the subordinate. If player

$i \to$ player j in the Figure, it is seen that player i is superior to player j. Player 1 and 2 have a same bargaining position in negotiations for the payoff allocation, but player 3 is in the weaker bargaining position than player 1 and 2.

(Figure 2)

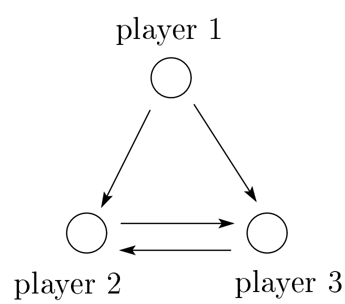


We denote the organizational form of $(t(1), t(2), t(3)) = (1, 1, 2)$ (or that of $(t(1), t(2), t(3)) = (1, 2, 1)$) by g^2 (or g^3) respectively.

(iii) Pyramidal hierarchy

The *pyramidal hierarchy* is expressed by $(t(1), t(2), t(3)) = (1, 2, 2)$. Player 1 is in the top tier, and player 2 and 3 belong to the second tier. In this organization, player 1 is a direct superior to player 2 and 3, and player 2 and player 3 are in the same position and have no subordinate. Figure 3 represents the pyramidal hierarchy of $(t(1), t(2), t(3)) = (1, 2, 2)$.

(Figure 3)



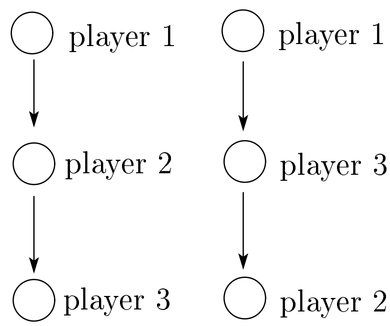
The pyramidal hierarchy is denoted by g^4.

(iv) Vertical hierarchy

The organizational forms such as $(t(1), t(2), t(3)) = (1, 2, 3)$ and $(t(1), t(2), t(3)) = (1, 3, 2)$ are called the *vertical hierarchy*. In the vertical hierarchy, all players are totally ordered. Alternatively, we denote the vertical hierarchy of $(t(1), t(2), t(3)) = (1, j, k)$ by $\{1 \to j \to k\}$. In the organizational form

$\{1 \to j \to k\}$, player 1 is in the top tier (tier 1), player j in the second tier (tier 2) and player k in the bottom tier (tier 3). Player 1 is a direct superior to player j, and player j is a direct superior to player k. Figure 4 represents the vertical hierarchies of $(t(1), t(2), t(3)) = (1, 2, 3)$ and $(t(1), t(2), t(3)) = (1, 3, 2)$.

(Figure 4)



We denote $(t(1), t(2), t(3)) = (1, 2, 3)$ by g^5 and $(t(1), t(2), t(3)) = (1, 3, 2)$ by g^6.

All organizational structures are contained above such that player 1 is in tier 1 and player 2 and 3 are assigned to either tier 1, 2, or 3. Denote by G^N the set of the organizational structures with all players. Thus, $G^N = \{g_1, g_2, \ldots, g_6\}$. We define that, for player i and player j, the set $G^{\{i,j\}}$ consists of $\{i \to j\}$, $\{j \to i\}$, and $\{i \leftrightarrows j\}$. In addition, $G^{\{i\}} = \{\emptyset\}$. The generic element in G^S, $S \subseteq N$, is denoted by g^S.

The rule of the bargaining game for allocating the return would change with the organizational forms. Furthermore, coalitional deviations in the bargaining game is restricted differently by the organizational form. We suppose that a player in the higher tier has a strong bargaining power and that players in the same tier occupy a same position in negotiations over the return. The difference of the organizational form can be interpreted as the difference of the distribution of the bargaining power among the players in the organization. The distribution of bargainig power affects incentives of human capital investment by players.

2.2 Human Capital Investment

Each player chooses the level of human capital investment $e_i \in \{0, 1\}$ at date 1. The cost of investment e_i to player i is represented by e_i itself. Each player has a binary choice of investment. If $e_i = 1$, player i makes an investment in human capital. If $e_i = 0$, i has no investment. Let us denote the triplet of investments of the players by $e = (e_1, e_2, e_3)$. The levels of e_1, e_2 and e_3 are observed by all players at the end of date 1.

2.3 The Bargaining Situations

At date 2, production and the return would be realized. Before production are actually conducted, the bargaining over the return takes place. We allow the possibility of production by subcoalitions of N at date 2. Coalition $\{i, j\}$ of two players i, j and a singleton-coalition $\{i\}$ of player i could realize some returns, although the feasible subcoalitions are limited by the organizational form. The possibility of production by subcoalitions affects the determination of the payoff allocation through negotiations. We assume that the selected organizational form and the level of human captial investment are observable among all players. The returns that coalitions of the players can achieve are commonly known by all players. Thus, a multilateral bargaining process is conducted under complete and symmetric information. We take a noncooperative game approach to this bargaining problem, while Hart and Moore (1990) have adopted the Shapley value in a cooperative game.

The return of the organization may generally depends on the member S, its structure g^S and the level of investment by the member, $e^S = (e_i)_{i \in S}$. It is denoted by $v(g^S, S|(e_i)_{i \in S})$ for $S \subseteq N$.

However, if the returns of the organization at date 2 are different across organizational forms, the choice of organizational form is affected by the scale of expected return. In order to focus on the relationship between the incentive of human capital investment and the choice of organizational structure, we assume that the revenue is same in all types of organizational structure if they contain same members and the same level of investments.

Assumption 1. The value of $v(g^S, S|e^S)$ does not depend on $g^S \in G^S$.

Let us define that $v(S|e^S) = v(g^S, S|e^S)$.

Assumption 2. For any $e = (e_i, e_j, e_k)$, the function v satisfies the following conditions:

$$v(N \mid (e_i, e_j, e_k)) \geq v(\{i, j\}|(e_i, e_j)) + v(\{k\}|e_k), \text{ and}$$
$$v(\{i, j\}|(e_i, e_j)) \geq v(\{i\}|e_i) + v(\{j\}|e_j), \text{ for } i, j, k = 1, 2, 3,$$

The above condition is called *superadditivity* of v.

Superadditivity means that if a coalition divides to partitions, the return by the coalition is greater than or equal to the aggregate return that the coalitions can generate by producing separately. In other words, partitions of a coalition are able to do at least as well acting together as they could do acting separately. The superadditivity assumption of the return was also made in Hart and Moore (1990).

Assumption 3. For all $S \subseteq N$, $v(S \mid (e_i)_{i \in S})$ is increasing in e_i.

Assumption 3 says that a human capital investment by a member of the coalition enhances the return of the coalition. This means that each human capital investment is beneficial to the members of the organization.

We make the following assumption about the marginal return on investments.

Assumption 4. For every $i, j = 1, 2, 3$, $i \neq j$, it is satisfied that

$$\begin{aligned} v(N|(1, e^{N\setminus\{i\}})) - v(N|(0, e^{N\setminus\{i\}})) \\ \geq v(\{i,j\}|(1, e_j)) - v(\{i,j\}|(0, e_j)) \\ \geq v(\{i\}|1) - v(\{i\}|0). \end{aligned}$$

Assumption 4 represents the increasing return to scale of investment. The marginal return on investment increases with the size of coalition.

The net return for each coalition $N = \{1, 2, 3\}$, $\{i, j\}$ and $\{i\}$, where $i, j = 1, 2, 3$ and $i \neq j$, is defined by

$$\begin{aligned} f(N|(e_1, e_2, e_3)) &= v(N|(e_1, e_2, e_3)) - \sum_{i=1}^{3} e_i, \\ f(\{i,j\}|(e_i, e_j)) &= v(\{i,j\}|(e_i, e_j)) - (e_i + e_j), \\ f(\{i\}|e_i) &= v(\{i\}|e_i) - e_i. \end{aligned}$$

We make the following assumption about the net return.

Assumption 5. The maximum of the net return for coalition N is attained at $(e_1, e_2, e_3) = (1, 1, 1)$. In other words, $f(N \mid (1, 1, 1)) \geq f(N \mid (e_1, e_2, e_3))$ for all $(e_1, e_2, e_3) \in \{0, 1\} \times \{0, 1\} \times \{0, 1\}$. Moreover, $f(N \mid (1, 1, 1)) > 0$.

Together with Assumption 2 and Assumption 5, it follows that the social surplus is maximized when all players make their human capital investments under a grand coalition N. We shall maintain all five assumptions throughout the paper.

2.4 Noncooperative Bargaining Games

The procedure of bargaining over the return at date 2 depends on the organizational form which is selected at date 0. The opportunity to make a proposal for an allocation of the return and the possibility of coalitional deviations are different in each organizational form. Basically, a player in the higher tier has a stronger bargaining power in allocating the return than one in the lower tier in the organizational form.

We assume that the order of proposal is determined by the rank in the hierarchy. A player in the higher tier is able to makes a proposal more precedently than a player in the lower tier. These bargaining procedures represent that player i is in a stronger position in negotiations than player j. According to a principal-agent model, a superior makes a take-it-or-leave-it offer to her subordinates. Thus, if player i is superior to player j, i.e., $i \to j$, player i makes a take-it-or-leave-it offer to player j. Since there is no informational asymmetry between players in our model, player i extracts all the net surplus from player j.

On the other hand, players in the same tier has a same bargaining power in negotiations over the return. In order to describe such a situation, we assume that players in the same tier have the same opportunity to propose a coalition and an allocation of the return. If player i and j are in tier 1; $i \leftrightarrows j$, one player between player i and j is selected as a proposer with equal probability of 1/2 in the bargaining procedure. If either player rejects the proposal, negotiations goes to the next round with a new randomly selected proposer, and the same process is repeated. There is no first-mover advantage among players in the same tier.

