# Evolutionary phenomena of indirect reciprocity in *n*-person games

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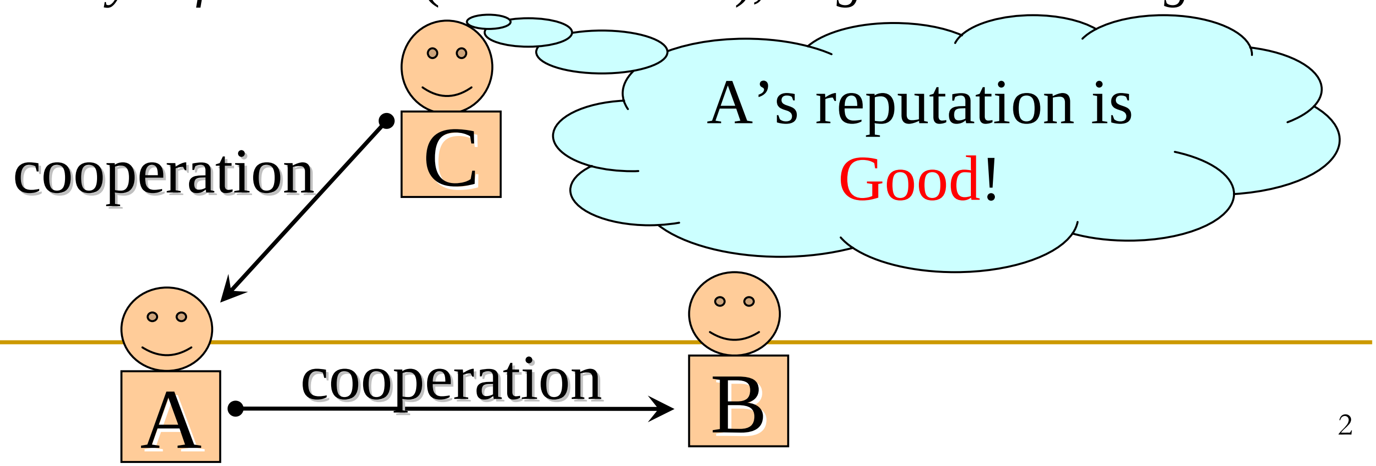
# Evolution of cooperation

- Kin-selection (Hamilton 1964)
- Multi-level selection (Wilson & Sober 1999)
- Reciprocity (Trivers 1971)
  - Direct reciprocity (Axelrod & Hamilton 1984)
  - *Indirect reciprocity* (Nowak & Sigmund 1998)
    - Referring reputation, a reciprocator can know which partners are cooperative and can reciprocate with a cooperator.
      - *Community enforcement* (Kandori 1992), or *general exchange*.



# Mathematical model of indirect reciprocity

- Nowak & Sigmund(1998):
  - The same individuals interact only a few times.
    - The individuals play a *2-person prisoner's dilemma* (or *giving*) game.
  - Image scoring is adopted as a reputation criterion.
    - Cooperation → *G* (good), Defection → *B* (bad)
  - *DIS* (*discriminating*) strategy: cooperates only with the opponents who have *good* reputation.

  → *DIS* can form indirectly reciprocal cooperation.

# Related issues

- Image scoring is not sufficient for the evolution of indirect reciprocity (e.g. Panchanathan & Boyd 2003, Ohtsuki & Iwasa 2007, Takahashi & Mashima 2006, Kandori 1992).
- What if in *n*-person games? (*n*>2)

Ostrom et al., 1999

Dugatkin, 1990

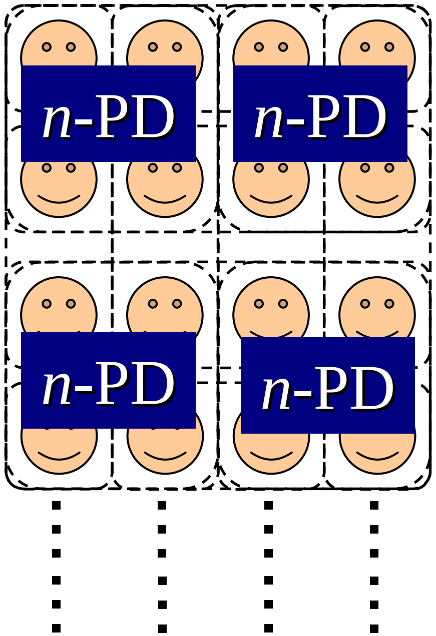
→We investigate the case of *n*-person games.

