The Evolution of Fairness under Assortative Matching in Ultimatum Mini Game *

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Abstract

This paper shows that, under assortative matching rule, the fair action can be maintained in ultimatum mini game on the evolutionary dynamics. If matching is random, then the selfish action generates higher payoff than fair action and the selfish individuals, who play rationally, are always survived on the replicator dynamics. If, however, matching is assortative, then fair responders are easy to encounter fair responders. They obtain higher payoff than the selfish individuals in case that there are the fair individuals more than the selfish individuals.

1 Introduction

Why would people do the fair action which decreases his/her own profit superficially? In the ultimatum game, the rational proposers should make offers that their share is almost the total amount, and the rational responders accept any offers. Many experimental data, however, suggest that people usually tend to divide the total amount equally (Binmore, McCarthy, Ponti, Samuelson, and Shaked (2002); Güth, Schmittberger, and Schwarze (1982)). This paper studies that problem using evolutionary approach and focusing on the matching rule.

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1 \ 2	Y	N
L	$3,1$	$0,0$
H	$2,2$	$2,2$

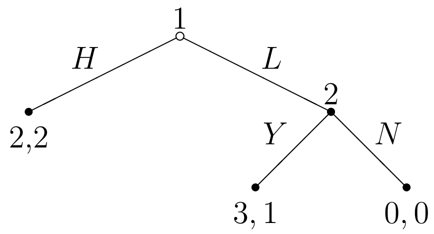

Figure 1: Ultimatum mini game

For example, we consider ultimatum mini game (figure 1). Let x_1 and x_2 be the frequency of individuals adopting strategy L and Y, respectively. The standard replicator dynamics is described as

$$\begin{aligned}\dot{x_1} &= g_1(x) = x(f_Y - \phi_1) \\ \dot{x_2} &= g_2(x) = y(f_Y - \phi_2)\end{aligned}$$

, where f_L and f_Y are the fitness of player using L and Y. ϕ_1 and ϕ_2 are the average fitness of population 1 and 2.

In ultimatum mini game (the payoff matrices of this game are in figure 1), the system under the random matching rule is

$$\begin{aligned}g_1(x) &= x_1(1 - x_1)(3x_2 - 2) \\ g_2(x) &= x_2(1 - x_2)x_1\end{aligned}$$

, where x_1 (x_2) is the frequency of strategy L (Y). Gale, Binmore, and Samuelson (1995) shows that it is only asymptotically stable state that all proposers will do the selfish offer and all responders will accept it ($x = (1, 1)$) by this system. If we consider deterministic dynamics, then subgame perfect equilibrium which derived by rationality is equal to that asymptotically stable state in ultimatum mini game. Thus, the random matching rule do not bridge the difference of theory and experimental data.

In case that there exists a noise or drift, Gale, Binmore, and Samuelson (1995) and Binmore and Samuelson (1999) introduce the perturbed selection dynamics

$$\dot{x} = g(x) + h(x).$$

This dynamics provides a perturbed selection process using the drift function h. If the function h is strictly decreasing in the difference between the largest and smallest expected payoff, there exists asymptotically stable state which is in the set of Nash equilibria and not subgame perfect equilibrium. These equilibria, however, depend on the form of h. For example, if h is not sensitive to payoffs then subgame perfect equilibrium is only asymptotically stable state.

This paper studies a role of matching rule, instead of considering the drift function h. We consider an assortative matching rule instead of a random matching rule. An assortative matching is a matching rule that sorts individuals into matches like players. Becker (1973, 1974) and Atkan (2006) show that complementarities in payoff (supermodularity of the payoff function) lead to assortative matching in symmetric case. The interaction rate between individuals is independent on their strategies under a random matching rule, but depends on their strategies under an assortative matching rule. This leads dynamics to replicator equations with nonlinear interaction rates. In symmetric payoff matrix game which have two strategies, Taylor and Nowak (2006) studies the more generalized rule and show the results under that rule. Bergstrom (2003) also defines another assortative matching rule in prisoner's dilemma game. The ultimatum mini game, however, has asymmetric payoff matrices. For this reason, we define another rule which has assortativity.