The possibility of coalitional deviations in the bargaining game also relies on the selected organizational form. A coalitional deviation means that a coalition becomes independent of the existing organization and is competing with the rest of the members of the organization in markets. It is assumed that a coalition consisting of players only with the relation '$\to$' can deviates from the organization in the bargaining. In the coalitional deviation, if a superior decides to deviate, his subordinates have no choice but to follow him. This assumption is consistent with the competing team in Rajan and Zingales (2001). If a manager in tier k decides to compete in the n-tier vertical hierarchy of Rajan and Zingales, $n-k$ subordinates follow the manager and produce together as a team. Demange (2005) also considered coalitional deviations to examine the stability of a hierarchical structure. In Demange's model, a 'team' is considered as a unit of deviations, and a coalition T is a team if and only if, for every i and j in T, either i is superior to j, j is superior to i, or a common superior exists in T to both j and i, and, in addition, all players between the tier with i and tier with j belong to T. Coalitional deviations in our model is consistent with the concept of blocking by teams in Demange (2005). However, we will require more strong stablity and allow more coalitional deviations than in Demange. In the two-tier pyramidal hierarchy, only a grand coalition N and singletons can deviate from the organization in Demange's setting. On the other hand, we allow deviations by the coalitions consisting of player 1 and one of his subordinates.

In addition, we do not allow coalitional deviations by players with the realtion '$\leftrightarrows$'; that is, we do not consider the possibility of collusion by players in the same tier. Demange (2005) also excluded coalitional deviations by players in the same tier because she only allow the block by 'team' and the team does not consist of players only in the same tier.

(i) Bargaining procedure in the horizontal organization

A noncooperative bargaining game in the horizontal organization at date 2 runs as follows. At every round $t = 1, 2, \ldots$, one player is selected as a proposer with equal probability among all players. The selected player i proposes either (a) a coalition N and an allocation of the return $v(N \mid e)$ for the memebers of N, or (b) a singleton coalition $\{i\}$. In the latter case, the game terminates and the vector of the return (v_1, v_2, v_3) of $(v(\{1\}|e_1), v(\{2\}|e_2), v(\{3\}|e_3))$ is realized, i.e., player i gets the share $v(\{i\}|e_i)$. In the former case, all other players in N either accept or reject the proposal sequentially. If all other players in N accept the proposal, then the agreed division of the return is enforced and game ends. If some players reject the proposal, the bargaining goes on to the next round and a new proposer is randomly selected by the same rule.

(ii) Bargaining procedure in the common agency

Let us explain a noncooperative bargaining game in the common agency. Focus on the organization g_2 in which player 1 and 2 is belonging to the top tier and player 3 is in the second tier. In the case of the common agency g_3, it is enough to interchange player 2 with player 3 at the following bargaining game.

At every round $t = 1, 2, \ldots$, one player is selected as a proposer with equal probability among players in the first tier. The selected player $i \in \{1, 2\}$ prososes either (a) a coalition N and a division of the return $v(N|(e_1, e_2, e_3))$, (b) a coalition $S = \{i, 3\}$ and a division of $v(\{i, 3\}|(e_i, e_3))$ for player i and player 3, or (c) a singleton coalition $\{i\}$. First, consider the case of (a). If all the other players in N accept the proposal, it is agreed upon and enforced. The game ends. If some players reject the proposal, negotiations continue to the next round and a new proposer is randomly selected among players in the first tier. Next, let us consider the case of (b). Player i makes a take-it-or-leave-it offer of a division of the return $v(\{i,3\}|(e_1,e_3))$ against player 3. If player 3 rejects the offer, the game ends with the allocation of the return $(v_1,v_2,v_3)=(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$. If player 3 accepts the offer, it is enforced and the game terminates. Finally, in the case of (c), the game ends with the vector of the returns $(v_1,v_2,v_3)=(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$.

(iii) Bargaining procedure in the pyramidal hierarchy

A bargaining procedure in the pyramidal hierarchy is carried out as follows. Player 1 is a proposer certaintly. Player 1 proposes either (a) a coalition N and a division of the return $v(N|(e_1,e_2,e_3))$, (b) a coalition $S=\{1,j\}$, $j=2,3$, and a division of $v(\{1,j\}|(e_1,e_j))$ between player 1 and player j, or (c) a singleton coalition $\{1\}$. In the case of (a), player 1 makes a take-it-or-leave-it offer for player 2 and player 3. If either player rejects the proposal, negotiations break down and the vector of the return $(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$ is realized. If both player 2 and 3 accept the offer, it is agreed upon and enforced. In the case of (b), player 1 makes a take-it-or-leave-it offer for player j. If player j accepts the proposal, it is enforced and the game ends. If player j rejects the proposal, the allocation of the return at a breakdown of negotiations of $(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$ is realized. When the proposal (c) is made, the game ends with the allocation of the return $(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$.

(iv) Bargaining procedure in the vertical hierarchy

A noncooperative bargaining game in the vertical hierarchy at date 2 runs as follows. Player 1 firstly proposes either (a) a coalition N and an allocation of the return $v(N|(e_1,e_2,e_3))$, (b) a singleton coalition $\{1\}$. In the case of (b), the return allocation $(v_1,v_2,v_3)=(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$ is realized. Note that in the case of (a), player 1 makes a take-it-or-leave-it offer to player 2 about the total of player 2 and 3's shares. If player 2 reject the proposal, then, player 2 chooses (i)a coalition $\{2,3\}$ or (ii)a singlton coalition $\{2\}$. In the former case of (i), player 2 makes a take-it-or-leave-it offer to player 3 about the division of $v(\{2,3\}|(e_2,e_3))$ and, then, player 3 accept or reject the proposal. If player 3 accepts the offer, it is enforced. If player 3 rejects the offer, negotiations break down and the vector of returns is reduced to $(v_1,v_2,v_3)=(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$. In the latter case of (ii), the return allocation $(v_1,v_2,v_3)=(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$ is realized and the game ends. If player 2 accepts the offer, then player 2 makes a take-it-or-leave-it offer to player 3 about a division of the total share proposed by player 1. If player 3 rejects the proposal, an allocation of the return becomes $(v(\{1\}|e_1),v(\{2\}|e_2),v(\{3\}|e_3))$. If player 3 accepts the proposal, it is agreed upon and enforced.

Each share of the return for the player is given as follows. When an allocation $(v_i)_{i \in N}$ of the return is agreed upon at round t, the return of player i is $\delta^{t-1} v_i$, where δ is a discount factor, satisfying $0 \leq \delta < 1$.

We shall apply a stationary subgame perfect equilibrium (SSPE) as a solution concept to the noncooperative bargaining games at date 2. An SSPE is a subgame perfect equilibrium with property that for every $t = 1, 2, \ldots$, the tth round strategy of every player depends only on the set of all active players at round t. It is well-known that in a noncooperative multilateral (more than three players) bargaining game, there are multiple subgame perfect equilibria when the discount factor is close to one. By this reason, the concept of an SSPE is adopted in almost all literature of noncooperative multilateral barganing model (Chatterjee et al., 1993, Gul, 1989, Okada, 1996, Ray and Vohra, 1999). Note that the stationarity of the solution concept applies to only the noncooperative bargaining games in the horizontal organization and the common agency and is irrelevant to the noncooperative bargaining games in the pyramidal and vertical hierarchy.

In this paper, we focus on the limit point of SSPE of each bargaining game as δ goes to 1.

3 Equilibrium Strategies

Let us characterize the equilibrium strategies at each date. The solution concept that we apply to the whole game consisting of date 0, date 1 and date 3 is a subgame perfect equilibrium (SPE). We restrict to SPEs satisfying the stationarity in the bargaining game at date 2. The equiilbrium strategies of the whole game can be obtained by the usual backward induction procedure in the theory of extensive games. All proofs are gathered in Appendix.

3.1 Bargaining Outcomes at Date 2

First, let us consider noncooperative bargaining games at date 2.

When the horizontal organization is selected at date 0 and the level of investment for all players at date 1 is given by $e = (e_1, e_2, e_3)$, the equilibrium strategies in a bargaining game at date 2 are characterized by Theorem 1 and 2.

Theorem 1. *If a discount factor δ is close to one and the following condition* *is satisfied:*

$$\begin{aligned} &v(N|e)/3 \geq v(\{1\}|e_1), \textit{and} \\ &v(N|e)/3 \geq v(\{2\}|e_2), \textit{and} \\ &v(N|e)/3 \geq v(\{3\}|e_3), \end{aligned} \tag{1}$$

then there exists an SSPE of the bargaining game in the horizontal organization.

In the SSPE, every player $i = 1, 2, 3$ *proposes a coalition* N *and an allocation of the return* $(v_1, v_2, v_3) = (v(N|e)/3, v(N|e)/3, v(N|e)/3))$ *at round 1. Moreover, the proposal is accepted in the SSPE.*

Notice that the return is divided to all players equally in the horizontal organization, which is independent of the contribution of each investments. We shall compare this allocation with the Shapley value in the end of this section.

Theorem 2. *If a discount factor* δ *is close to one and the condition (1) does not be satisfied, then there is no SSPE of the bargaining game in the horizontal organization.*

In the common agency, we obtain the following three theorems.

Theorem 3. *If a discount factor* δ *is close to one and the following condition is satisfied:*

$$\begin{aligned} &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{1,3\}|(e_1, e_3)) - v(\{3\}|e_3), \ \textit{and} \\ &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{2,3\}|(e_2, e_3)) - v(\{3\}|e_3), \end{aligned} \tag{2}$$

then there exists an SSPE of the bargaining game in the common agency.