- Suzuki & Akiyama 2005, 2007

# Summary of the results

- Indirect reciprocity can be an *ESS* in *n*-person games under *image scoring*.
  - In 2-person games, indirect reciprocity can not be an ESS (Panchanathan & Boyd 2003, Ohtsuki & Iwasa 2005).
- Indirect reciprocal cooperation can be maintained as (sometimes chaotic) oscillation under *image scoring*.
  - As mutation rate increases, evolutionary dynamics change: convergence to a fixed point → oscillation → *chaotic* oscillation → oscillation → convergence.

# Overview of the model




Groups are reformed randomly in each round.

Round

Generation

① Consider a population consisting of infinite number of individuals.

② All individuals are divided into groups consisting of *n*-individuals.

③ They play an *n*-person prisoner's dilemma game in each group.

④ Each individual is assigned her reputation based on her action.

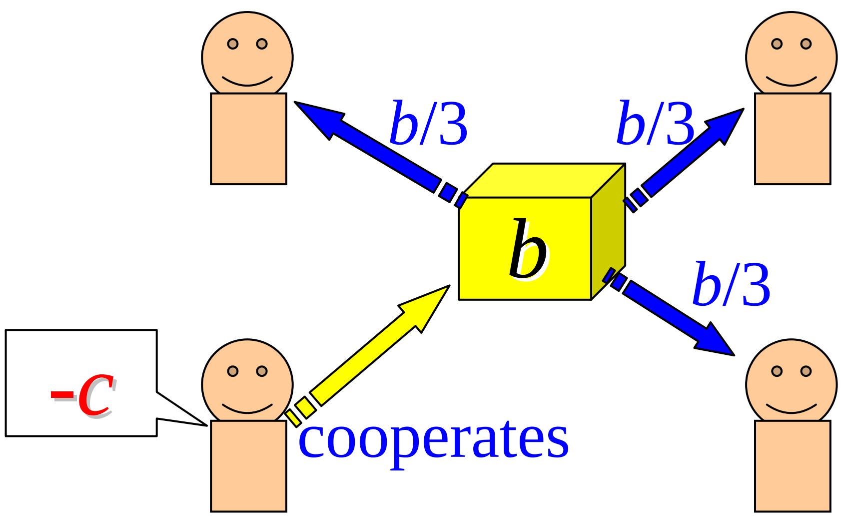
⑤ A round (②-④) is repeated.

⑥ Each individual leave her offspring depending on her fitness (*natural selection*).

⑦ A generation (②-⑥) is repeated.

# $n$-person prisoner's dilemma game

- Payoff for a cooperator: $bk/(n-1)-c$
- Payoff for a defector: $bk/(n-1)$
  - $c$: cost of cooperation, $b$: benefit of cooperation, $k$: the number of opponents cooperating in the group.



# Other settings

- The number of rounds in a generation:
  - After the first round, each of the subsequent rounds occurs with probability ***w*** ($0 < w < 1$).