2 The model

There exist two populations, the proposers (population 1) and the responders (population 2). The sizes of these two populations are the same. They encounter another player in each other population and make a pair according to the matching rule for dividing a given total surplus of which size is n. When all individuals make a pair, they play ultimatum mini game with a partner. Each player is programmed for only one of two possible strategies, and can not change this strategy. Each one of proposers can make a high offer (H) or low offer (L). If he/she adopts strategy H, it is assumed that responders always accept it. If he/she adopts strategy L (H), responders may accept Y or reject N it. In this paper, without loss of generality, we define $n = 4$. The game tree and strategic form of ultimatum mini game are in the figure 1.

Now, we define an assortative matching rule. Let x_1 ($(1 - x_1)$) denote the proportion of the selfish (fair) proposer in population 1, and x_2 ($(1 - x_2)$) denote the proportion of the selfish (fair) responder in population 2, respectively.

Selfish (fair) proposers always do strategy L (H), and selfish (fair) responders do strategy Y (N). Let p_i be the probability that population i's selfish player meets population j's selfish player and q_i be the probability that population i's fair player meets population j's fair player.

We suppose that the probability that the proposer encounters responder (p_i, q_i) depends both on his/her own strategy and the distribution of others' strategies in two populations. Therefore, encounters do not occur randomly, and we define the assortative matching as the following.

Definition 1. *A matching rule is an assortative matching rule if,*

for all i, the probability p_i, $q_i \in [0,1]$ satisfy that for all $(x_i, x_j) \in [\epsilon, 1-\epsilon] \times [\epsilon, 1-\epsilon]$,

(i) $\lim_{x_i \to 0} p_i(x) > x_j$,$\lim_{x_j \to 0} p_i(x) = 0$,
$\lim_{x_i \to 1} p_i(x) = x_j$,$\lim_{x_j \to 1} p_i(x) = 1$
$(\lim_{x_i \to 1} q_i(x) > 1 - x_j$, $\lim_{x_j \to 1} q_i(x) = 0$,
$\lim_{x_i \to 0} q_i(x) = 1 - x_j$, $\lim_{x_j \to 0} q_i(x) = 1)$

(ii) monotone nonincreasing in x_i (x_j), monotone nondecreasing in x_j (x_i)

(iii) $p_i(x_i, x_j) > x_j$, $q_i(x_i, x_j) > 1 - x_j$

(iv)

$$x_i p_i = x_j p_j,\ (1 - x_i)q_i = (1 - x_j)q_j \tag{1}$$

$$(1 - p_i)x_i = (1 - q_j)(1 - x_j) \tag{2}$$

Equations (1-2) are parity equations which imply that probability function p_i and q_i and the matchings are consistent. All players can be pair as long as these equations are satisfied. Definition 1 have two characters. First, by the condition (ii) and (iii), the probability p_i (q_i) are increasing (decreasing) and higher than 45° line as the frequency of their same type opponent x_j ($(1 - x_j)$) is increasing. Second, the growth of their own type x_i ($(1 - x_i)$) causes the decrease of the probability p_i (q_i). We also assume that p_i and q_i are Lipschitz continuous and that x_i is in $[\epsilon, 1 - \epsilon]$ for all i at any state (ϵ is sufficiently small), because the $p_i(0, 0)$ and $q_i(1, 1)$ do not converge from the condition (i). This means that there is at least one person using each strategy at all time.

