The Evolution of Fairness under Assortative Matching in Ultimatum Mini Game

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Introduction

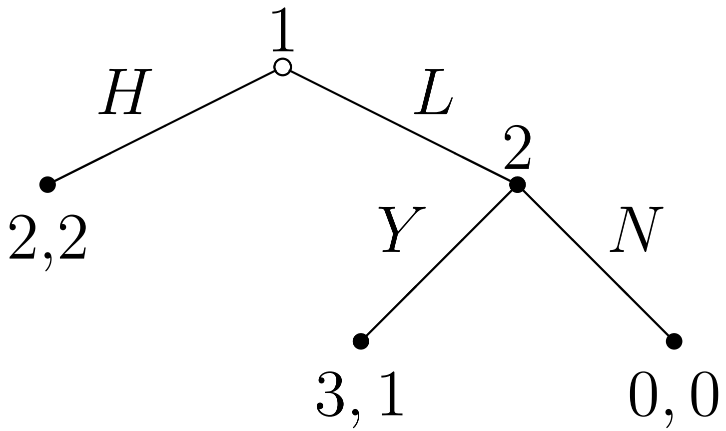
People often care about "fairness."

- Ultimatum bargaining model
  - Theoretically,
    all offers are nearly 100%, and all responders accept.
  - However, such outcomes are rarely observed.
- Why do people choose such fair behaviors?
- Do the fair behaviors survive?

→ This paper focuses on "**Matching rules**."

Ultimatum mini game

		2	
		Y	N
1	L	$3, 1$	$0, 0$
	H	$2, 2$	$2, 2$



- 1: proposer, 2: responder
- (Pure) Nash Equilibrium (NE): $(L, Y), (H, N)$
- Subgame Perfect Equilibrium (SPE): (L, Y)

Results of the experiment: Binmore et al. (2002)

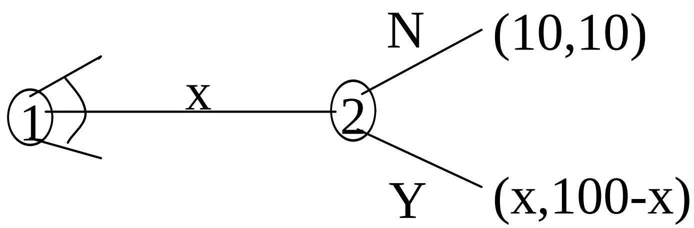
- 100% offer is rare.

  → Many proposers' offers are not optimal.

- Rejection exists.

  → Responses are not best replies.

Why do people take account of "fairness"?



The result of Binmore et al. (2002)

Rounds	Observations	Mean offer	Median	5th %tile	95th %tile
1-10	400	64.9	65	50	80
11-20	400	66.8	68	55.5	76.5

	All demands		Demands in [70, 80]	
Rounds	Observations	Rejection %	Observations	Rejection %
1-10	400	24	111	48
11-20	400	19	146	34

Related Literature

- Gale et al. (1995)
  - Ultimatum mini game with Random matching
  - The SPE (L,Y) is asymptotically stable.
- Cressman and Schlag (1998)
  - 2 populations extensive-form game
  - Any SPE is asymptotically stable.

Basic Model

- Evolutionary game (Replicator Dynamics)
- 2 populations: proposers (1) and responders (2)
- Selfish strategy; L,Y
- Fair strategy; H,N
- $x_1(x_2)$: the proportion of selfish strategy L (Y)

1 \ 2	Y	N
L	$3, 1$	$0, 0$
H	$2, 2$	$2, 2$

Notations

Matching Probabilities;

$$
\begin{aligned}
p_1 &= Pr(L \text{ meets } Y) \\
q_1 &= Pr(H \text{ meets } N) \\
p_2 &= Pr(Y \text{ meets } L) \\
q_2 &= Pr(N \text{ meets } H)
\end{aligned}
$$

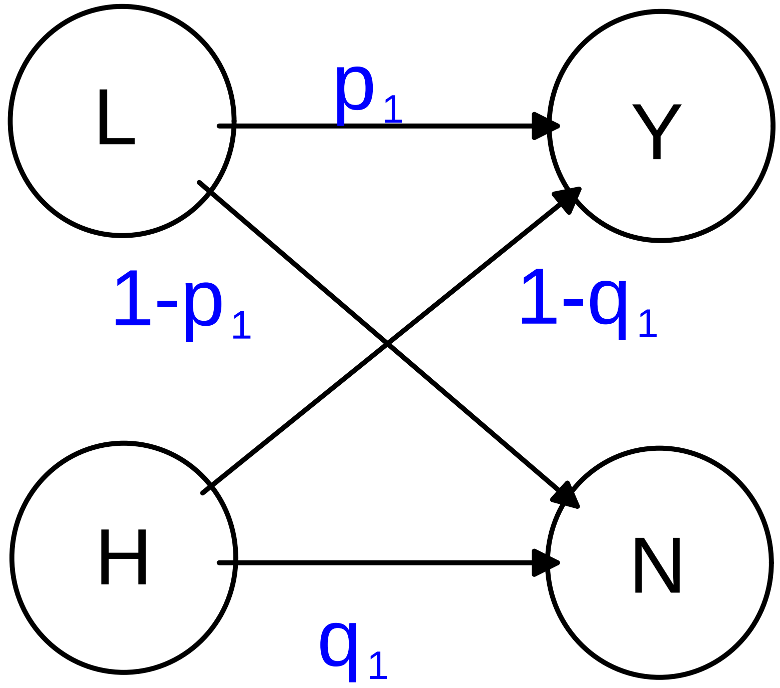


Figure 1: Matching probabilities of population 1

Notations

- f: Average payoffs of strategies

$$f_L = p_1 3 + (1 - p_1)0, \quad f_H = (1 - q_1)2 + q_1 2$$
$$f_Y = p_2 1 + (1 - p_2)2, \quad f_N = (1 - q_2)0 + q_2 2$$

- ϕ: Average payoffs of populations

$$\phi_1 = x_1 f_L + (1 - x_1) f_H, \quad \phi_2 = x_2 f_Y + (1 - x_2) f_N$$

The Dynamics

Replicator Dynamics

$$\begin{aligned}\dot{x_1} &= x_1(f_L - \phi_1) \\ &= x_1(1 - x_1)(3p_1 - 2)\end{aligned}$$