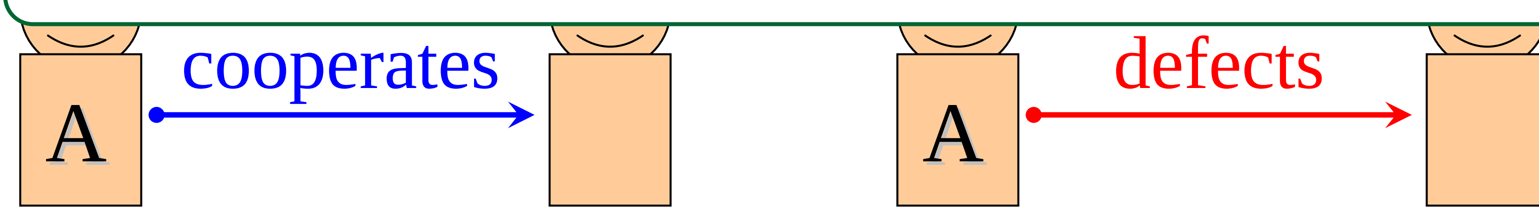
  → The expected value of the number of rounds in a generation is $1/(1-w)$.
- Implementation error (action noise):
  - With the small probability $\varepsilon$, an individual who intends to cooperate fails to cooperate.

# Reputation criterion

Bshary 2001, 2006

- Image scoring (Nowak & Sigmund 1998):
  - At the first round, all individuals have Good reputation.
  - Cooperation → Good.
  - Defection → Bad.

This requires only the past action of the opponent.



# Strategies (*n*-person games)

- Each individual decides her own action based on the number of *Good*-opponents in the group.
- The strategies are represented as an *n*-dimensional binary vector $\in \{0,1\}^n$.
  - e.g., in 4-person game, (0,1,1,0).
  - 0: defection, 1: cooperation.

→ There exist $2^n$ strategies in total.

# Important strategies in 4-person games

Action when the number of *G*-opponents is 0.
the number of *G*-opponents is 1.
the number of *G*-opponents is 2.
the number of *G*-opponents is 3.

- $S_0$=(0,0,0,0) called *ALLD* *always* defects.
- $S_1$=(0,0,0,1) called *strictDIS* cooperates only when *all* the opponents are *Good*.
- $S_3$=(0,0,1,1) called *generousDIS1* cooperates only when *at least two* opponents are *Good*.
- $S_7$=(0,1,1,1) called *generousDIS2* cooperates only when *at least one* opponents is *Good*.
- $S_{15}$=(1,1,1,1) called *ALLC* *always* cooperates.

→ $2^4$=16 strategies in total.

# Fitness for the strategies

- Share of the strategies: $\boldsymbol{x} = (x_0, \ldots, x_{2^n-1})$.
- Fitness for strategy $S_i$ is $f_i(\boldsymbol{x})$,
  - which is defined as an average total payoff during the generation.
- Replicator dynamics:

$$x_i(t+1) = x_i(t) \frac{\overbrace{f_i(\mathbf{x})}^{\text{Fitness for strategy } i}}{\underbrace{\sum_{j=0}^{15} x_j(t) f_j(\mathbf{x})}_{\text{Average fitness over the population}}}$$

In addition, we consider the effect of *mutation*.