3 Results

In this section, we discuss dynamics under a general assortative matching rule. We assume that p_i and q_i are differentiable and dynamics has monotonicity and regularity. Thus, monotonicity conditions of matching rule (the condition (ii)) are equal to

$$\frac{\partial p_i(x_i, x_j)}{\partial x_i} \leq 0 \quad \text{and} \quad \frac{\partial p_i(x_i, x_j)}{\partial x_j} \geq 0 \tag{3}$$

$$\frac{\partial q_i(x_i, x_j)}{\partial x_i} \geq 0 \quad \text{and} \quad \frac{\partial q_i(x_i, x_j)}{\partial x_j} \leq 0, \tag{4}$$

and the system is described as

$$\begin{aligned} \dot{x_1} = g_1(x) &= (x_1 - \epsilon)(1 - \epsilon - x_1)(3p_1 + 0(1 - p_1) - 2(1 - q_1) - 2q_1) \quad (5) \\ \dot{x_2} = g_2(x) &= (x_2 - \epsilon)(1 - \epsilon - x_2)(p_2 + 2(1 - p_2) - 0(1 - q_2) - 2q_2). \quad (6) \end{aligned}$$

Let $\bar{\phi}_i$ denote the difference between fair and selfish individuals of average payoff in population i as

$$\begin{aligned} \bar{\phi}_1(x) &= 3p_1(x) - 2 \\ \bar{\phi}_2(x) &= 2 - p_2(x) - 2q_2(x). \end{aligned}$$

We, first, find the set of stable points of the system (5)-(6) under some assortative matching rule.

Proposition 1. *Let $\mathcal{R}$ be the the set of rest points of the system (5)-(6) under some assortative matching rule. The $\mathcal{R}$ has eight limit points*

$$\begin{aligned} x = (\epsilon, \epsilon), (\epsilon, 1 - \epsilon), (1 - \epsilon, \epsilon), (1 - \epsilon, 1 - \epsilon), \\ (x_1', x_2'), (x_1'', \epsilon), (\epsilon, x_2''), (1 - \epsilon, x_2''') \end{aligned}$$

, where $\bar{\phi}_1(x_1', x_2') = 0$, $\bar{\phi}_2(x_1', x_2') = 0$, $\bar{\phi}_1(x_1'', \epsilon) = 0$, $\bar{\phi}_2(\epsilon, x_2'') = 0$, and $\bar{\phi}_2(1 - \epsilon, x_2''') = 0$.

Proof. According to the form of system (5)-(6), The nine points have possibility to be rest points. The following lemma, however, shows that $(x_1', 1 - \epsilon)$ is not rest point.

Lemma 1. *$(x_1', 1 - \epsilon)$ such that $\bar{\phi}_1(x) = 0$ does not exist under any assortative matching rule.*

Proof. Suppose there exists $(x_1', 1-\epsilon)$ such that $\bar{\phi}_1(x) = 0$. By $\bar{\phi}_1 = 0$, $p_1(x_1', 1-\epsilon) = \frac{2}{3}$. This implies $p_2(x_1', 1-\epsilon) = \frac{2}{3(1-\epsilon)}x_1$ by parity equations (1-2). However, this contradicts with condition (iii) ($p_2 > x_1$) when ϵ is sufficiently small. □

While, there exist some assortative matching rule in rest cases. □

Each rest point only has stability. Some rest points are not robust over small perturbations. Next, we check the asymptotical stability of rest points.

Proposition 2. *Let $\mathcal{A}$ be the set of asymptotically stable points under some assortative matching rule. $x^* \in \mathcal{A}$ is asymptotically stable, if and only if*

(a) there exists (x_1^*, x_2^*) such that $\bar{\phi}_1 = 0$, $x_2 = \epsilon$ and an assortative matching rule which satisfies

$$\frac{\partial p_1}{\partial x_1}(x^*) < 0 \text{ and } p_2(x^*) + 2q_2(x^*) > 2.$$

(b) there exists (x_1^*, x_2^*) such that $x_1^* = \epsilon$, $\bar{\phi}_2 = 0$, and an assortative matching rule which satisfies

$$p_1(x^*) < \frac{2}{3} \text{ and } \frac{\partial p_2(x^*)}{\partial x_2} > -2\frac{\partial q_2(x^*)}{\partial x_2}.$$