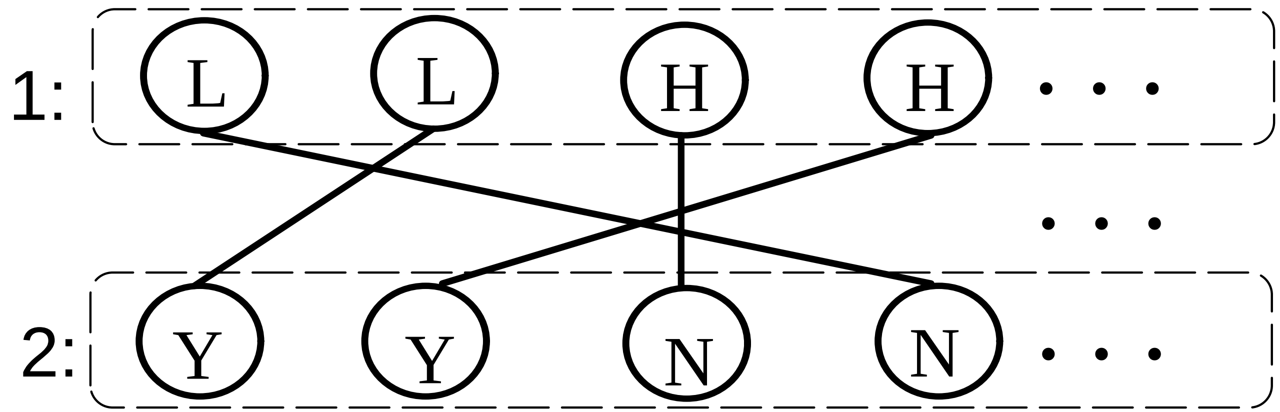
$$\begin{aligned}\dot{x_2} &= x_2(f_Y - \phi_2) \\ &= x_2(1 - x_2)(2 - p_2 - 2q_2)\end{aligned}$$

The Random Matching rule; Gale et al. (1995)

Each player encounters a partner "at random."

$\rightarrow$ The matching rates depend only on partners' distribution.

$p_1^R = Pr(L \text{ meets } Y) = x_2$, $p_2^R = Pr(Y \text{ meets } L) = x_1 \ldots$



$$p_1 = Pr(L \textbf{ meets } Y)$$

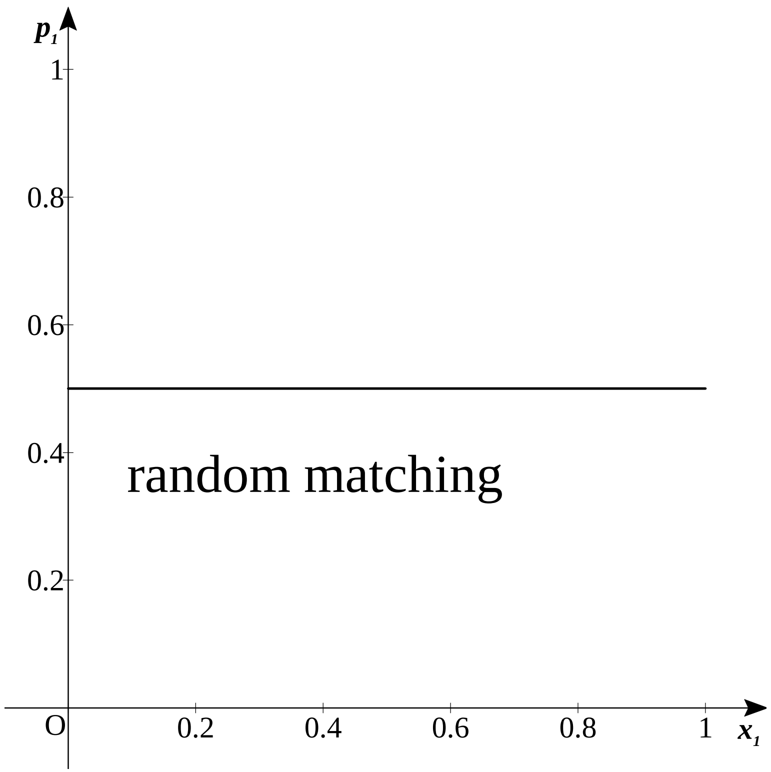


Figure 2: when $x_2 = 0.5$

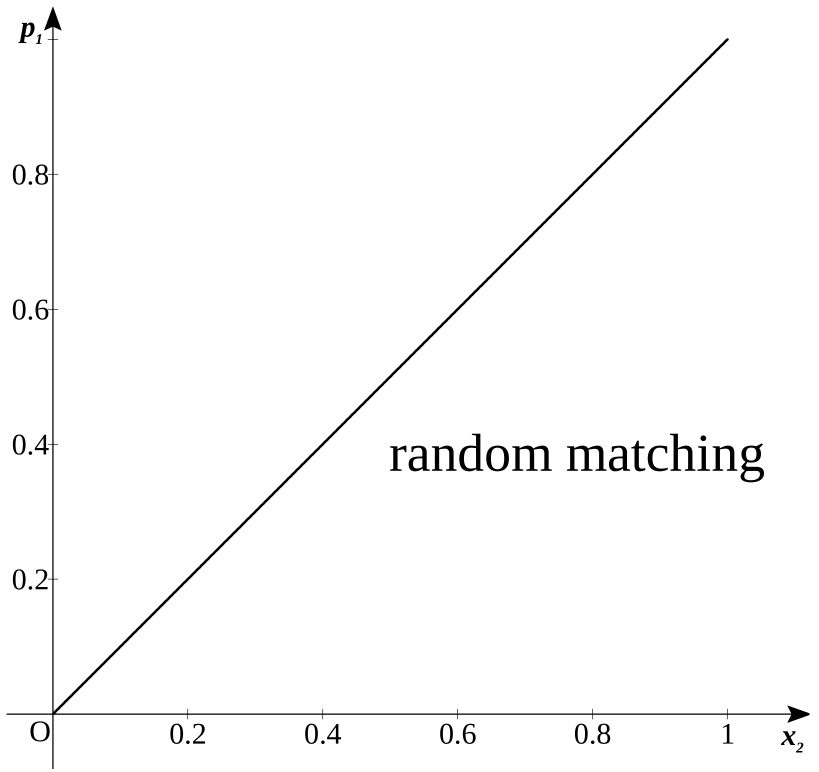


Figure 3: when $x_1 = 0.5$

The Results

The Dynamics

$$\begin{aligned}\dot{x_1} &= x_1(1 - x_1)(3x_2 - 2)\\ \dot{x_2} &= x_2(1 - x_2)x_1\end{aligned}$$