In the SSPE, a proposer $i = 1, 2$ *offers a coalition* N *and an allocation of the return* $(v_1, v_2, v_3) = ((v(N|e) - v(\{3\}|e_3))/2, (v(N|e) - v(\{3\}|e_3))/2, v(\{3\}|e_3))$ *at round 1. Moreover, the proposal is accepted in the SSPE.*

Theorem 4. *If a discount factor* δ *is close to one and the following condition is satisfied:*

$$\begin{aligned} &v(\{1,3\}|(e_1, e_3)) \geq \\ &\qquad v(N|e) - \frac{1}{2}v(\{2,3\}|(e_2, e_3)) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3), \ \textit{and} \\ &v(\{2,3\}|(e_2, e_3)) \geq \\ &\qquad v(N|e) - \frac{1}{2}v(\{1,3\}|(e_2, e_3)) - \frac{1}{2}v(\{1\}|e_1) - \frac{1}{2}v(\{3\}|e_3), \end{aligned} \tag{3}$$

then there exists an SSPE of the bargaining game in the common agency.

In the SSPE, player 1 offers as a proposer a coalition $\{1,3\}$ and the vector of the return $(v_1, v_3) = (v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$ at round 1, and player 2 offers as a proposer at round 1 a coalition $\{2,3\}$ and the vector of the return $(v_2, v_3) = (v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$. Moreover, the proposals are accepted in the SSPE.

Note that the expected equilibrium share of return in the above SSPE (Theorem 4) is given by

$$\begin{aligned} v_1^* &= \frac{1}{2}\left(v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{1\}|e_1), \\ v_2^* &= \frac{1}{2}\left(v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{2\}|e_2), \\ v_3^* &= v(\{3\}|e_3). \end{aligned}$$

The following theorem shows non-existence of an SSPE in the common agency.

Theorem 5. *If a discount factor δ is close to one and the condition (2) and (3) are not satisfied, then there is no SSPE of the bargaining game in the common agency.*

Next, let us consider the pyramidal hierarchy. In this case, there always exists a subgame perfect equilibrium because the bargaining game is a finite-length extensive form game. The stationarity of the equilibrium strategy is irrelevant here. Moreover, a strategy of each player in the subgame perfect equilibrium is uniquely determined for any δ when Assumption 1 - 5 are satisfied.

Theorem 6. *There exists a subgame perfect equilibrium of the bargaining game in the pyramidal hierarchy. In the SPE, player 1 proposes a coalition N and an allocation of the returns of $(v_1, v_2, v_3) = (v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3), v(\{2\}|e_2), v(\{3\}|e_3))$. Moreover, player 2 and player 3 are accept the proposal in the SPE.*

Finally, consider the bargaining game in the vertical hierarchy. We can obtain an unique subgame perfect equilibrium of the bargaining game in the vertical hierarchy.

Theorem 7. *There exists a subgame perfect equilibrium of the bargaining game in the vertical hierarchy g_5. In the SPE, player 1 proposes to player 2 a share in the return of $v(\{2,3\}|(e_2,e_3))$ for player 2 and 3 , and player 2 accepts the proposal and, then proposes to player 3 a division of the share such as $(v_2, v_3) = (v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$. Player 3 accepts the proposal by player 2 in the SPE.*

In the above SPE, the expected return of each player is given by

$$\begin{aligned} v_1^* &= v(N|e) - v(\{2,3\}|(e_2,e_3)), \\ v_2^* &= v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), \\ v_3^* &= v(\{3\}|e_3). \end{aligned}$$

It is easy to prove the same theorem in the case of the vertical hierarchy g_6 by replacing player 2 with player 3. In the SPE of the bargaining game in g_6, player 2 gets $v(\{2\}|e_2)$ and player 3 does $v(\{2,3\}|(e_2,e_3)) - v(\{2\}|e_2)$.

Remark. (Comparisons to the Shapley value) Hart and Moore (1990) take a cooperative game approach to the bargaining problem of a return allocation, adopting the Shapley value as a solution concept. In our bargaining problem for three players, the Shapley value of each player is given by

$$\begin{aligned} B_1(e) =& \frac{1}{3}(v(N|e) - v(\{2,3\}|(e_2,e_3))) + \frac{1}{6}(v(\{1,2\}|(e_1,e_2)) - v(\{2\}|e_2)) \\ &+ \frac{1}{6}(v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3)) + \frac{1}{3}v(\{1\}|e_1), \\ B_2(e) =& \frac{1}{3}(v(N|e) - v(\{1,3\}|(e_1,e_3))) + \frac{1}{6}(v(\{1,2\}|(e_1,e_2)) - v(\{1\}|e_1)) \\ &+ \frac{1}{6}(v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)) + \frac{1}{3}v(\{2\}|e_2), \\ B_3(e) =& \frac{1}{3}(v(N|e) - v(\{1,2\}|(e_1,e_2))) + \frac{1}{6}(v(\{1,3\}|(e_1,e_3)) - v(\{1\}|e_1)) \\ &+ \frac{1}{6}(v(\{2,3\}|(e_2,e_3)) - v(\{2\}|e_2)) + \frac{1}{3}v(\{3\}|e_3). \end{aligned}$$

In our noncooperative bargaining game under the common agency, the pyramidal hierarchy and the vertical hierarchy, a player k in the bottom tier gain only the stand-alone return $v(\{k\}|e_k)$ in the equilibrium. Therefore, each equilibrium return allocation in the organizational form excepting the horizantal hierarchy is always different from the Shapley value. If three players are perfectly symmetric in the contributions to the return; $v(\{1,2\}|(e_1,e_2)) = v(\{1,3\}|(e_1,e_3)) = v(\{2,3\}|(e_2,e_3))$ and $v(\{1\}|e_1) = v(\{2\}|e_2) = v(\{3\}|e_3)$, then, the Shapley value is reduced to the vector $(v(N|e)/3, v(N|e)/3, v(N|e)/3)$. Thus, only if all players are perfectly symmetric and identical, the Shapley value coincides with the equilibrium return vector of a noncooperative bargaining game in the horizontal organization. However, both allocations are divergent in general.

3.2 Decision of Human Capital Investments

At date 1, each players decide whether to invest or not, maximizing his/her expected payoff. The expected SSPE return for player i ($i = 1, 2, 3$) in date 2 is denoted by $v_i^*(e_1, e_2, e_3; g_j)$, which is determined according to the bargaining procedure at date 2 in the organizational structure g_j. We denote by e_{-i} the combination of the human capital investments for all players except player i.

Definition 1. Give organizational structure g_j, the vector of $e^* = (e_1^*, e_2^*, e_3^*)$ is an *equilibrium pair of investments at date 1* if it satisfies, for all $i = 1, 2, 3$,

$$v_i^*(e_i^*, e_{-i}^*; g_j) - e_i^* \geq v_i^*(e_i, e_{-i}^*; g_j) - e_i \text{ for all } e_i \in \{0, 1\}$$

According to the equilibrium strategies of investments at date 1 and of the bargaining game at date 2, player 1 selects an organizational form g_j so as to maximize her payoff.

4 Results on the Organizational Structure

In this section, we show what kind of organization is chosen in the relation to the human capital investments of the players. All proofs of the propositions in this section are gathered in Appendix.

Definition 2. The human capital investments e_1, e_2, e_3 are *perfectly complementary* if it satisfies the following conditions: For all $e = (e_1, e_2, e_3)$ containing $e_i = 0$,

$$v(N|e) = 0, \tag{4}$$

and, for all $S \subset N$ such that $S \neq N$ and for all $(e_i)_{i \in S}$,

$$v(S|(e_i)_{i \in S}) = 0. \tag{5}$$

Perfectly complementary investemnts imply that three human capitals generate no value if they do not use together. Condition (4) means that no return occurs at date 2 if some player does not make a human capital investment, and condition (5) says that even if (sub-)coalitions are formed, there would be no return in the coalition at date 2.

The following proposition characterize the situations such that the horizontal form is selected.

Proposition 1. *If human capital investments* e_1, e_2, e_3 *are perfect complementary, the horizontal form is chosen in equilibrium.*

It is optimal for player 1 to choose the horizontal form (partnership) because the return to player 2 and 3 are not enough to invest in the other organizational forms and it is essential to induce player 2 and player 3 to invest when the human capital investments are perfectly complementary. The similar result has been obtained in Hart and Moore (1990). Hart and Moore showed that two assets should be owned or controlled together if they are unproductive unless they are used together. Proposition 1 says that player 1 should give player 2 and 3 equal authorities to themselves. The hierarchy structures which have a boss and subordinates are not optimal when investments of all members are complement and essential for the firm.

Next we shall consider the common agency. Many papers pointed out that the organization with two boss (two principals) is not desirable from the point of view such as the information process (Radner, 1993, Bolton and Dewatripont, 2004), the authority delegation under incomplete contracts (Hart and Moore, 2005) and the group stability (Demange, 2005).

The common agency may, however, be optimal if the relationship is considered between incentives of the human capital investment and the distribution of bargaining power in the organization. We next show in what conditions the common agency is selected in equilibrium.

Definition 3. A human capital investment e_i is *marketable* if player i has an incentive to make the human capital investment independently; $v(\{i\}|1)-1 \geq v(\{i\}|0) - 0$.

This condition means that the human capital is valuable in the market by itself.When the investment is general, it is just as valuable with an alternative firm . Therefore this condition is likely satisfied for the general investments.

Proposition 2. *There exists an equilibrium in which the common agency is chosen and the efficient level of the human capital investment* $e^* = (1, 1, 1)$ *is implemented.*

In the proof of Proposition 2, we give an example that the human capital investments e_1 and e_2 of player 1 and 2 are perfect complementary and the human capital investment of player 3 e_3 is marketable. By adding some conditions to the example, we can present the case such that common agency can implement all players to invest and player 1 selects the common agency in equilibrium.