(c) there exists (x_1^*, x_2^*) such that $x_1^* = 1-\epsilon$, $\bar{\phi}_2 = 0$, and an assortative matching rule which satisfies

$$p_1(x^*) > \frac{2}{3} \text{ and } \frac{\partial p_2(x^*)}{\partial x_2} > -2\frac{\partial q_2(x^*)}{\partial x_2}.$$

Proof. By proposition 1, asymptotically stable points are subset of the $\mathcal{R}$. First, we will remove asymptotically stable points under no assortative matching rule. We obtain the following lemmas.

Lemma 2. *There exists no assortative matching rule that makes $(\epsilon, \epsilon), (\epsilon, 1-\epsilon), (1-\epsilon, \epsilon), (1-\epsilon, 1-\epsilon)$ are asymptotically stable.*

Proof. We will show the case of $x = (\epsilon, \epsilon)$. The Jacobian

$$\frac{\partial g}{\partial x}(\epsilon,\epsilon) = \begin{pmatrix} (1-2\epsilon)(3p_1(\epsilon,\epsilon)-2) & 0 \\ 0 & (1-2\epsilon)(2-p_2(\epsilon,\epsilon)-2q_2(\epsilon,\epsilon)) \end{pmatrix}.$$

Thus, if (ϵ, ϵ) is asymptotically stable then $p_1(\epsilon,\epsilon) < 2/3$ and $2(1-q_2(\epsilon,\epsilon)) < p_2(\epsilon,\epsilon)$. However, this contradicts with parity equations (1-2)[1]. Other rest points are also not asymptotically stable by the same argument. □

Lemma 3. *If there exists (x_1', x_2') which satisfies $\bar{\phi}_1(x') = 0$ and $\bar{\phi}_2(x') = 0$, then this is a saddle point under any assortative matching rule.*

Proof. By $\bar{\phi}_1 = 0$ and the parity equations (1-2), $p_1(x') = \frac{2}{3}$. Then, $p_2 = \frac{2x_1'}{3x_2'}$, and $q_2 = 1 - \frac{x_1'}{3(1-x_2')}$ by parity equations (1-2). This implies $x_2' = \frac{1}{2}$ by $\bar{\phi}_2 = 0$. Thus, the Jacobian

$$\begin{aligned}\frac{\partial g}{\partial x}(x_1', x_2') &= \begin{pmatrix} (x_1'(1-x_1')-\epsilon(1-\epsilon))(\frac{\partial \phi_1(x')}{\partial x_1}) & (x_1'-\epsilon)(1-\epsilon-x_1')(\frac{\partial \phi_1(x')}{\partial x_2}) \\ (x_2'-\epsilon)(1-\epsilon-x_2')(\frac{\partial \phi_2(x')}{\partial x_1}) & (x_2'(1-x_2')-\epsilon(1-\epsilon))(\frac{\partial \phi_2(x')}{\partial x_2}) \end{pmatrix}, \\ &= \begin{pmatrix} -c & (x_1'-\epsilon)(1-\epsilon-x_1')(\frac{\partial \phi_1(x')}{\partial x_2}) \\ 0 & 0 \end{pmatrix}\end{aligned}$$

since $\frac{\partial \phi_1(x')}{\partial x_1} = 3\frac{\partial p_1(x')}{\partial x_1} \leq 0$, $\frac{\partial \phi_2(x')}{\partial x_1} = \frac{\partial \phi_2(x')}{\partial x_2} = 0$ ($c \geq 0$ is constant). Therefore, (x_1', x_2') is a saddle point □