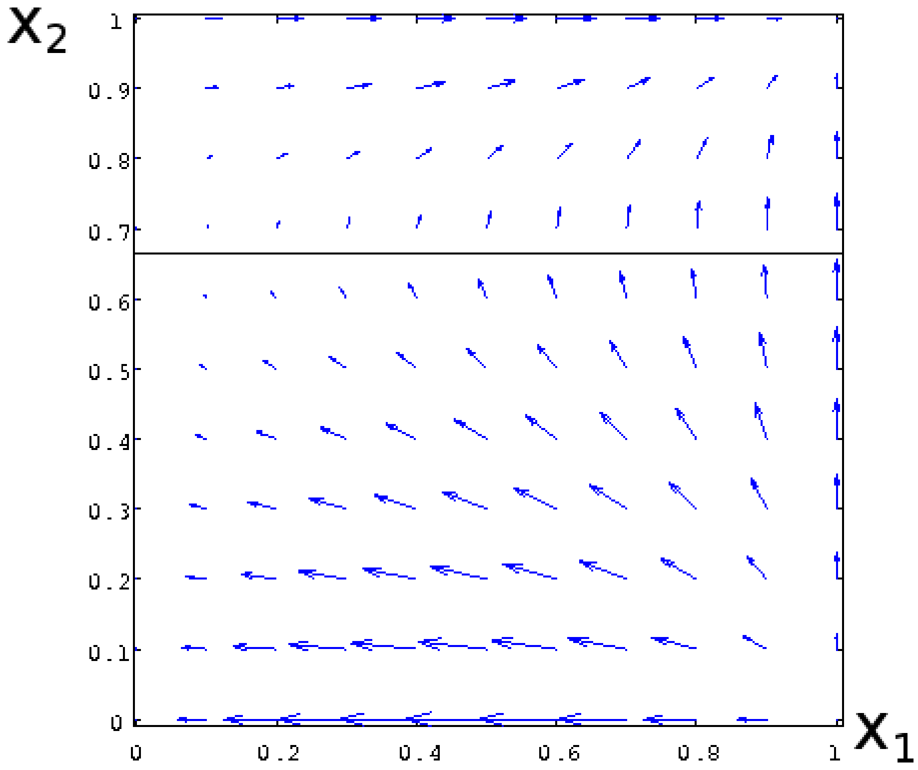
Theorem 1. *(Gale et al. (1995))*

With the exception of (0, 2/3), the Nash equilibria are local attractors.

The $(x_1, x_2) = (1, 1)$ *is the unique asymptotic attractor.*

$\rightarrow$ Asymptotically stable point = SPE point.

The Phase Diagram



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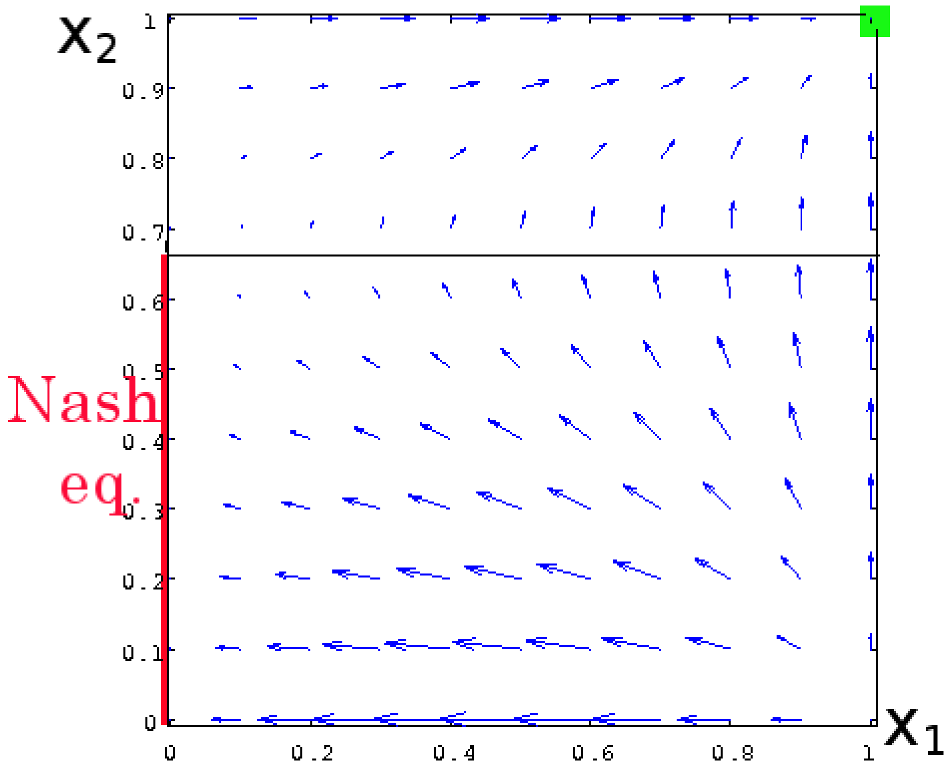

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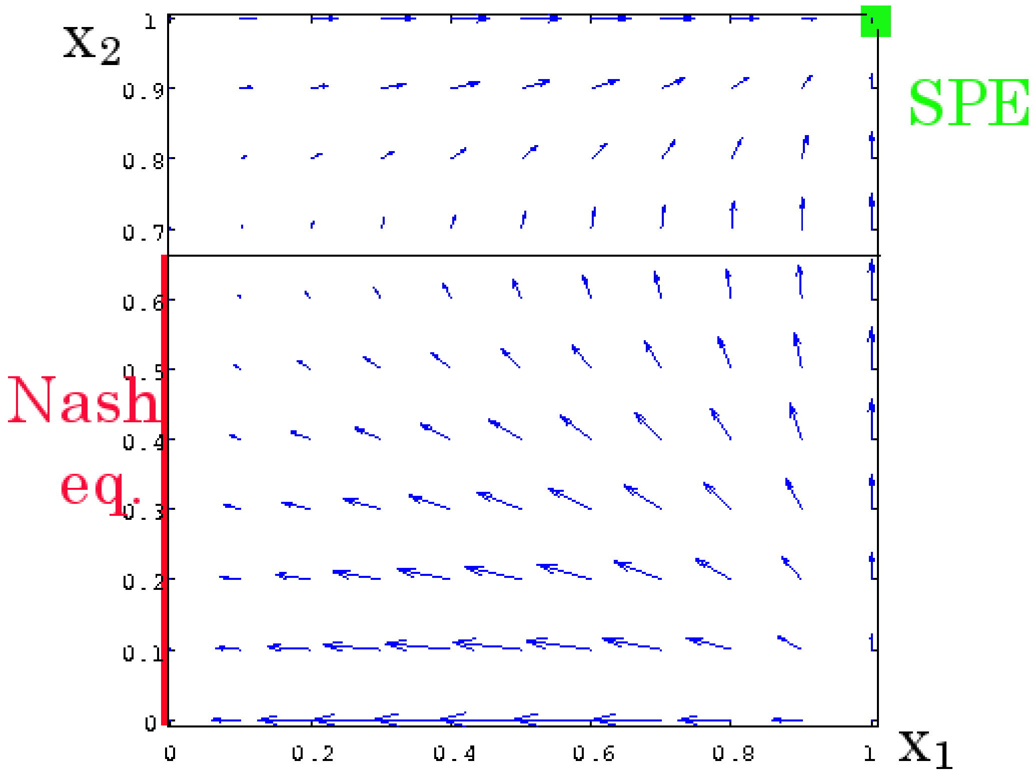