# Mutation

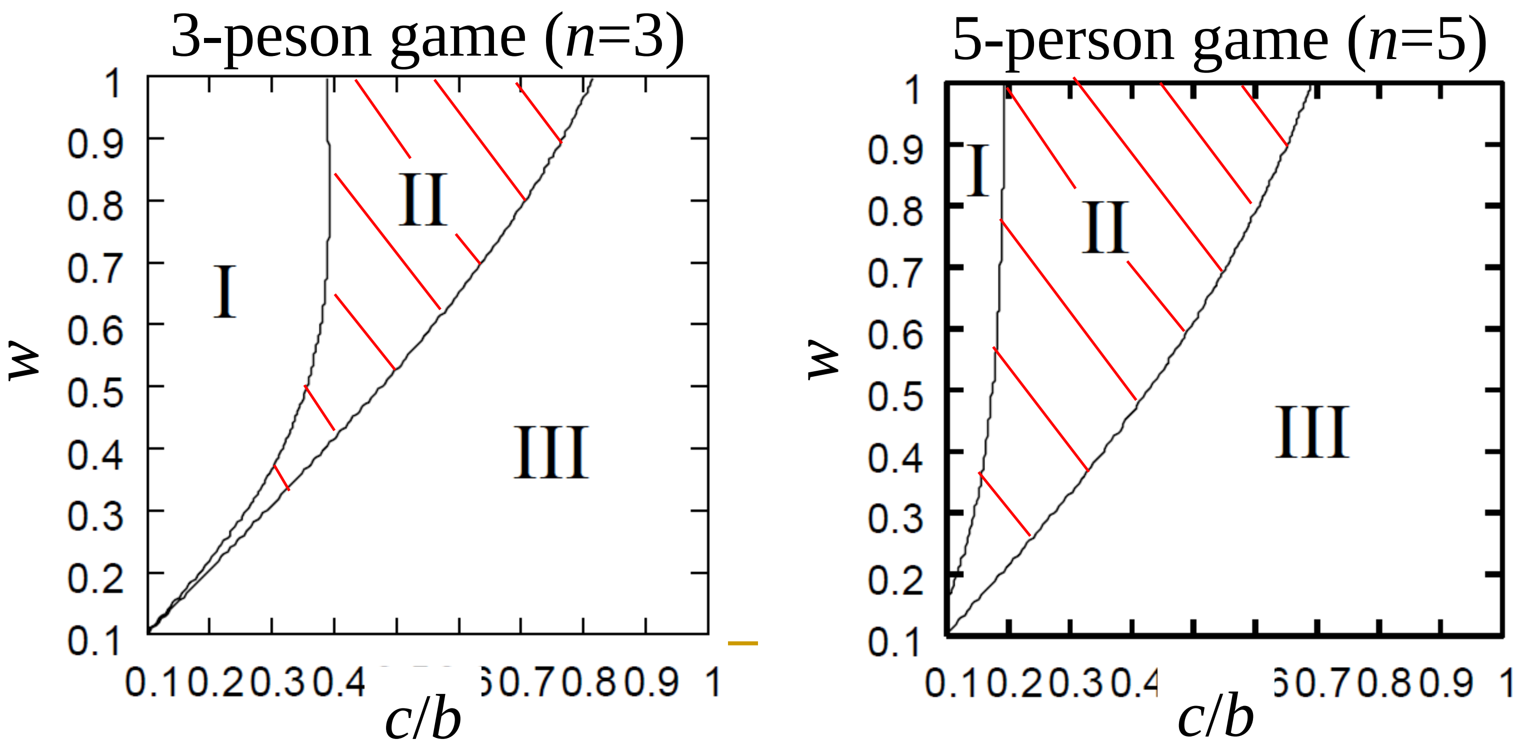
- Probability that one bit of a strategy vector inverts is denoted as $\mu$. That is,

| | |
|---|---|
| (0011) ↓ the prob is $\mu$. (0010) | (0011) ↓ the prob is $\mu^2$. (1010) |
| (0011) ↓ the prob is $\mu^3$. (1110) | (0011) ↓ the prob is $\mu^4$. (1100) |

- Replicator-mutator dynamics: $x_i(t+1) = \dfrac{\sum_{j=0}^{15} x_j(t) f_j(\mathbf{x}) q_{ji}}{\sum_{j=0}^{15} x_j(t) f_j(\mathbf{x})}$
  - $q_{ji}$: the probability that mutation of strategy $j$ give rise to strategy $i$.