By lemmas 1, 2, 3, we only investigate the conditions that $x = (x_1'', \epsilon)$, (ϵ, x_2''), $(1-\epsilon, x_2''')$ are asymptotically stable. First, we test $x = (\epsilon, x_2'')$. The Jacobian

$$\frac{\partial g}{\partial x}(\epsilon, x_2'') = \begin{pmatrix} (1-2\epsilon)(3p_1(\epsilon,x_2'')-2) & 0 \\ (x_2''-\epsilon)(1-\epsilon-x_2'')(\frac{\partial \phi_2(\epsilon,x_2'')}{\partial x_1}) & (x_2''(1-x_2'')-\epsilon(1-\epsilon))(\frac{\partial \phi_2(\epsilon,x_2'')}{\partial x_2}) \end{pmatrix}.$$

Thus, if an assortative matching rule satisfies $p_1(\epsilon, x_2'') < \frac{2}{3}$ and $\frac{\partial \phi_2(\epsilon,x_2'')}{\partial x_2} = -\frac{\partial p_2(\epsilon,x_2'')}{\partial x_2} - 2\frac{\partial q_2(\epsilon,x_2'')}{\partial x_2} < 0$, then both eigenvalue λ are negative. Hence, (ϵ, x_2'') is asymptotically stable if and only if this conditions are satisfied. The conditions that other two points $(x_1'', \epsilon), (1-\epsilon, x_2'''))$ are asymptotically stable, are calculated by the same way. □

[1] By (1-2), $p_1 = p_2$, then $q_2 > 2/3$. Therefore, $\epsilon/3 < (1-p_1)\epsilon = (1-q_2)(1-\epsilon) < (1-\epsilon)/3$. It is a contradiction because ϵ is sufficiently small.

By proposition 2 and parity equations (1-2), if $\epsilon \simeq 0$ then

(a)

$$\begin{aligned} x_1^* &= \frac{x_2^*}{p_1(x^*)} p_2(x^*) \text{ (by (1-2))} \\ &\leq \frac{3}{2}\epsilon \text{ (by } \bar{\phi}_1 = 0, p_2 \leq 1) \\ &\simeq 0 \end{aligned}$$

(b)

$$\begin{aligned} x_2^* &= \frac{x_1^*}{p_2(x^*)} p_1(x^*) \text{ (by (1-2))} \\ &< \frac{2}{3} \text{ (by } \frac{2}{3} > p_1 > x_2,\ p_2 > x_1) \end{aligned}$$

(c)

$$\begin{aligned} 2 &= p_2(x^*) + 2q_2(x^*) \text{ (by } \bar{\phi}_2 = 0) \\ &= p_2(x^*) + 2q_1(x^*)\frac{1 - x_1^*}{1 - x_2^*} \text{ (by (1-2))} \\ &\leq 1 + 2\frac{\epsilon}{1 - x_2^*} \text{ (by } p_2, q_1 \leq 1), \end{aligned}$$

then $x_2^* \geq 1 - 2\epsilon \simeq 1$.

Thus, the state $x = (1,1), (0,c)$ are asymptotically stable points under some assortative matching rule when fluctuation is sufficiently small ($c(< \frac{2}{3})$ is constant). Hence, each Nash equilibrium except $x = (0, 2/3)$ is asymptotically stable under some assortative matching rules.

4 An Example

In this section, we define a simple assortative matching rule and calculate which state is stable. One simple assortative matching rule which satisfies the conditions (i), (ii), (iii) and parity equations (1-2) of the definition 1 is defined as the following.