General Result under Random Matching

Γ: 2 populations extensive-form game

Theorem 2. *Cressman and Schlag (1998)*

- *Any Nash eq. that is in the interior of the Nash eq. set of* Γ *relative to the set of rest points of the replicator dynamic is stable. Moreover, a pure strategy profile is a NE if and only if it is stable.*
- *For any* Γ *in which any path has at most one decision point off the subgame perfect equilibrium path and this point has (at most) two possible choices, the SPE component is the unique minimal interior asymptotically stable set.*

Summary: the Random matching

- SPE is the unique asymptotically stable point.
- Fair behaviors do not survived.

Why are the fair behaviors observed?

→**Assortative matching**.

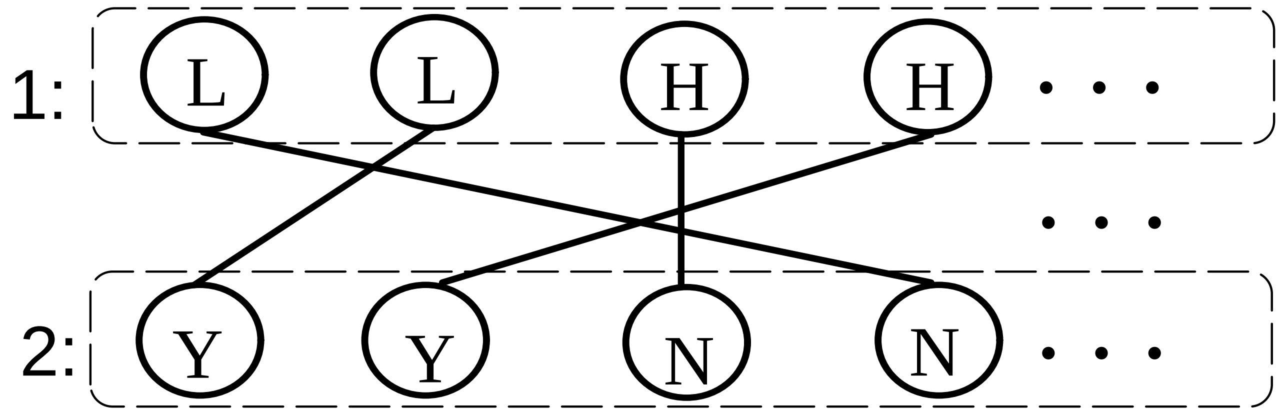
The Assortative Matching Rule

Matchings are easy to be made between same type players.

$$\frac{\partial p_i^A}{\partial x_i} \leq 0\ ,\ \ \frac{\partial q_i^A}{\partial x_i} \geq 0$$

$$p_i^A, q_i^A > p_i^R, q_i^R \forall i = 1, 2$$

$\rightarrow$ The matching rates also depend on their own strategy.



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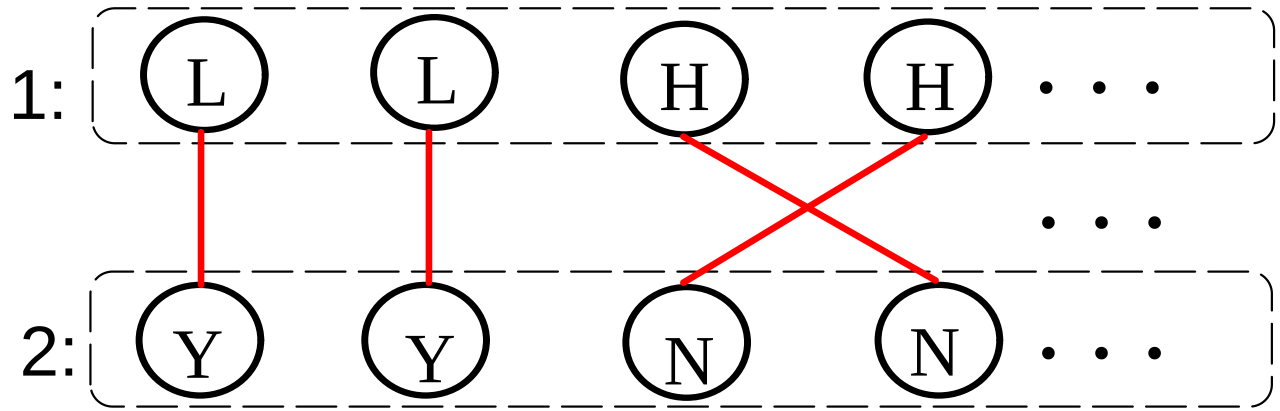


Related Literature 2

- Bergstrom (2003)
  - Prisoner's dilemma with assortative matching
  - (Cooperate, Cooperate) is also asymptotically stable
- Taylor and Nowak (2006)
  - 2×2 symmetric strategic form game with assortative matching

An Example

$$\epsilon \leq x_1, x_2 \leq 1 - \epsilon$$

$$p_1 = \begin{cases} \frac{x_2}{x_1} & \text{if } x_1 \geq x_2 \\ 1 & \text{otherwise} \end{cases} \quad q_1 = \begin{cases} \frac{1-x_2}{1-x_1} & \text{if } x_1 < x_2 \\ 1 & \text{otherwise} \end{cases}$$

$$p_2 = \begin{cases} \frac{x_1}{x_2} & \text{if } x_1 < x_2 \\ 1 & \text{otherwise} \end{cases} \quad q_2 = \begin{cases} \frac{1-x_1}{1-x_2} & \text{if } x_1 \geq x_2 \\ 1 & \text{otherwise} \end{cases}$$

This rule maximizes #(same type pair).