For example, player 1 concentrates on technical improvements and player 2 concentrates on management for the firm. Then both of the human capital investments are needed to the firm. In order to provide incentives to investment for player 2, it is optimal for player 1 to give equal authority to player

2 and built a strong partnership. Honda, Google and Yahoo! and so on are the successful examples of common agency.

In our model, a subcoalition(subgroup) can be formed as in Theorem 4 if player 1 chooses the common agency at date 0. Next proposition, however, says that the organization of subcoalition is dominated by other organizational forms.

Proposition 3. *A subcoalition cannot be formed in equilibrium.*

Proposition 3 shows that the division of firm or the boundary of firm does not matter in the game with supereadditivity and no externality. Player 1 cannot acquire the benefit from player 2 at all, forming a subcoalition $\{1,3\}$. Since the return is assumed to be superadditive and to be a increasing return to scale, player 1 can get a larger payoff in the pyramidal hierarchy than in a subcoalition {1,3} even if player 2 does not invest.

Next, let us consider the tier-assingment problem of player 2 and player 3. In what follows, we assume that player 1 has already acquired a human capital ($e_1 = 1$) and omit the incentive problem for player 1.

Proposition 4. *If player 2 and 3 invest to human capital in the pyramidal hierarchy, player 1 chooses the pyramidal hierarchy at date 0.*

Player 1 can acquire all of the surplus in the pyramidal hierarchy. On the other hand, the player in the middle tier has some bargaining power in the vertical hierarchy. The common agency gives player 2 the same bargaining power as player 1 and the horizontal organization gives the equal bargaining power to all of members. Therefore, the optimal organizational form for player 1 is the pyramidal hierarchy if it can implement player 2 and player 3 to invest.

Let us consider the optimal (tier-)assignment in the vertical hierarchy. If player 2 and player 3 are asymmetric, which player should be assigned to the middle tier? This problem depends on the marketability of their investments and the firm-specificity of their investments.

When both of investments are marketable, Proposition 4 implies that the vertical hierarchy is dominated by the pyramidal hierarchy from the viewpoint of player 1. Therefore, we will consider two cases in which one of investments is marketable and in which both of investments are not marketable.

Suppose that the investment of player 2 is not marketable, but that of player 3 is marketable;

$$v(\{2\}|1) - v(\{2\}|0) < 1 \tag{6}$$

$$v(\{3\}|1) - v(\{3\}|0) \geq 1 \tag{7}$$

Proposition 5. *The organization g_5 dominates the organization g_6 under conditions (6) and (7).*

Because of the incompleteness of contracts, the return from the investments of the players depends on the marketability of their human capital investments and the distribution of bargaining power. If the investment is sufficient general and valuable in the market, the player has incentive to invest voluntarily. On the other hand, if the investment is specific, the hold-up problem arises and player 1 can mitigate this problem to give player 2 a better bargaining position. The hierarchical structure in which a player with marketable investment is posted to the bottom tier and a player with specific investment is posted to the middle tier dominates the hierarchical structure in which the assignment of players is in the reverse order. For example, the investments of computer programmers are highly marketable and, then, they are usually assigned to the bottom rank.

Definition 4. A human capital investment e_i is *firm-specific* with repect to player 1 if it is not marketable and the following condition

$$\begin{aligned} &v(\{1,i,j\}|(1,1,e_j)) - v(\{1,i,j\}|(1,0,e_j)) \\ &\qquad > v(\{i,j\}|(1,e_j)) - v(\{i,j\}|(0,e_j)) \end{aligned} \tag{8}$$

is satisfied.

Conidtion (8) is same with assumption 4 except it contains an equall sign. Conidtion (8) implies that player i's marginal invest return is higher in the grand coalition which contains player 1 than in the subcoalition which does not contain player 1. The larger difference between the left-hand side and the right-hand side of (8) means that e_i is more valuable with player 1. We call the raito of the left hand to right hand $\Delta_i(e_j)$ as a degree of firm-specific.

$$\Delta_i(e_j) = \frac{v(\{1,i,j\}|(1,1,e_j)) - v(\{1,i,j\}|(1,0,e_j))}{v(\{i,j\}|(1,e_j)) - v(\{i,j\}|(0,e_j))}$$

Large $\Delta_i(e_j)$ implies that the investment of player i is more specific in a marginal sense.

Proposition 6. *If investments of player 2 and player 3 are not marketable and the investment of player 2 contributes more to the firm's return than that of player 3 in the following sense:*

$$v(N|(1,1,0)) \geq v(N|(1,0,1)), \tag{9}$$
$$v(\{2,3\}|(1,0)) = v(\{2,3\}|(0,1)), \tag{10}$$

then, the organization g_5 dominates the organization g_6.

If both of investments are not marketable, then a player whose investment contributes more important to the firm's value should be posted to the upper tier than the other player whose investment has less important contribution.

Using the degree of firm-specific, conditions (9) and (10) imply $\Delta_2(0) \geq \Delta_3(0)$. Proposition 6 suggests that if the human capital investment of player 2 is higher degree of firm-specific than that of player 3, player 2 should be assigned to the middle tier.

Proposition 5 and Proposition 6 clarify which agents the owner should assign to the middle tier in the hierarchical organization if the agents are asymmetric. Che and Ishiguro (2006) also approached this problem. Che and Ishiguro showed that, if two agent has same cost function, the agent who has the higher success probabiltiy of project and has the higher marginal probability on his human capital investment should be in the middle tier because the agent can be better motivated through empowerment. Their result is similar to Proposition 6, although the tier-assignment in our model depends not only on the revenue of the firm but also the market value of each human capital investment.

Next we compare the vertical hierarchy g_5 with the pyramidal hierarchy g_4. Propositon 4 says that if both investments of player 2 and 3 are marketable, player 1 prefers the pyramidal hierarchy to the vertical hierarchy. Proposition 5 implies that if one of the investment is marketable, a player with marketable investment should be assigned to the bottom tier. Hence, it is sufficient to compare the pyramidal hierarchy with the vertival hirarchy where the investment of player 2 who is assigned to middle player is not marketable.

Proposition 7 is concerned with the case where player 2 does not invest in the pyramidal hierarchy but in the vertical hierarchy ($v(\{2,3\}|(1,e_3)) - v(\{2,3\}|(0,e_3)) \geq 1$). Proposition 8 treats with the case where player 2 does not invest in both types of the organization ($v(\{2,3\}|(1,e_3)) - v(\{2,3\}|(0,e_3)) < 1$).

Proposition 7. *Assume that a player in the middle tier will invests in the vertical hierarchy but the player will not invest in the pyramidal hierarchy. The vertical hierarchy dominates the pyrammidal hierarchy if and only if*

$$\begin{aligned} & v(N|(1,1,e_3)) - v(N|(1,0,e_3)) \\ & \qquad \geq v(\{2,3\}|(1,e_3)) - v(\{2\}|0) - v(\{3\}|e_3), \end{aligned} \tag{11}$$

where $e_3 \in \{0,1\}$.

The intuition for this result is straightforward. The benefit of adopting the vertical hierarchy is that the owner can motivate player 2 to make his human capital investment. The left-hand side of (11) represents the increased return by the investment of player 2. On the other hand, the vertical hierarchy gives player 2 bargaining power over the ex post return. The increased payment for player 2 is equal to the right-hand side of (11). Player 1 prefers the vertical hierarchy to the pyramidal hierarchy if and only if the investment of a player in the middle tier is not marketable and the benefit from the vertical hierarchy outweighs its cost.

Assumption 2 implies that $v(\{2,3\}|(1,e_3))-v(\{2\}|0)-v(\{3\}|e_3) \geq v(\{2,3\}|(1,e_3))-v(\{2,3\}|(0,e_3))$. Assumption 4 implies that $v(N|(1,1,e_3))-v(N|(1,0,e_3)) \geq v(\{2,3\}|(1,e_3))-v(\{2,3\}|(0,e_3))$. Given the value of $v(\{2,3\}|(1,e_3))-v(\{2,3\}|(0,e_3))$, large $\Delta_i(e_j)$ implies that $v(N|(1,1,e_3))-v(N|(1,0,e_3))$ is large. Thus, (11) is satisfied when $\Delta_2(e_3)$ is enough large. Therefore, Proposition 7 says that the vertical hierarchy is preferred to the pyramidal hierarchy if the investment of the player assigned to the middle tier is sufficiently firm-specific. Our model suggests that the steeper hierarchy is adapted for organizations which need firm-specific human capital investments.

When $e_3 = 1$, the vertical hierarchy can implement the efficient outcome $e = (1,1,1)$. But, in the case that e_2 is not enough specific to satisfy (11), player 1 prefers the pyramidal hierarchy even if she can motivate all subordinates to invest in the vertical hierarchy. Therefore, Proposition 7 shows the possibility that low powered incentive organization is realized.

Proposition 8. *If a player in the middle tier does not invest in the vertical hierarchy, then the vertical hierarchy is dominated by the pyramidal hierarchy.*

Propositon 7 and Proposition 8 imply that the vertical hierarchy can be optimal only if the owner can motivate players to exert firm-specific investment by assigning them to the middle tier in the hierarchy. The steeper (vertical) hierarchy can motivate the firm-specific human capital investment for the player in the upper tier than the flat hierarchy.