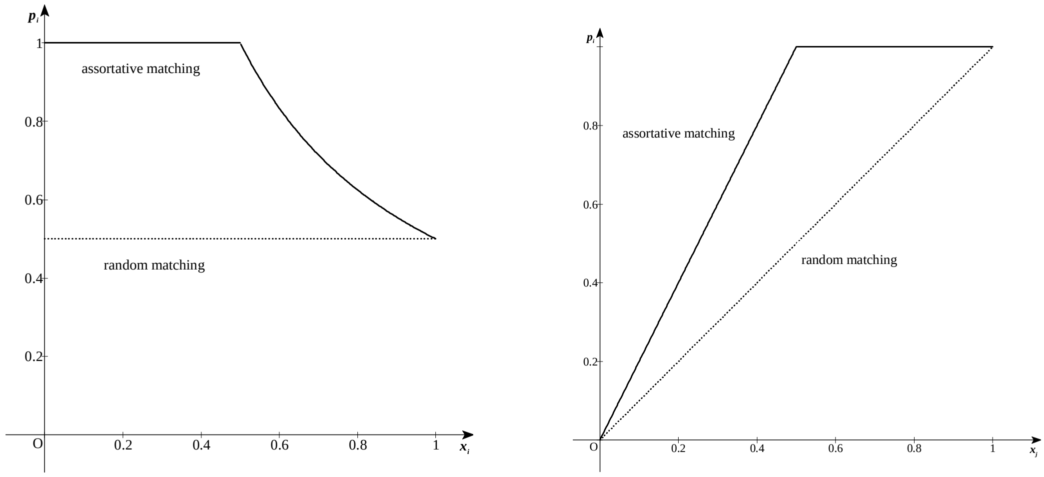

Figure 2: The probability p_i

The population 1's probabilities are

$$p_1 = \begin{cases} \frac{x_2}{x_1} & \text{if } x_1 > x_2 \\ 1 & \text{otherwise} \end{cases}$$

$$q_1 = \begin{cases} \frac{1-x_2}{1-x_1} & \text{if } x_1 \leq x_2 \\ 1 & \text{otherwise} \end{cases}$$

and the population 2's probabilities are

$$p_2 = \begin{cases} \frac{x_1}{x_2} & \text{if } x_1 \leq x_2 \\ 1 & \text{otherwise} \end{cases}$$

$$q_2 = \begin{cases} \frac{1-x_1}{1-x_2} & \text{if } x_1 > x_2 \\ 1 & \text{otherwise} \end{cases}$$

The figure 4 shows the probability that selfish proposers meet selfish responders under this assortative matching rule and the random matching rule. As can be seen in the figure 4, fair proposers tend to meet a fair responder rather than a selfish proposer. Of course, this matching rule is one of matching rules which have the assortativity, but is the simple rule to consider the effect of it.

Let us assume that the x_i is differentiable functions in time t. We also assume that the per capita rate of growth is given by the difference between the payoff for strategy x_i and the average payoff in the populations, and that the growth rate is continuous on state space. These two conditions are defined as monotonicity and regularity by Binmore and Samuelson (1999). Therefore, the selection dynamics is described as the following dynamics.

For the characteristic of this assortative matching rule, there exists two cases in the selection dynamics of this model.

case 1: $x_1 > x_2$

$$\begin{aligned} \dot{x_1} = g_1(x) &= (x_1 - \epsilon)(1 - \epsilon - x_1)(3\frac{x_2}{x_1} - 2) && (7) \\ \dot{x_2} = g_2(x) &= (x_2 - \epsilon)(1 - \epsilon - x_2)(1 - 2\frac{1 - x_1}{1 - x_2}) && (8) \end{aligned}$$

case 2: $x_1 \leq x_2$

$$\begin{aligned} \dot{x_1} = g_1(x) &= (x_1 - \epsilon)(1 - \epsilon - x_1)(3 - 2) && (9) \\ \dot{x_2} = g_2(x) &= (x_2 - \epsilon)(1 - \epsilon - x_2)(\frac{x_1}{x_2} + 2 - 2\frac{x_1}{x_2} - 2). && (10) \end{aligned}$$

Indeed, these selection equations under this assortative matching rule form nonlinear system. The figure 4 is the phase diagram of this system (7)-(10).

Using the system (7)-(10), we will find stationary strategy distributions of ultimatum mini game under an assortative matching rule. We obtain the following results from this model.