An Example: $p_1 = Pr(L \text{ meets } Y)$

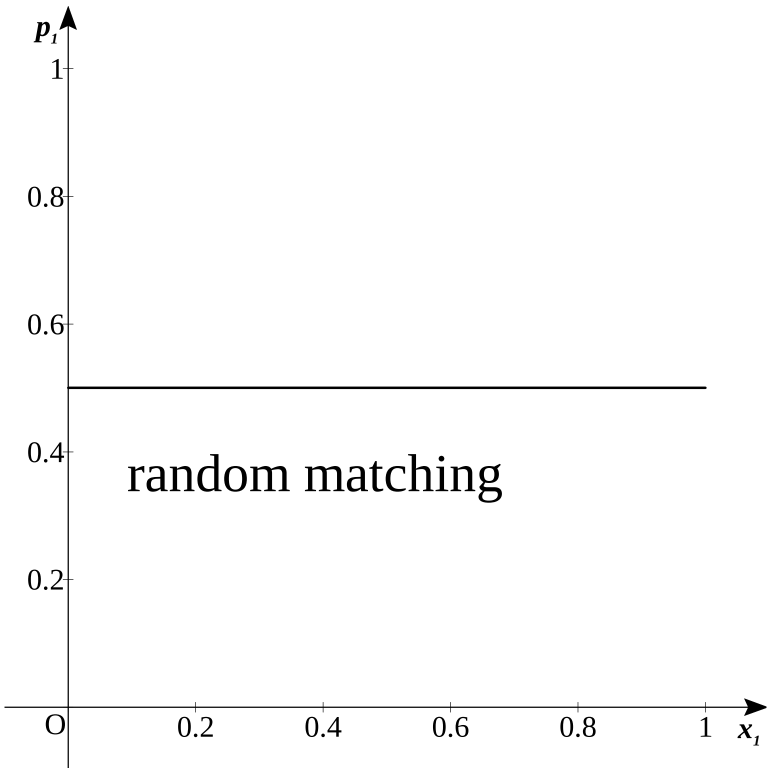


Figure 4: $p_i(x_i, 0.5)$

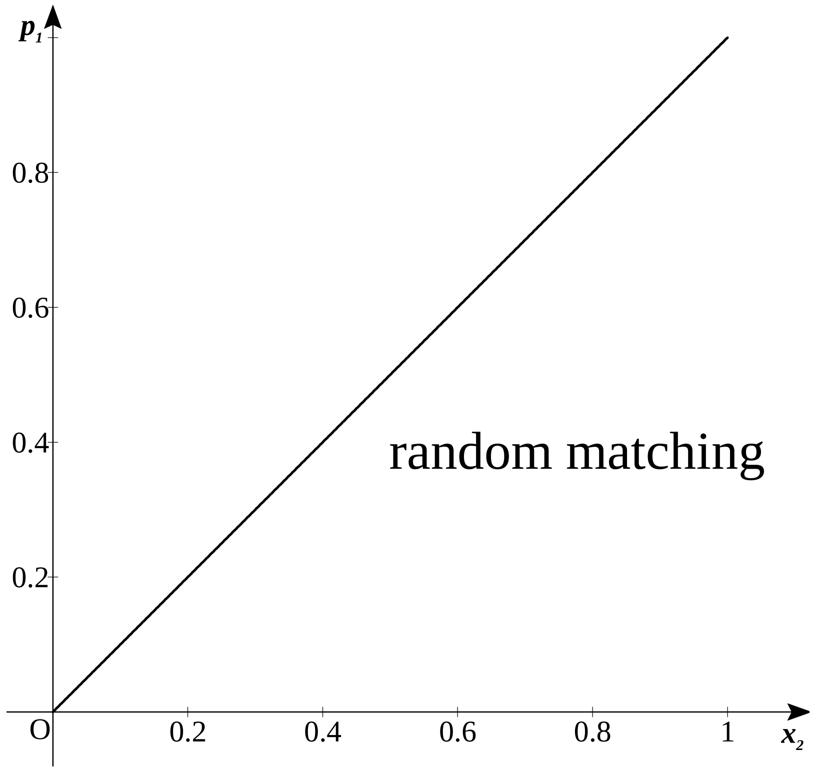


Figure 5: $p_i(0.5, x_j)$

An Example: $p_1 = Pr(L \textbf{ meets } Y)$

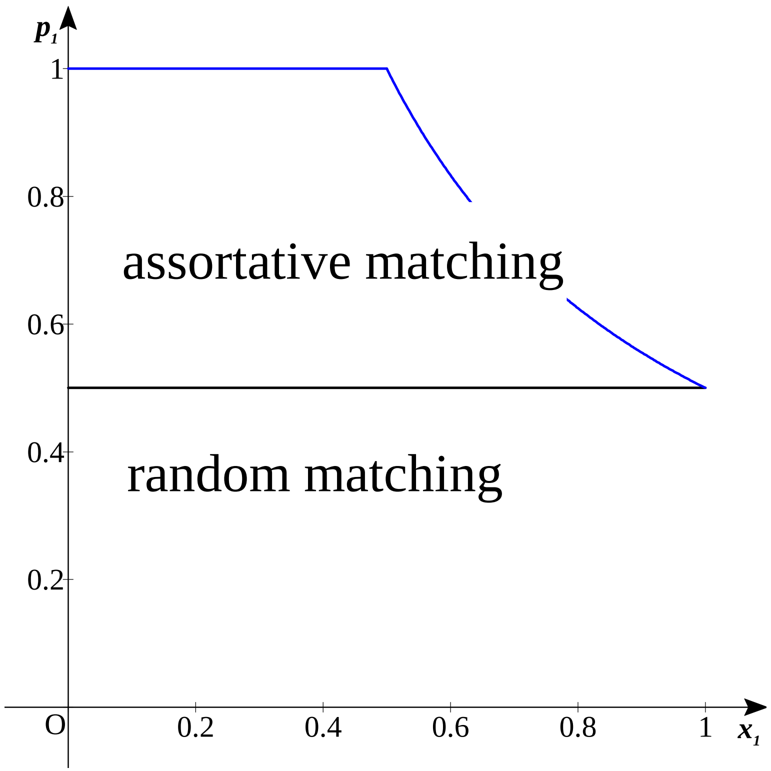


Figure 6: $p_i(x_i, 0.5)$

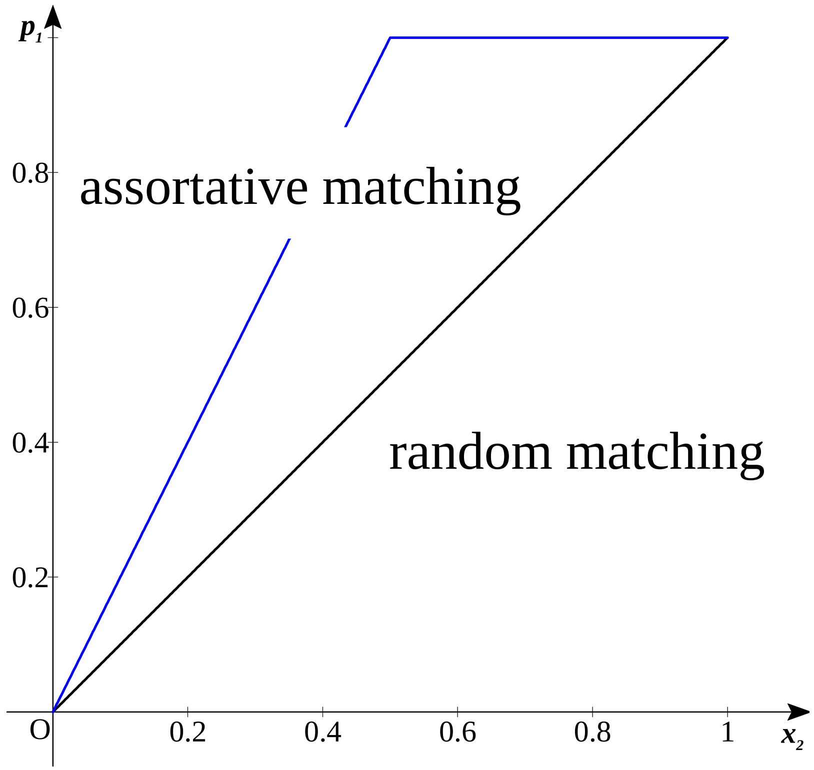


Figure 7: $p_i(0.5, x_j)$

The Dynamics

$\dot{x_1} = (x_1 - \epsilon)(1 - \epsilon - x_1)(3p_1 - 2)$