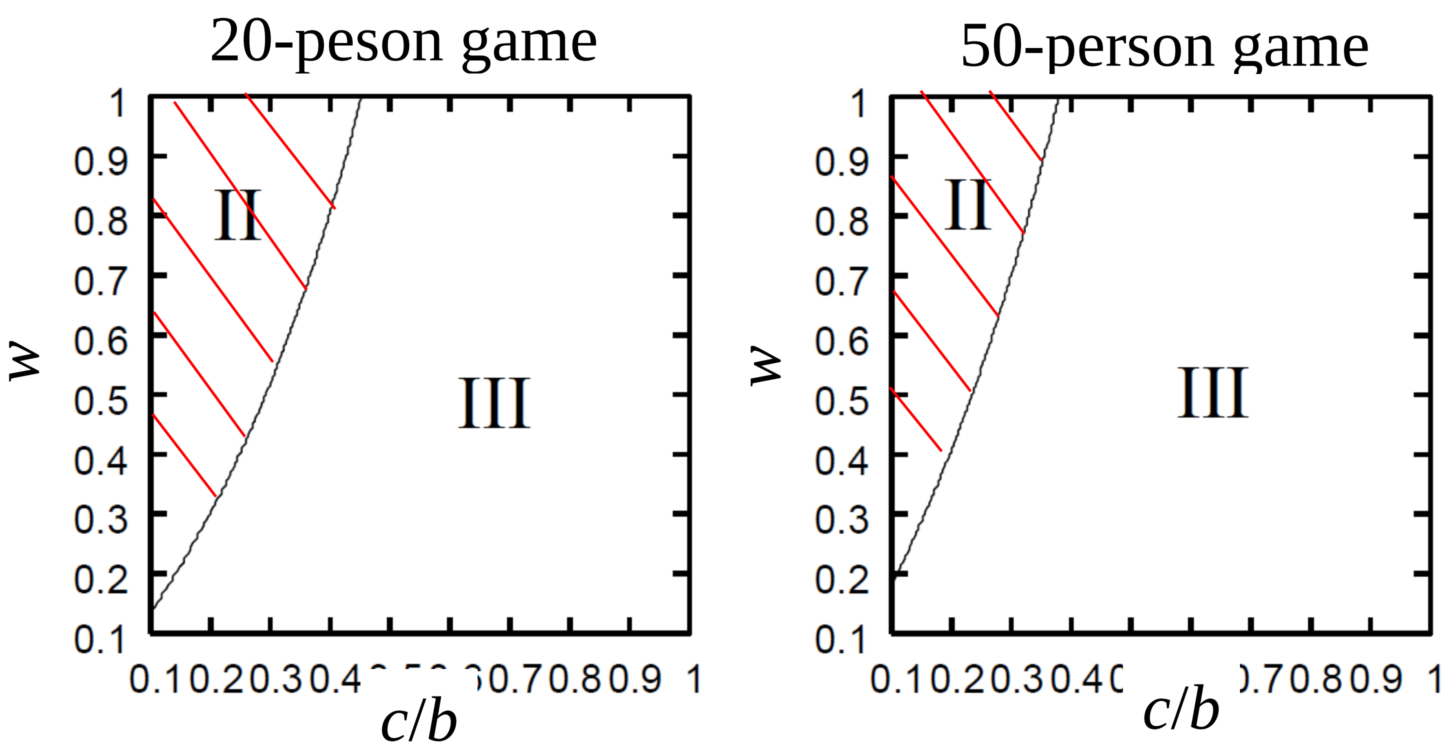
# Evolutionary stability of *Discriminator*

- (0,…,0,1) called *strictDIS* can be an ESS.
  - Region I: *strictDIS* can be invaded by cooperative strategies.
  - Region II: ***strictDIS* is an ESS.**
  - Region III: *strictDIS* can be invaded by defective strategies.



# Evolutionary stability of *Discriminator*

- When $n$ is sufficiently large,
  - *strict DIS* is an ESS, if $c/b < w\hat{\varepsilon}^{n-1} / \left(1 + w\hat{\varepsilon}^{n-1}\right)$.
    - Region II: *strict DIS* is an ESS.
    - Region III: *strict DIS* can be invaded by defective strategies.