Lemma 4. *Let $\bar{\mathcal{R}}$ be the set of rest points of the system (7)-(10). $\bar{\mathcal{R}}$ has seven points*

$$\begin{aligned} x = (\epsilon, \epsilon), (\epsilon, 1 - \epsilon), (1 - \epsilon, \epsilon), (1 - \epsilon, 1 - \epsilon), \\ (\frac{3}{2}\epsilon, \epsilon), (1 - \epsilon, 1 - 2\epsilon)(3/4, 1/2). \end{aligned}$$

Proof. Omitted □

Proposition 3. *Let $\bar{\mathcal{A}}$ be the set of asymptotically stable points of the system (7)-(10). $\bar{\mathcal{A}}$ has two points $x = (\frac{3}{2}\epsilon, \epsilon), (1 - \epsilon, 1 - 2\epsilon)$.*

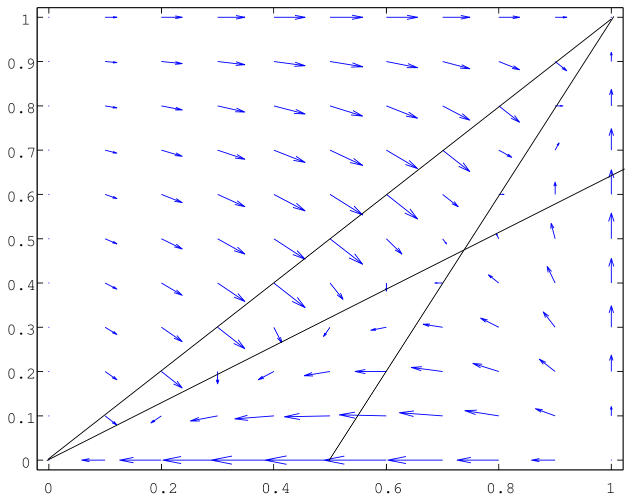

Figure 3: Phase diagram

Proof. We consider the linear approximations of neighborhoods of all stable points, since it is sufficient by the definition to examine whether the stable point is the asymptotically stable.

To verify that, we check the eigenvalue of Jacobian. In case of $x = (1-\epsilon, \epsilon)$, we only consider the system of case 1. On the neighborhood of the stable point $(1-\epsilon, \epsilon)$, the Jacobian is

$$\frac{\partial g}{\partial x}(1-\epsilon, \epsilon) = \begin{pmatrix} (\frac{3\epsilon}{1-\epsilon} - 2)(2\epsilon - 1) & 0 \\ 0 & (1 - \frac{2\epsilon}{1-\epsilon})(1 - 2\epsilon) \end{pmatrix}.$$

Then, eigenvalue $\lambda \simeq 1, 2 > 0$ when $\epsilon \simeq 0$. Therefore, $(1-\epsilon, \epsilon)$ is not asymptotically stable. Similarly, we will show that $x = (\epsilon, 1-\epsilon)$ is not asymptotically stable.

In case of $x = (1-\epsilon, 1-\epsilon)$, it is not sufficient to consider any one case of the system, since two case of the system is surely included the neighborhood of $(1-\epsilon, 1-\epsilon)$. Now, the Jacobian of the system (7)-(10) on $(1-\epsilon, 1-\epsilon)$ is

$$\frac{\partial g}{\partial x}(1-\epsilon, 1-\epsilon) = \begin{cases} \begin{pmatrix} 2\epsilon - 1 & 0 \\ 0 & 1 - 2\epsilon \end{pmatrix} & \text{if } x \text{ in case1} \\ \begin{pmatrix} 2\epsilon - 1 & 0 \\ 0 & 1 - 2\epsilon \end{pmatrix} & \text{if } x \text{ in case2} \end{cases}.$$

Therefore, $(1-\epsilon, 1-\epsilon)$ is a saddle point because $\lambda \simeq -1, 1$ in all cases when $\epsilon \simeq 0$. Thus, this point is not asymptotically stable. We will also show that $x = (3/4, 1/2), (\epsilon, \epsilon)$ is not asymptotically stable but a saddle point by the same argument.