$\dot{x_2} = (x_2 - \epsilon)(1 - \epsilon - x_2)(2 - p_2 - 2q_2)$

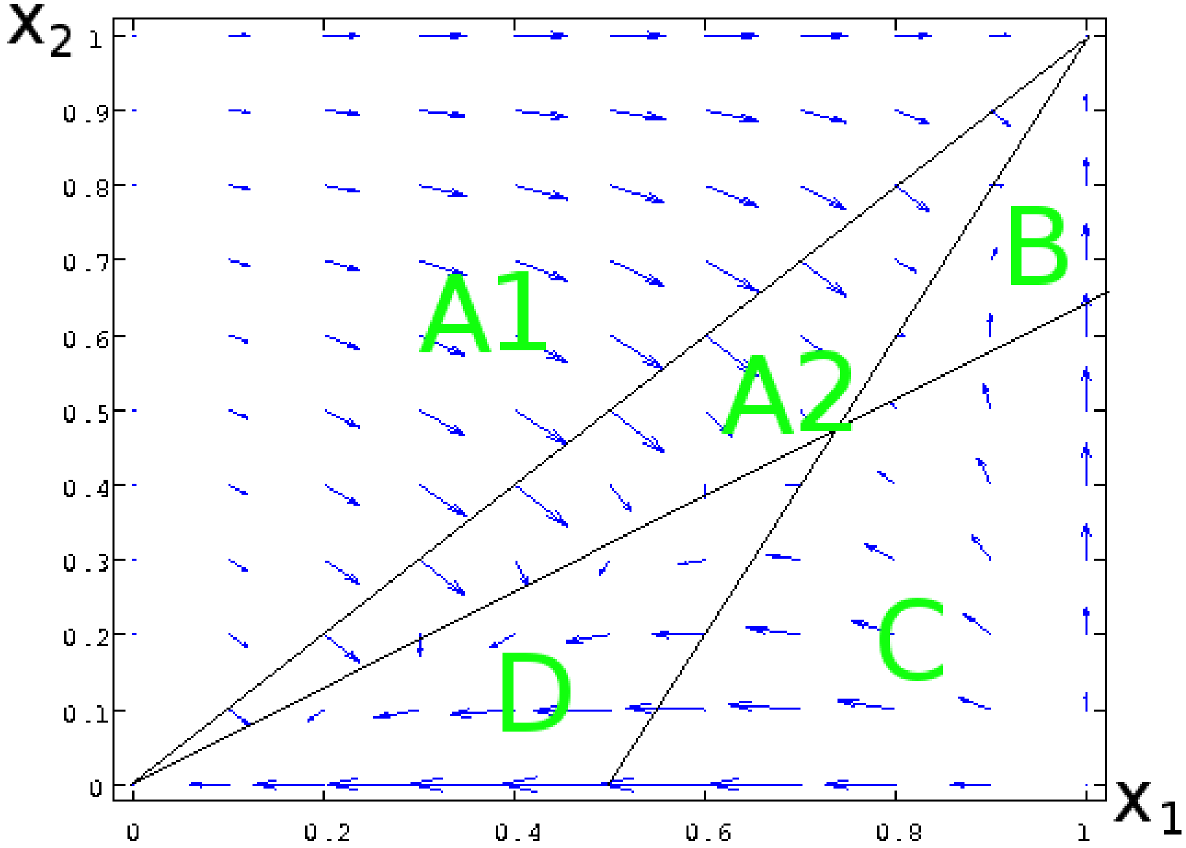
if $x_1 < x_2$

$$\dot{x_1} = (x_1 - \epsilon)(1 - \epsilon - x_1)(3 - 2) \quad (1)$$
$$\dot{x_2} = (x_2 - \epsilon)(1 - \epsilon - x_2)(-\frac{x_1}{x_2}) \quad (2)$$

if $x_1 \geq x_2$

$$\dot{x_1} = (x_1 - \epsilon)(1 - \epsilon - x_1)(3\frac{x_2}{x_1} - 2) \quad (3)$$
$$\dot{x_2} = (x_2 - \epsilon)(1 - \epsilon - x_2)(1 - 2\frac{1 - x_1}{1 - x_2}) \quad (4)$$