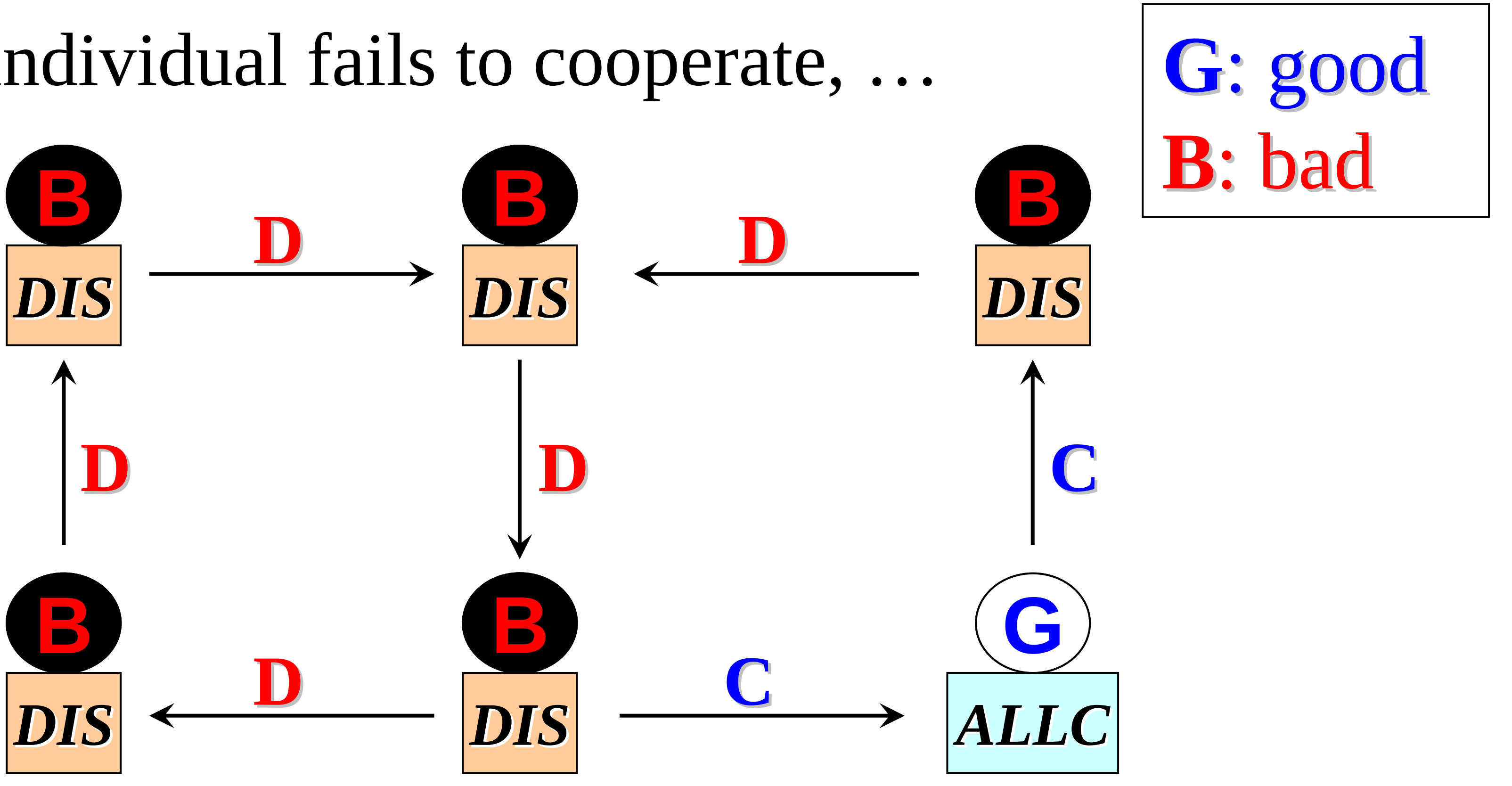
# Evolutionary stability of *Discriminator*

- In *n*-person games, *Discriminating* strategy can be an ESS under image scoring.
  - On the other hand, it cannot be an ESS in 2-person games (e.g. Panchanathan & Boyd 2003, Ohtsuki & Iwasa 2007).
- Why?