Finally, we compare the vertical hierarchy and the common agency from the point of view of player 2's incentives. If player 1 wants to motivate player 2 to invest firm-specific human capital, one way is that she chooses the vertical hierarchy and give a subordinate to player 2. There is another way which is that player 1 chooses the common agency and gives player 2 an equal bargaining position. Which is better for player 1, giving a subordinate or giving the bargaining position equal to player 1? The answer is depend on incentives and consts. If

$$\begin{aligned}\frac{1}{2}\{v(N|(1,1,e_3)) - v(N|(1,0,e_3))\} \\ \geq v(\{2,3\}|(1,e_3)) - v(\{2,3\}|(0,e_3)),\end{aligned}$$

it can implement stronger incentive for player 2 to invest in the common agency than the vertical hierarchy. This condition is hold when the degree of firm-specific is high enough. But if the degree of firm-specific is high, the cost is larger in the common agency than in the vertical hierarchy. Which organization is better for player 1 contingents on the characteristic of the investments of player 2 as we have seen in Proposition 2.

5 Conclusion

This paper examined how the choice of organizational forms depends on the characteristics of human capital investments. We compared four types of organization and showed that every organizational form can be optimal.

The pyramidal hierarchy is the lowest cost but the most difficult to provide incentives to invest for player 2 and player 3, because incentives to invest human capitals are derived only from the allocation of the return through the ex post bargaining. However the owner can uses various instruments to provide incentives for employees in the real world. For example, tournaments or relative payments can be used if subordinates engage in the same task. Furthermore we assume that there is no externality between coalitions. However, suppose instead that there are externalities, a subcoalition may be realized in the pyramidal hierarchy where the player who does not invest is eliminated. Then the owner may implement improved incentive by choosing the pyramidal hierarchy. Further research for externalities between coalitions would clarify the design of organization.

Appendix

Proofs of Theorems in Section 3

Proof of Theorem 1.

We provide the following two lemmas in order to prove Theorem 1. The lemmas hold for any discount factor δ.

Lemma 1. *In every SSPE of the bargaining game in the horizontal organization where the expected return vector of the players is (v_1, v_2, v_3) and each player i proposes a coalition S_i on the equilibrium plays, every player i proposes a solution (S_i, y^i) of the maximization problem:*

$$\max_{S,y}(v(S|(e_j)_{j\in S}) - \sum_{j\in S} y_j) \qquad \text{subject to } y_j \geq \delta v_j, \text{for all } j \in S, j \neq i. \tag{A1}$$

The proposal (S_i, y^i) is accepted in the SSPE.

Proof. Let $x^i = (x_1^i, x_2^i, x_3^i)$ be the expected equilibrium return vector when player i becomes the proposer at round 1. Because each player is selected as a proposer with probability 1/3 in the bargaining game under the horizontal organization, $v_i = \sum_{k=1}^{3} x_i^k/3$ for $i = 1, 2, 3$. We denote m^i by the maximum value of (A1). We will prove $x_i^i = m^i$.

Let us start to prove ($x_i^i \leq m^i$). Suppose that player i proposes $(S, \hat{y})$ such that $\hat{y}_i > m^i$. Note that S is either N or $\{i\}$ in the case of this bargaining game. Since m^i is the maximum value of (A1), $\hat{y}_j < \delta v_j$ for some $j \in S$ with $j \neq i$. It is optimal for j to reject i's proposal because j's continuation return is δv_j when he rejects the proposal. Then, the game goes on to round 2. As a result, player i obtains the discount payoff δv_i. It is follow from the superadditivity of v that $\sum_{j=1}^{3} x_j^k \leq v(N|e)$ for all $k = 1, 2, 3$. Therefore, $v_1 + v_2 + v_3 \leq (\sum_{j=1}^{3} \sum_{k=1}^{3} x_j^k)/3 \leq v(N|e)$. Thus, the proposal with a coalition N and the return vector (v_1, v_2, v_3) is feasible. This implies that $v_i \leq m^i$. Because $\delta < 1$, we have $\delta v_i \leq v_i \leq m^i$. Player i gets only δv_i even if he demands a return greater than m^i. This proves $x_i^i \leq m^i$.

Next, let us prove ($x_i^i \geq m^i$). Any solution (S, y) of the problem (A1) satisfies $m^i = v(S|(e_j)_{j \in S}) - \sum_{j \in S, j \neq i} y_j$, where $y_j = \delta v_j$. For any $\varepsilon > 0$, define z such that

$$z_i = m^i - \varepsilon, \quad z_j = \delta v_j + \frac{\varepsilon}{|S| - 1}.$$

If player i proposes (S, z), then it is accepted. Therefore, $x_i^i \geq z_i = m^i - \varepsilon$. By taking ε small enough, we can obtain $x_i^i \geq m^i$. Then, we have $x_i^i = m^i$.

Finally, since $\delta v_i < m^i$, player i proposes a coalition S_i and the return vector $(m^i, (\delta v_j)_{j \in S_i, j \neq i})$ at round 1. □

Lemma 2. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is (v_1, v_2, v_3) and player 1, 2 and 3 propose a coalition N on the plays of the equilibrium if and only if*

(i) for every i such that $i, j, k = 1, 2, 3$, $i \neq j \neq k$,

$$v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i). \tag{A2}$$

(ii) the expected return vector (v_1, v_2, v_3) satisfies

$$\begin{aligned} v_1 &= \frac{1}{3}(v(N|e) - \delta v_2 - \delta v_3) + \frac{2}{3}\delta v_1, \\ v_2 &= \frac{1}{3}(v(N|e) - \delta v_1 - \delta v_3) + \frac{2}{3}\delta v_2, \\ v_3 &= \frac{1}{3}(v(N|e) - \delta v_1 - \delta v_2) + \frac{2}{3}\delta v_3. \end{aligned} \tag{A3}$$

Proof. (only-if). In the SSPE, the expected return vector is (v_1, v_2, v_3) and all players propose the grand coalition N. Player i can propose either N or $\{i\}$ when he becomes a proposer. By appling Lemma 1 to the SSPE, we can obtain

$$v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i) \text{ for } i = 1, 2, 3, i \neq j \neq k.$$

Every player i proposes the return allocation $(x_j^i)_{j \in N}$ such that

$$x_i^i = v(N|e) - \delta v_j - \delta v_k, \quad x_j^i = \delta v_j, \quad x_k^i = \delta v_k.$$

This proposal is accepted at round 1. Therefore, by the definition of the bargaining game in the horizontal organization, the expected return vector (v_1, v_2, v_3) is given by (A3).

(if). Consider the strategy combination such that, player i proposes a coalition N and the return vector $(v(N|e) - \delta v_j - \delta v_k, \delta v_j, \delta v_k)$, and accepted any proposal y^i for player i if and only if $y^i \geq \delta v_i$. It is easy to see that the above strategy is a locally optimal choice for every player under condition (i) and (ii) in Lemma 2. □

Proof of Thoerem 1. By Lemma 2, the expected equilibrium return vector (v_1, v_2, v_3) which satisfies (A3) converges to $(v(N|e)/3, v(N|e)/3, v(N|e)/3)$ as δ goes to 1. In addition, the condition (i) in Lemma 2 becomes

$$v(N|e)/3 \geq v(\{1\}|e_1),\ v(N|e)/3 \geq v(\{2\}|e_2),\ v(N|e)/3 \geq v(\{3\}|e_3).$$

These conditions are corresponding to (1) in Theorem 1. Lemma 2 (combining with Lemma 1) implies that in the SSPE, player 1, 2 and 3 all propose at round 1 a coalition N and the return vector $(v(N|e)/3, v(N|e)/3, v(N|e)/3)$ when δ is sufficiently close to one. The proposal is accepted in the SSPE.

Proof of Theorem 2.

We can prove the following lemmas about the existence of an SSPE in the same way as Lemma 2. We omit proofs of Lemma 3, 4 and 5.

Lemma 3. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is (v_i, v_j, v_k) and player i and j propose a coalition N and player k proposes a coalition $\{k\}$ on the plays of the equilibrium if and only if*

(i)

$$\begin{aligned} & v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i) \quad \text{for } i \in N = \{1,2,3\}, \\ & v(N|e) - \delta v_i - \delta v_k \geq v(\{j\}|e_j) \quad \text{for } j \in N, \\ & v(\{k\}|e_k) \geq v(N|e) - \delta v_i - \delta v_j. \quad \text{for } k \in N. \end{aligned}$$

(ii) the expected return vector (v_i, v_j, v_k) satisfies

$$\begin{aligned} v_i &= \frac{1}{3}\left(v(N|e) - \delta v_j - \delta v_k\right) + \frac{1}{3}\delta v_i + \frac{1}{3} v(\{i\}|e_i), \\ v_j &= \frac{1}{3}\left(v(N|e) - \delta v_i - \delta v_k\right) + \frac{1}{3}\delta v_j + \frac{1}{3} v(\{j\}|e_j), \\ v_k &= \frac{1}{3} v(\{k\}|e_k) + \frac{2}{3}\delta v_k. \end{aligned}$$

Lemma 4. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is (v_i, v_j, v_k) and player i proposes a coalition N and player j and k propose a coalition $\{j\}$ and a coalition $\{k\}$ on the plays of the equilibrium if and only if*

(i) for $i, j, k \in N = \{1,2,3\}$ with $i \neq j \neq k$,

$$\begin{aligned} & v(N|e) - \delta v_j - \delta v_k \geq v(\{i\}|e_i) \text{ and }, \\ & v(\{j\}|e_j) \geq v(N|e) - \delta v_i - \delta v_k \text{ and }, \\ & v(\{k\}|e_k) \geq v(N|e) - \delta v_i - \delta v_j. \end{aligned}$$

(ii) the expected return vector (v_i, v_j, v_k) satisfies

$$v_i = \frac{1}{3}\left(v(N|e) - \delta v_j - \delta v_k\right) + \frac{2}{3}v(\{i\}|e_i),$$
$$v_j = \frac{2}{3}v(\{j\}|e_j) + \frac{1}{3}\delta v_j,$$
$$v_k = \frac{2}{3}v(\{k\}|e_k) + \frac{1}{3}\delta v_k.$$