The Phase Diagram



The Dynamics when $x_1 < x_2$

A1: for all (x_1, x_2): $x_1 \uparrow$, $x_2 \downarrow$

$$
\begin{aligned}
p_1 &= Pr(L \text{ meets } Y) = 1 \rightarrow \; f_L > f_H \\
p_2 &= Pr(Y \text{ meets } L) > 0, \\
q_2 &= Pr(N \text{ meets } H) = 1 \rightarrow \; f_Y < f_N
\end{aligned}
$$

1 \ 2	Y	N
L	$3, 1$	$0, 0$
H	$2, 2$	$2, 2$

The Dynamics when $x_1 \geq x_2$

A2: $p_1 > \frac{2}{3}$, $q_2 > \frac{1}{2}$: $x_1 \uparrow$, $x_2 \downarrow$

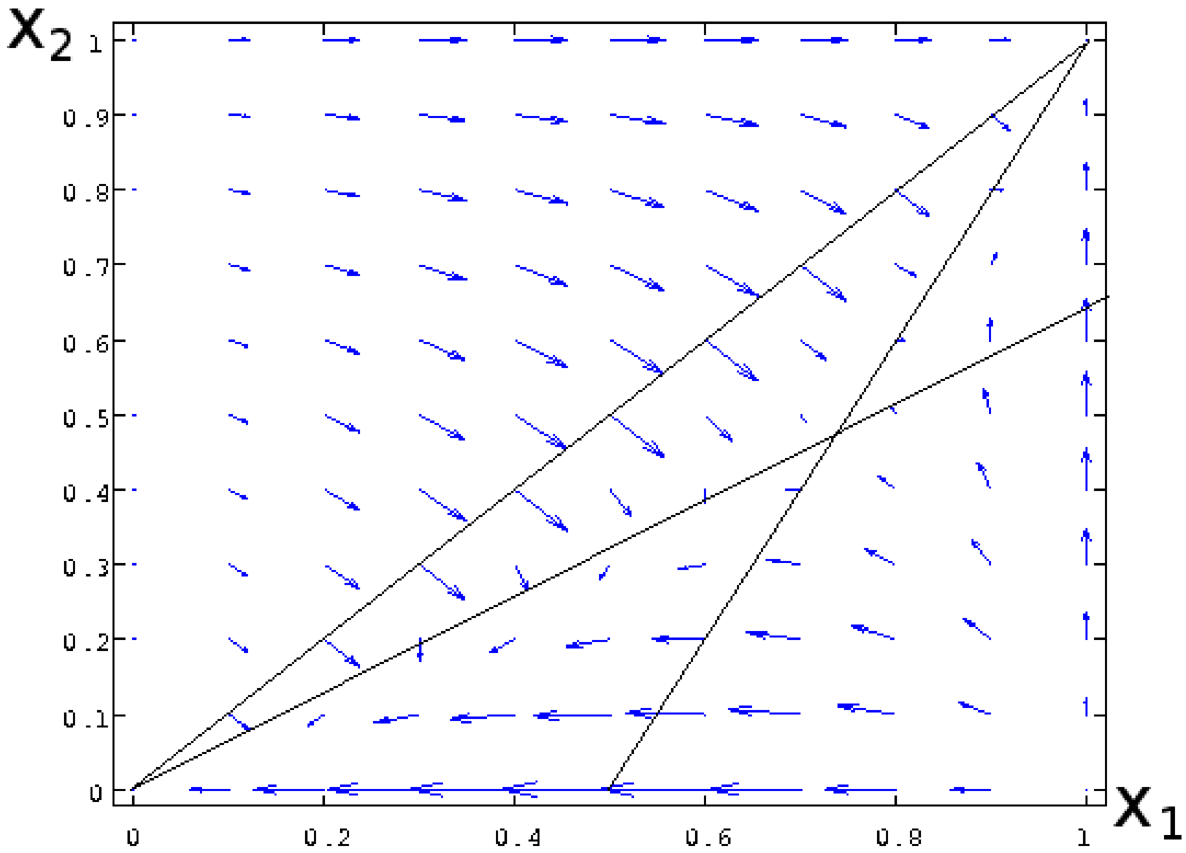
B: $p_1 > \frac{2}{3}$, $q_2 \leq \frac{1}{2}$: $x_1 \uparrow$, $x_2 \uparrow$

C: $p_1 \leq \frac{2}{3}$, $q_2 > \frac{1}{2}$: $x_1 \downarrow$, $x_2 \downarrow$

D: $p_1 \leq \frac{2}{3}$, $q_2 \leq \frac{1}{2}$: $x_1 \downarrow$, $x_2 \uparrow$

$$\begin{aligned} p_1 &= \frac{x_2}{x_1} \geq \frac{2}{3} \rightarrow f_L \geq f_H \\ p_2 &= 1 \\ q_2 &= \frac{1-x_1}{1-x_2} \geq \frac{1}{2} \rightarrow f_Y \leq f_N \end{aligned}$$