Finally, we check the case of $x = (\frac{3}{2}\epsilon, \epsilon)$. This case only be tested in case 1 because of $x_1 > x_2$. The Jacobian of the system (7)-(10) of $(\frac{3}{2}\epsilon, \epsilon)$ is

$$\frac{\partial g}{\partial x}(\frac{3}{2}\epsilon, \epsilon) = \begin{pmatrix} -\frac{2}{3} + \frac{5}{3}\epsilon & 1 - \frac{5}{2}\epsilon \\ 0 & \frac{-1+2\epsilon}{1-\epsilon} \end{pmatrix}.$$

Thus, $\lambda \simeq -\frac{2}{3}, -1$ when $\epsilon \simeq 0$. Hence, $(\frac{3}{2}\epsilon, \epsilon)$ is asymptotically stable. Similarly, $(1-\epsilon, 1-2\epsilon)$ is asymptotically stable because $\lambda \simeq -1$ when $\epsilon \simeq 0$

□

By the proposition 3, there are the selfish equilibrium ($x = (1, 1)$) and the fair one ($x = (0, 0)$) if fluctuation is sufficiently small ($\epsilon \simeq 0$).

By assortativity, fair responders are easy to encounter fair proposers than selfish responders. Hence, if proposers are almost fair ($x \simeq 0$), then strategy N generates higher expected payoff than strategy Y (under a random matching rule, in contrast, a strategy N is weekly dominated by strategy Y). Therefore, fair and selfish equilibria coexist.

5 Conclusion

In this paper, we study a role of a matching rule on evolutionary game dynamics in ultimatum mini game. If encounters are random, then Gale, Binmore, and Samuelson (1995) shows that subgame perfect equilibrium is only asymptotically stable point on replicator dynamics although experimental data do not support this result.

One possible explanation for these fair or inequity aversion actions, they like not to do selfish action because their preferences depend not only on their own payoff but on fairness or equity (Bolton and Ockenfels (2000); Fehr and Schmidt (1999)). However, it is not clear that people have such fair and interdependent preferences. Second, in repeated situation, if there is something like reputation, they worry about to get bad reputations and will act fairly (Nowak and Sigmund (1998); Ohtsuki and Iwasa (2004); Nowak, Page, and Sigmund (2000)). If proposers obtain higher payoff at one-shot, then their opposite players will have bad feeling and reject to punish those action irrationally. If responders accept any offer which is the dominant strategy at one-shot and this may become known, the next proposer will make unfair offer. Therefore, to act fairly improves their long-term payoffs even if their short-term payoffs decrease. In this case, however, the players are less anonymous because they have reputations. They do not play the same opponent in all time but they are labeled individually.

Here we assume that matching rule is assortative. The assortative matching rule leads to nonlinear system and thus expand the set of stable point. For the ultimatum mini game, there exist some assortative matching rule supporting each Nash equilibrium except one equilibrium point to be asymptotically stable. If encounters are assortative, then fair agents have higher probability to meet fair agents. In this case, the average payoff of fair actions sometime becomes higher than selfish action on dynamics depending on the mass of fair agents. Therefore, the strategies offering equal division and rejecting selfish offer are survived and maintained. The main result of this paper is that once we assume assortativity, these matching rule can maintain the fair actions as an asymptotically stable point without fair preference or reputation.

However, our study has two limitations. First, the subgame perfect equilibrium is also asymptotically stable. The path is determined by a initial state and an assortative matching rule. Thus, the fair actions are not always achieved and sustained by this model. Second, it is not obvious that the same result may occur on other dynamics. Furthermore, we only analyze the ultimatum "mini" game which have only two strategies. The behavior of dynamics is also ambiguous in case of the general ultimatum game which have infinite strategies. These extensions of model are interesting to research.

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