Lemma 5. *There exists an SSPE of the bargaining game in the horizontal organization where the expected return vector of players is (v_i, v_j, v_k) and player i, j and k propose a singleton coalition $\{i\}$, $\{j\}$ and $\{k\}$ respectively on the plays of the equilibrium if and only if*

(i) for $i, j, k \in N = \{1, 2, 3\}$ with $i \neq j \neq k$,

$$v(\{i\}|e_i) \geq v(N|e) - \delta v_j - \delta v_k \text{ and },$$
$$v(\{j\}|e_j) \geq v(N|e) - \delta v_i - \delta v_k \text{ and },$$
$$v(\{k\}|e_k) \geq v(N|e) - \delta v_i - \delta v_j.$$

(ii) the expected return vector (v_i, v_j, v_k) satisfies

$$v_i = v(\{i\}|e_i),\ v_j = v(\{j\}|e_j), v_k = v(\{k\}|e_k).$$

From condition (ii) in Lemma 3 (also, Lemma 4, Lemma 5), we can derive the expected return vector of the players (v_1^*, v_2^*, v_3^*) as δ goes to one. By substituting (v_1^*, v_2^*, v_3^*) for condition (i) in Lemma 3 (also, Lemma 4, Lemma 5), we can easily see that the condition (i) contradicts the superadditivity of v as $\delta \to 1$. This implies that there is no SSPE of the bargaining game in the horizontal organization when the discount factor is close to one. We complete the proof of Theorem 2.

Proof of Theorem 3.

In the common agency g_2, player 1 and 2 belong to tier 1 and have an equal opportunity (probability) to make a proposal in the bargaining game. We provide the following lemma. The lemma is proved in the same way as in Lemma 2. Therefore, we abbreviate the proof of Lemma 6

Lemma 6. *There exists an SSPE of the bargaining game in the common agency g_2 where the expected return vector of the players is (v_1, v_2, v_3) and player 1 and player 2 propose a coalition N on the plays of the equilibrium if and only if*

(i) for player 1,

$$v(N|e) - \delta v_2 - \delta v_3 \geq v(\{1,3\}|(e_1, e_3)) - \delta v_3 \text{ and},$$
$$v(N|e) - \delta v_2 - \delta v_3 \geq v(\{1\}|e_1),$$

and for player 2,

$$v(N|e) - \delta v_1 - \delta v_3 \geq v(\{2,3\}|(e_2, e_3)) - \delta v_3 \text{ and},$$
$$v(N|e) - \delta v_1 - \delta v_3 \geq v(\{2\}|e_2).$$

(ii) the expected return vector (v_1, v_2, v_3) satisfies

$$\begin{aligned} v_1 &= \frac{1}{2}\left(v(N|e) - \delta v_2 - \delta v_3\right) + \frac{1}{2}\delta v_1, \\ v_2 &= \frac{1}{2}\left(v(N|e) - \delta v_1 - \delta v_3\right) + \frac{1}{2}\delta v_2, \\ v_3 &= v(\{3\}|e_3). \end{aligned}$$

As in Lemma 1, it can be shown that in the above SSPE, player 1 proposes at round 1 a coalition N and the return vector $(v(N|e) - \delta v_2 - \delta v_3, \delta v_2, \delta v_3)$ for player 1, 2 and 3, and player 2 proposes at round 1 a coalition N and the return vector $(\delta v_1, v(N|e) - \delta v_1 - \delta v_3, \delta v_3)$. Moreover, these proposals always been accepted at round 1 in the SSPE.

If a discount factor δ goes to one, the expected return vector (v_1, v_2, v_3) in the SSPE converges to (v_1^*, v_2^*, v_3^*) such that

$$\begin{aligned} v_1^* &= \frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right), \\ v_2^* &= \frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right), \\ v_3^* &= v(\{3\}|e_3). \end{aligned}$$

In addition, condition (i) in Lemma 6 is rewritten as

$$\begin{aligned} &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{1,3\}|(e_1, e_3)) - v(\{3\}|e_3), \\ &\frac{1}{2}\left(v(N|e) - v(\{3\}|e_3)\right) \geq v(\{2,3\}|(e_1, e_3)) - v(\{3\}|e_3). \end{aligned}$$

Therefore, we can obtain Theorem 3 from Lemma 6 as $\delta \to 1$.

Proof of Theorem 4.

We can provide the following lemma. We omit the proof of Lemma 7 because it can be proved in the same way as in Lemma 2.

Lemma 7. *There exists an SSPE of the bargaining game in the common agency g_2 where the expected return vector of the players is (v_1, v_2, v_3) and player 1 proposes a coalition $\{1,3\}$ and player 2 proposes a coalition $\{2,3\}$ on the plays of the equilibrium if and only if*

(i) for player 1,

$$\begin{aligned} &v(\{1,3\}|(e_1, e_3)) - \delta v_3 \geq v(N|e) - \delta v_2 - \delta v_3 \text{ and,} \\ &v(\{1,3\}|(e_1, e_3)) - \delta v_3 \geq v(\{1\}|e_1), \end{aligned}$$

and for player 2,

$$\begin{aligned} &v(\{2,3\}|(e_2, e_3)) - \delta v_3 \geq v(N|e) - \delta v_1 - \delta v_3 \text{ and,} \\ &v(\{2,3\}|(e_2, e_3)) - \delta v_3 \geq v(\{2\}|e_2). \end{aligned}$$

(ii) the expected return vector (v_1, v_2, v_3) satisfies

$$\begin{aligned} v_1 &= \frac{1}{2}\left(v(\{1,3\}|(e_1, e_3)) - \delta v_3\right) + \frac{1}{2}\delta v(\{1\}|e_1), \\ v_2 &= \frac{1}{2}\left(v(\{2,3\}|(e_2, e_3)) - \delta v_3\right) + \frac{1}{2}\delta v(\{2\}|e_2), \\ v_3 &= v(\{3\}|e_3). \end{aligned}$$

If a discount factor δ goes to one, the expected return vector of the players in the SSPE converges to (v_1^*, v_2^*, v_3^*) such that

$$v_1^* = \frac{1}{2}\left(v(\{1,3\}|(e_1,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{1\}|e_1),$$
$$v_2^* = \frac{1}{2}\left(v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)\right) + \frac{1}{2}v(\{2\}|e_2),$$
$$v_3^* = v(\{3\}|e_3).$$

Then, the condition (i) in Lemma 7 becomes

$$\begin{aligned}
&v(\{1,3\}|(e_1,e_3)) \geq \\
&\quad v(N|e) - \frac{1}{2}v(\{2,3\}|(e_2,e_3)) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3), \text{ and} \\
&v(\{2,3\}|(e_2,e_3)) \geq \\
&\quad v(N|e) - \frac{1}{2}v(\{1,3\}|(e_2,e_3)) - \frac{1}{2}v(\{1\}|e_1) - \frac{1}{2}v(\{3\}|e_3),
\end{aligned}$$

as δ is sufficiently close to one. This condition is same as (3) in Theorem 4. Thus, Lemma 7 implies Theorem 4 as $\delta \to 1$.

Proof of Theorem 5.

In order to prove Theorem 5, we follow the same procedures as the proof in Theorem 2. We must provide several lemmas about the existence of an SSPE of the bargaining game in the common agency. These lemmas gives a necessary and sufficient condition for the existence of an SSPE as in Lemma 2. In the bargaining game for the common agency g_2, the following SSPE should be considered except in Lemma 6 and 7: an SSPE in which (i)player 1 proposes a coalition N and player 2 proposes a coalition $\{2,3\}$ on the plays of the equilibrium, (ii)player 1 proposes a coalition $\{1,3\}$ and player 2 proposes a coalition N, (iii)player 1 proposes a coalition N and player 2 proposes a singleton coalition $\{2\}$, (iv)player 1 proposes a coalition $\{1\}$ and player 2 proposes a coalition N, (v)player 1 proposes a coalition $\{1,3\}$ and player 2 proposes a coalition $\{2\}$, (vi)player 1 proposes a coalition $\{1\}$ and player 2 proposes a coalition $\{2,3\}$, and (vii)player 1 and player 2 proposes a singleton coalition $\{1\}$ and $\{2\}$. Corresponding to each SSPE, the lemma is provided. Thus, seven lemmas would be provided. We does not describe these lemmas in full detail, and we also omit the proof of the lemmas.

We can see that each necessary and sufficient condition for the existence of the SSPE does not satisfied if a discount factor δ is sufficiently close to one under Assumption 1-5. Then, Theorem 5 is obtained.

Proof of Theorem 6.

We can determine a subgame perfect equilibrium of the bargaining game by backward induction procedures since the bargaining game in the pyramidal hierarchy is finite game with perfect information. Let us start with the response strategies for player 2 and 3 in tier 2 of the organization. If player 2 reject an offer from player 1, then negotiations break down and player 2 obtains the payoff of $v(\{2\}|e_2)$. Therefore, player 2 accepts a proposal y_2 such that $y_2 \geq v(\{2\}|e_2)$. Similarly, player 3 accepts a proposal y_3 such that $y_2 \geq v(\{3\}|e_3)$. Taking into accounts of the response of player 2 and 3, player 1 makes a take-it-or-leave-it offer of a division of the return $v(N|e)$ among the players. If player 1 offers $v(\{2\}|e_2)$ for player 2 and $v(\{3\}|e_3)$ for player 3, then he obtains the return of

$(v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3))$. This return is the maximum return that player 1 can obtain in an acceptable offer. Furthermore, if player 1 makes any offer that is rejected, then he obtains at most $v(\{1\}|e_1)$, which is less than $v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3)$ by the superadditivity of v. Hence, player 1 proposes at round 1 a coalition N and an allocation of $(v_1^*, v_2^*, v_3^*) = (v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3), v(\{2\}|e_2), v(\{3\}|e_3))$. Moreover, the proposal is accepted at round 1.