The Phase Diagram



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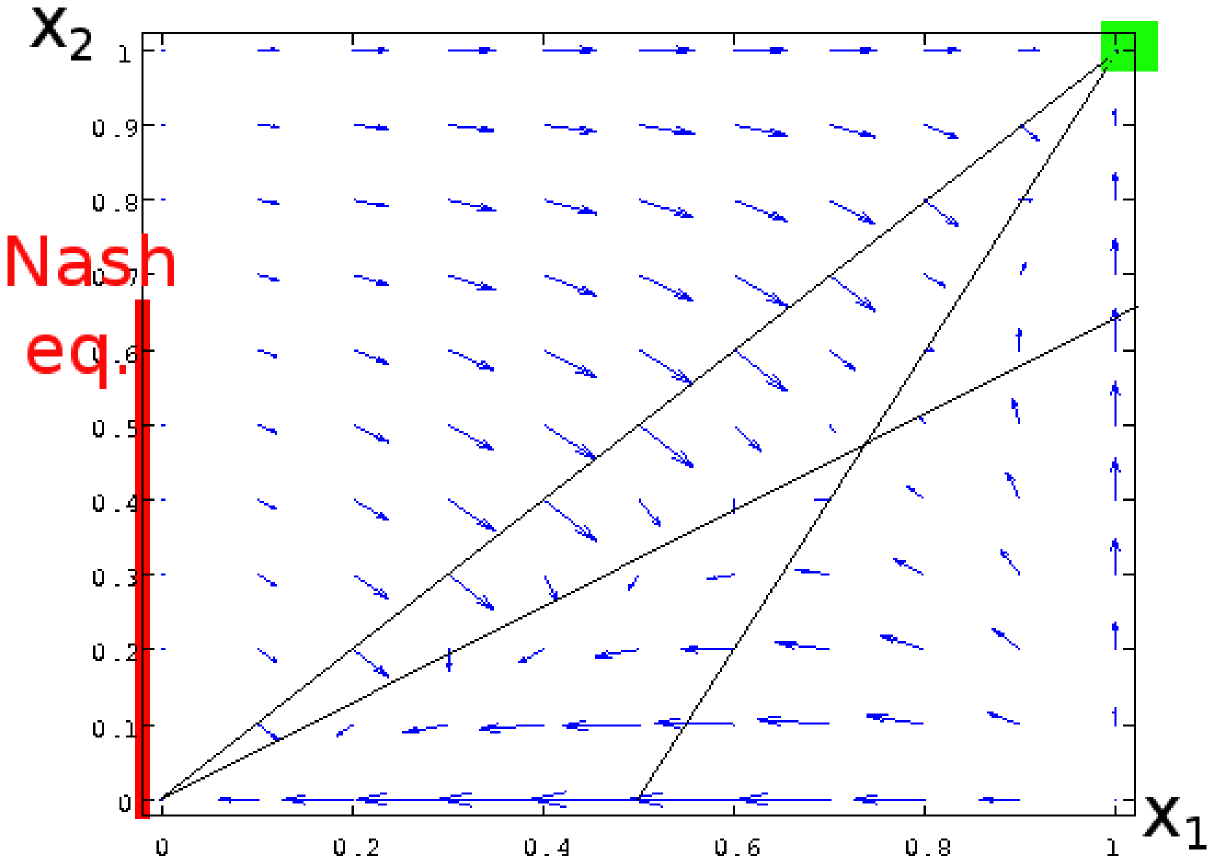

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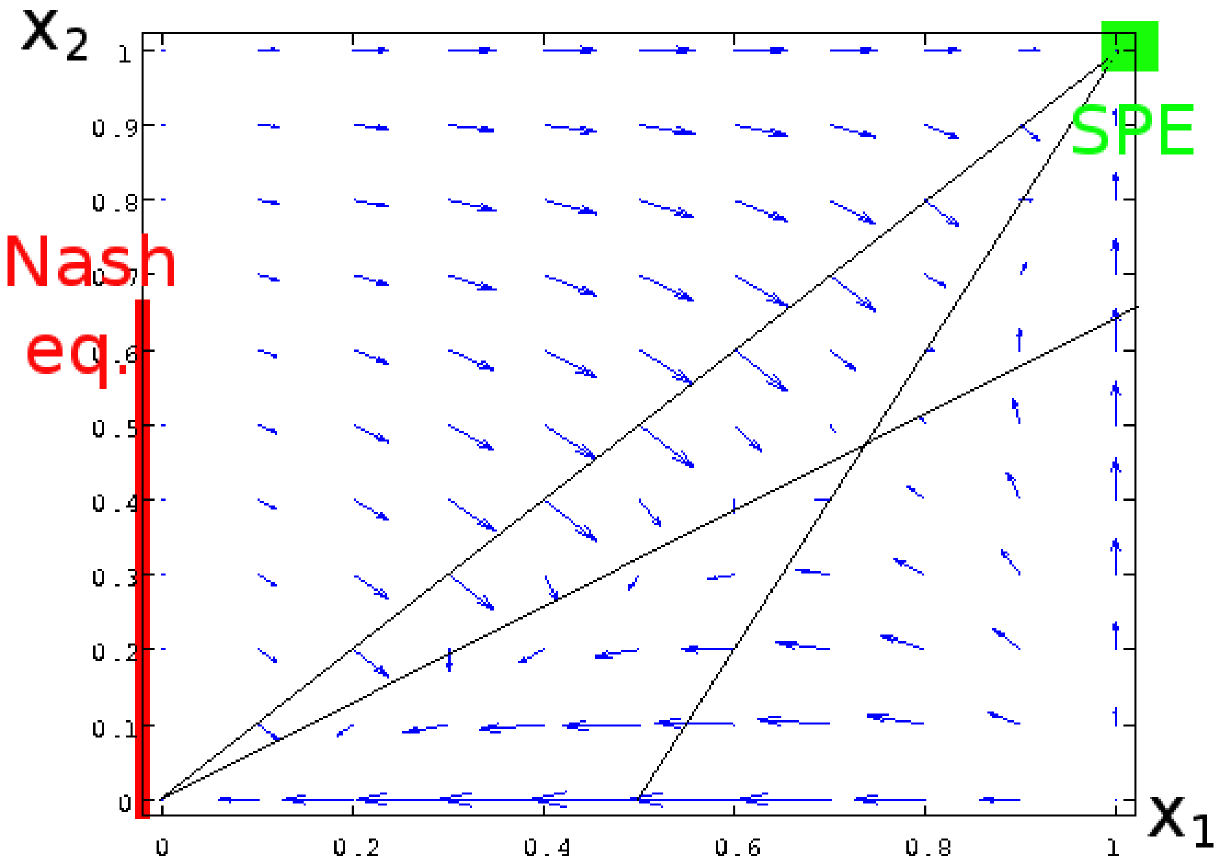

The Result of the example

Proposition 1. $(3\epsilon/2, \epsilon)$ *and* $(1-\epsilon, 1-2\epsilon)$ *are the asymptotically stable of the system (1)- (4)*

As $\epsilon \to 0,$

$$(x_1, x_2) \to (0,0), (1,1)$$

On the eq.,

fair actions (H,N) survive.

(selfish actions (L,Y) also survive.)

The Result

Consider all assortative matching rules.

Proposition 2. *If* $\epsilon \simeq 0$,

$$(x_1, x_2) = (1,1), (0,c) \;\; (0 \leq c < 2/3)$$

are asymptotically stable under some Assortative matching rules.

The set of Nash eq. is $x = (1,1), (0,c) \;\; (0 \leq c \leq 2/3)$

Thus, except $x = (0, 2/3)$, **each Nash eq. is asymptotically stable under the Assortative matching rule.**

Conclusion

In Ultimatum mini game,

- under the Random matching rule, SPE is the only asymptotically stable point.
- However, under the Assortative matching rule, each Nash eq. is asymptotically stable.
- In example, especially, the state all people act fairly is asymptotically stable.

→People behave fairly to maximize their own payoff, if matchings are assortative

References

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