Proof of Theorem 7.

The equilibrium strategies of each player are derived by the backward induction procedure. If player 3 rejects an offer by player 2, player 3 obtains the return of $v(\{3\}|e_3)$. Therefore, player 3 accepts an offer y_3 if and only if $y_3 \geq v(\{3\}|e_3)$. In the vertical hierarchy, player 2 has the alternative of deviating from the organization by coalition $\{2,3\}$. If player 2 does so, player 2 obtains $v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)$ and player 3 obtains $v(\{3\}|e_3)$. This implies that player 2 accepts a return y_2 such that $y_2 \geq v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)$. Since $v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3) \geq v(\{2\}|e_2)$ by superadditivity of v, player 2 does not reject the return $v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3)$. Therefore, player 1 proposes a coalition N and offers $v(\{2,3\}|(e_2,e_3))$ as a share of player 2 and player 3. Then, player 2 accepts the proposal and offers $v(\{3\}|e_3)$ for player 3. This offer is also accepted. In the equilibrium, the expected return of each player is given by $(v_1^*, v_2^*, v_3^*) = (v(N|e) - v(\{2,3\}|(e_2,e_3)), v(\{2,3\}|(e_2,e_3)) - v(\{3\}|e_3), v(\{3\}|e_3))$.

Proofs of Propositions in Section 4

Proof of Proposition 1.

By Assumption 5, we have

$$v(N|(1,1,1)) > \sum_{i=1}^{3} e_i = 3.$$

When the organizational structure is the horizontal organization, the allocation of return for player i $(i = 1,2,3)$ in the SSPE is $v_i^* = v(N|e)/3$ by Theorem 1. Since $v(N|(1,1,1))/3 - 1 > v(N|(0,1,1))/3 = 0$, player 1 chooses $e_1 = 1$ at date 1. Since player 2 and player 3 face the same incentive problem, there exists an equilibrium that all players invest and a grand coalition is formed in the horizontal organization. The payoff of player 1 in the horizontal organization becomes $\pi_1^H = v(N|e)/3 - 1 > 0$.

Next, we consider the common agency. Because Theorem 3 holds when (e_1, e_2, e_3) is perfectly complementary, $v_3^* = 0$ by (5). Then, player 3 chooses $e_3 = 0$. This makes $v_1^* = v_2^* = 0$ and $e_1 = e_2 = e_3 = 0$. Hence, the payoff of player 1 in the common agency π_1^C is zero; $\pi_1^C = 0$.

In the pyramidal hierarchy, Theorem 6 implies that $v_2^* = v_3^* = 0$. Since player 2 and 3 make no investment, $(e_2 = e_3 = 0)$, it follows that $v(N|(e_1,0,0)) = 0$ and player 1 also does not invest; $e_1 = 0$. Hence, the payoff of player 1 in the pyramidal hierarchy is zero; $\pi_1^P = 0$. In the vertical hierarchy, we can obtain $\pi_1^V = 0$ by the same argument as in the pyramidal hierarchy, where π_1^V is the payoff of player 1 in the vertical hierarchy.

Therefore, it is optimal for player 1 to choose the horizontal organization.

Proof of Proposition 2.

We give an example that satisfies Assumption 1-5 and holds Proposition 2. Assume that $v(\{1\}|0) = v(\{2\}|0) = v(\{3\}|0)$.

We consider the case where only the investment of player 3 is marketable; $v(\{3\}|1) - v(\{3\}|0) \geq 1$, and investments of player 1 and player 2 are not marketable. In addition, it is assumed that

$$v(\{1\}|1) = v(\{1\}|0) = 0, \text{and } v(\{2\}|1) = v(\{2\}|0) = 0.$$

We assume that the firm's value is equal to $v(\{3\}|e_3)$ if player 1 and 2 do not invest. The additional return is generated only if the human capital investments of player 1 and player 2 would be together, that is,

$$v(N|(1,1,e_3)) > v(N|(1,0,e_3)) = v(N|(0,1,e_3)) = v(\{3\}|e_3), \tag{B1}$$
$$v(\{1,3\}|(e_1,e_3)) = v(\{2,3\}|(e_2,e_3)) = v(\{3\}|e_3). \tag{B2}$$

At first, we consider the incentive problem in the common agency g_2. Since the return of player 3 is $v(\{3\}|e_3)$, player 3 chooses $e_3 = 1$. Theorem 3 holds under conditions (B1) and (B2). Then, the proposer offers a coalition N in equilibrium. Let us consider the case in which the following conditions are satisfied:

$$\frac{1}{2}(v(N|(1,1,1)) - v(\{3\}|1)) - \frac{1}{2}(v(N|(0,1,1)) - v(\{3\}|1)) \geq 1,$$
$$\frac{1}{2}(v(N|(1,1,1)) - v(\{3\}|1)) - \frac{1}{2}(v(N|(1,0,1)) - v(\{3\}|1)) \geq 1.$$

In this case, player 1 and player 2 make their human capital investments. From (B1), both of conditions are reduced to $(v(N|(1,1,1)) - v(\{3\}|1))/2 \geq 1$. Then, the equilibrium payoff of player 1 when the common agency is selected is given by

$$\pi_1^C = \frac{1}{2}(v(N|(1,1,1)) - v(\{3\}|1)) - 1 > 0. \tag{B3}$$

Since $v(\{2\}|1) = 0$, player 2 does not invest in the pyramidal hierarchy. Because $v(N|(e_1,0,1)) - v(\{2\}|0) - v(\{3\}|1) - e_1 = -e_1$, player 1 chooses $e_1 = 0$. Thus, the equilibrium payoff of player 1 if she chooses the pyramidal hierarchy is $\pi_1^P = v(N|(0,0,1)) - v(\{2\}|0) - v(\{3\}|1) = 0$. Therefore, player 1 prefers the common agency to the pyramidal hierarchy.

Next consider the incentive for player 2 in the vertical hierarchy. Note that $(v(\{2,3\}|(1,1)) - v(\{3\}|1)) - (v(\{2,3\}|(0,1)) - v(\{3\}|1)) = 0 < 1$. This implies that player 2 will choose $e_2 = 0$ at date 1. It is easy to see that player 1 also chooses $e_1 = 0$ if $e_2 = 0$. Thus, the equilibrium payoff of player 1 if she chooses the vertical hierarchy becomes $\pi_1^V = v(N|(0,0,1)) - v(\{2,3\}|(0,1)) = 0$, and, then, she prefers the common agency to the vertical hierarchy.

Finally, the incentive constraint of investment for player 1 in the horizontal organization is represented by

$$\frac{1}{3}v(N|(1,e_2,e_3)) - 1 \geq \frac{1}{3}v(N|(0,e_2,e_3)). \tag{B4}$$

If $e_2 = 0$, this condition (B4) is violated and player 1 does not invest, i.e., $e_1 = 0$. If $e_2 = 1$, the condition (B4) becomes

$$\frac{1}{3}\left\{v(N|(1,1,e_3) - v(3|e_3)\right\} \geq 1, \tag{B5}$$

where $e_3 \in \{0,1\}$. The payoff of player 1 in the case of $e = (1,1,1)$ is

$$\pi_1^H = \frac{1}{3}v(N|(1,1,1)) - 1. \tag{B6}$$

From (B3) and (B6), it follows that $\pi_1^C \geq \pi_1^H$ if $v(N|(1,1,e_3))/3 \geq v(3|e_3)$. If $v(N|(1,1,e_3))/3 < v(\{3\}|e_3)$, no SSPE exists in the horizontal organization by Theorem 2. The payoff of π_1^H in the case of $e = (1,1,0)$ under the horizontal organization is always smaller than that in the case of $e = (1,1,1)$.

There exists no SSPE in which $e = (0,0,1)$ is implemented because $v_1^* = v_2^* = v_3^* = v(N|(0,0,1))/3 < v(\{3\}|1)$. When $e = (0,0,0)$, $\pi_1^H = 0$. Therefore, the horizontal organization is dominated by the common agency with $e = (1,1,1)$.

Proof of Proposition 3.

We shall show that the organization of a subcoalition in Theorem 4 is dominated by the pyramidal hierarchy.

From Theorem 4 and Theorem 6, we can see that the incentive problem for player 3 is same in both the common agency and the pyramidal hierarchy. Player 1 invests in the common agency if

$$\frac{1}{2}v(\{1,3\}|(1,e_3)) - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{1\}|1) - \frac{1}{2}v(\{1\}|0) \geq 1. \tag{B7}$$

In the pyramidal hierarchy, player 1 invests if

$$v(N|(1,e_2,e_3)) - v(N|(0,e_2,e_3)) \geq 1. \tag{B8}$$

Assumption 4 implies that the left-hand side of (B8) is larger than that of (B7). Then, there are three cases in which (i)player 1 invest in both organizations, (ii)player 1 does not invest in both organizations and (iii)player 1 invests in the pyramidal hierarchy but not in the common agency. In the cases of (i) and (ii), the level of e_1 is same in both organizations. When player 1 chooses the same investment level, we can obtain that

$$\begin{aligned}
\pi_1^P - \pi_1^C &= v(N|(e_1,e_2,e_3)) - v(\{2\}|e_2) - v(\{3\}|e_3) \\
&\qquad - \frac{1}{2}v(\{1,3\}|(e_1,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|e_1) \\
&\geq \frac{1}{2}v(N|(e_1,e_2,e_3)) + \frac{1}{2}v(\{1,3\}|(e_1,e_3)) + \frac{1}{2}v(\{2\}|e_2) \\
&\qquad - v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1,3\}|(e_1,e_3)) - \frac{1}{2}v(\{1\}|e_1) \\
&= \frac{1}{2}v(N|(e_1,e_2,e_3)) - \frac{1}{2}v(\{1\}|e_1) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3) \geq 0
\end{aligned}$$

This implies that the payoff of player 1 in the pyramidal hierarchy is greater than that in the common agency. In the case of $e_1 = 0$ in the common agency and $e_1 = 1$ in the pyramidal hierarchy, we have

$$\begin{aligned}\pi_1^P - \pi_1^C &= v(N|(1,e_2,e_3)) - v(\{2\}|e_2) - v(\{3\}|e_3) - 1\\ &\qquad - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|0)\\ &\geq v(N|(0,e_2,e_3)) + 1 - v(\{2\}|e_2) - v(\{3\}|e_3) - 1\\ &\qquad - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|0)\\ &\geq \frac{1}{2}v(N|(0,e_2,e_3)) + \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{2\}|e_2) - v(\{2\}|e_2)\\ &\qquad - v(\{3\}|e_3) - \frac{1}{2}v(\{1,3\}|(0,e_3)) + \frac{1}{2}v(\{3\}|e_3) - \frac{1}{2}v(\{1\}|0)\\ &= \frac{1}{2}v(N|(0,e_2,e_3)) - \frac{1}{2}v(\{1\}|0) - \frac{1}{2}v(\{2\}|e_2) - \frac{1}{2}v(\{3\}|e_3) \geq 0.\end{aligned}$$

This means that the payoff in the pyramidal hierarchy is greater than that in the common agency. Therefore, the payoff for player 1 in forming a subcoalition under the common agency is always smaller than that in the pyramidal hierarchy.

Proof of Proposition 4.

According to Theorem 6, if the pyramidal hierarchy is chosen, the equilibrium return at date 2, given $e = (e_1, e_2, e_3)$, is represented by

$$\begin{aligned}v_1^*(e) &= v(N|e) - v(\{2\}|e_2) - v(\{3\}|e_3),\\ v_2^*(e) &= v(\{2\}|e_2),\\ v_3^*(e) &= v(\{3\}|e_3).\end{aligned}$$

Since both of player 2 and player 3 will choose to invest, the following conditions must be satisfied:

$$v(\{2\}|1) - v(\{2\}|0) \geq 1, \tag{B9}$$

$$v(\{3\}|1) - v(\{3\}|0) \geq 1. \tag{B10}$$

The equilibrium payoff of player 1 in the pyramidal hierarchy is given by

$$\pi_1^P(1,1,1) = v(N|(1,1,1)) - v(\{2\}|1) - v(\{3\}|1). \tag{B11}$$

Next we consider the vertical hierarchy. From (B10), player 3 chooses $e_3 = 1$. Since $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) \geq 1$ (by Assumption 4), player 2 chooses $e_2 = 1$. Therefore, the equilibrium payoff of player 1 in the vertical hierarchy is given by

$$\pi_1^V(1,1,1) = v(N|(1,1,1) - v(\{2,3\}|(1,1)). \tag{B12}$$

By Assumption 2, we can obtain that $\pi_1^P(1,1,1) \geq \pi_1^V(1,1,1)$.

Let us consider the horizontal organization. Since there is an SSPE in the horizontal organization only if $v(N|e)/3 \geq v(\{i\}|e_1)$ for $i = 1,2,3$, it is enough to restrict to such the case. Player 1 can get the maximum payoff at $(e_1, e_2, e_3) = (1,1,1)$. That is,

$$\pi_1^H(1,1,1) = v(N|(1,1,1))/3. \tag{B13}$$

in the horizontal organization. Using the condition that $v(N|e)/3 \geq v(\{i\}|e_i)$, $i = 1,2,3$, we can obtain that $\pi_1^P(1,1,1) \geq \pi_1^H(1,1,1)$.

Finally, we compare the common agency with the pyramidal hierarchy. From Proposition 3, it is sufficient to show that the common agency in the case of Theorem 3 is dominated by the pyramidal hierarchy. Since the incentive problem of player 3 is same in both organizational forms, player 3 chooses $e_3 = 1$ in the common agency. The payoff of player 1 is maximized at $e = (1, 1, 1)$ in the common agency. That is,

$$\pi_1^C(1,1,1) = \frac{1}{2}\{v(N|(1,1,1)) - v(\{3\}|1)\}. \tag{B14}$$

From the condition in Theorem 3, it follows that $v(N|(1,1,1))/2 \geq v(\{2,3\}|(1,1) - v(\{3\}|1)/2$. This implies that $\pi_1^P(1,1,1) \geq \pi_1^C(1,1,1)$. Therefore, if $(e_2, e_3) = (1,1)$ can be implemented under the pyramidal hierarchy, then player 1 chooses the pyramidal hierarchy at date 2.

Proof of Proposition 5.

If conditions (6) and (7) are satisfied, $e_3 = 1$ in g_5 and $e_2 = 0$ in g_6.

If $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) \geq 1$, then, $e_2 = 1$ in g_5. Then, the payoff of player 1 in g_5 is given by

$$v(N|(1,1,1)) - v(\{2,3\}|(1,1)). \tag{B15}$$

The payoff of player 1 in g_6 is

$$v(N|(1,0,e_3)) - v(\{2,3\}|(0,e_3)), \tag{B16}$$

where $e_3 \in \{0,1\}$. Then, by Assumption 5, we obtain that $(B15) \geq (B16)$.

If $v(\{2,3\}|(1,1)) - v(\{2,3\}|(0,1)) < 1$, then $e_2 = 0$ in g_5. Therefore, the payoff of player 1 in g_5 is

$$v(N|(1,0,1)) - v(\{2,3\}|(0,1)). \tag{B17}$$

The payoff of player 1 in g_6 is given by (B16). If $e_3 = 1$ in g_6, then, we have that $(B17) = (B16)$. If $e_3 = 0$ in g_6, $(B17) \geq (B16)$ by Assumption 5. Hence, g_5 dominates g_6.

Proof of Proposition 6.

Since e_2 and e_3 are not marketable, a player in the bottom tier does not invest. Thus, $e_3 = 0$ in g_5 and $e_2 = 0$ in g_6.

Under Condition (10), a player in the middle tier of g_5 and g_6 has a same incentive to invest the human capital. If a player in the middle tier invest in either g_5 and g_6, the payoff of player 1 in g_5 is

$$v(N|(1,1,0)) - v(\{2,3\}|(1,0)), \tag{B18}$$

and that in g_6 is given by

$$v(N|(1,0,1)) - v(\{2,3\}|(0,1)). \tag{B19}$$

From (9) and (10), it follows that (B18) $\geq$ (B19).

If a player in the middle tier does not invest in either g_5 and g_6, the payoff of player 1 is same in g_5 and g_6 and that is given by $v(N|(1,0,0)) - v(\{2,3\}|(0,0))$. Hence, player 1 prefers g_5 to g_6.

Proof of Proposition 7.

If the investment of player 3 is marketable but that of player 2 is not marketable, Proposition 5 implies the player 2 is superior to player 3 in equilibrium. Since the pair of $e_2 = 0$ and $e_3 = 1$ is implemented in the pyramidal hierarchy, the payoff of player 1 is given by

$$\pi_1^P = v(N|(1,0,1)) - v(2|0) - v(3|1).$$

Since $e_2 = e_3 = 1$ in the vertical hierarchy, the payoff of player 1 is

$$\pi_1^V = v(N|(1,1,1)) - v(\{2,3\}|(1,1)).$$

Hence, $\pi_1^V \geq \pi_1^P$ if and only if

$$v(N|(1,1,1)) - v(N|(1,0,1)) \geq v(\{2,3\}|(1,1)) - v(\{2\}|0) - v(\{3\}|1),$$

When both of investments are not marketable, the payoff of player 1 from the pyramidal hierarchy is represented by

$$\pi_1^P = v(N|(1,0,0)) - v(2|0) - v(3|0),$$

because $e_2 = e_3 = 0$ in equilibrium. Since $e_2 = 1$ and $e_3 = 0$ are implemented in the vertical hierarchy, the payoff of player 1 is given by

$$\pi_1^V = v(N|(1,1,0)) - v(\{2,3\}|(1,0)).$$

Therefore, $\pi_1^V \geq \pi_1^P$ if and only if

$$v(N|(1,1,0)) - v(N|(1,0,0)) \geq v(\{2,3\}|(1,0)) - v(\{2\}|0) - v(\{3\}|0).$$

This completes the proof.

Proof of Proposition 8.

First, if both of the investments are marketable, Proposition 5 implies that the vertical is dominated by the pyramidal hierarchy.

Next, if the investment of player 2 is not marketable and that of player 3 is marketable, the payoff of player 1 is given by

$$\pi_1^P = v(N|(1,0,1)) - v(2|0) - v(3|1)$$

in the pyramidal hierarchy, and that in the vertical hierarchy is

$$\pi_1^V = v(N|(1,0,1)) - v(\{2,3\}|(0,1)).$$

By Assumption 2, we can obtain that $\pi_1^P \geq \pi_1^V$.

Finally, if both of investments are not marketable, the payoff of player 1 is

$$\pi_1^P = v(N|(1,0,0)) - v(2|0) - v(3|0)$$

in the pyramidal hierarchy and that in the vertical hierarchy g_5 is given by

$$\pi_1^V = v(N|(1,0,0)) - v(\{2,3\}|(0,0)).$$

By Assumption 2, we have $\pi_1^P \geq \pi_1^V$. Hence, the vertical hierarchy in which a player in the middle tier does not invest is dominated by the pyramidal hierarchy.

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