→ The mechanism for this is as follows…

# 2-person games

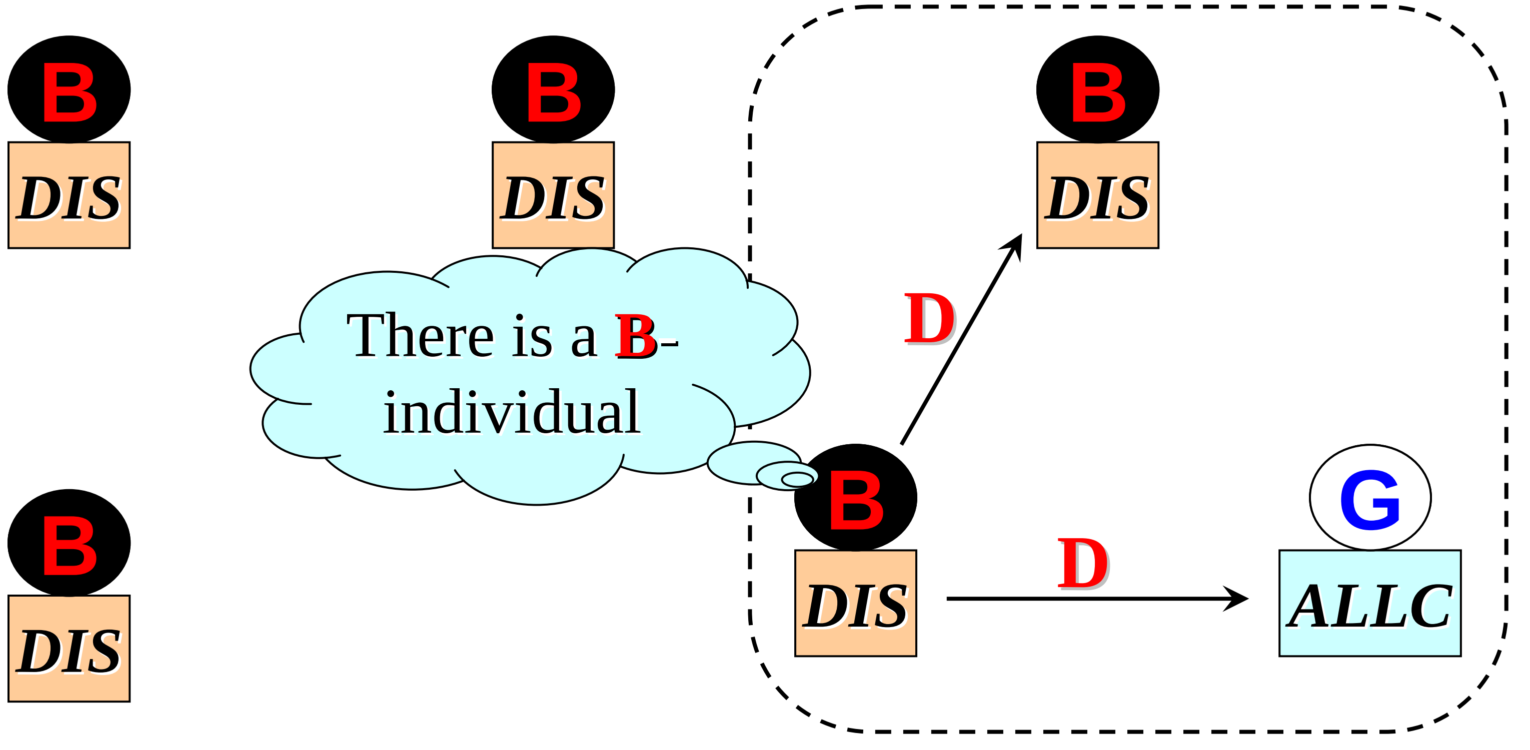
- If an individual fails to cooperate, …




- Only *ALLC* is not drawn into the chain of the retaliative defections. → *ALLC* can invade!

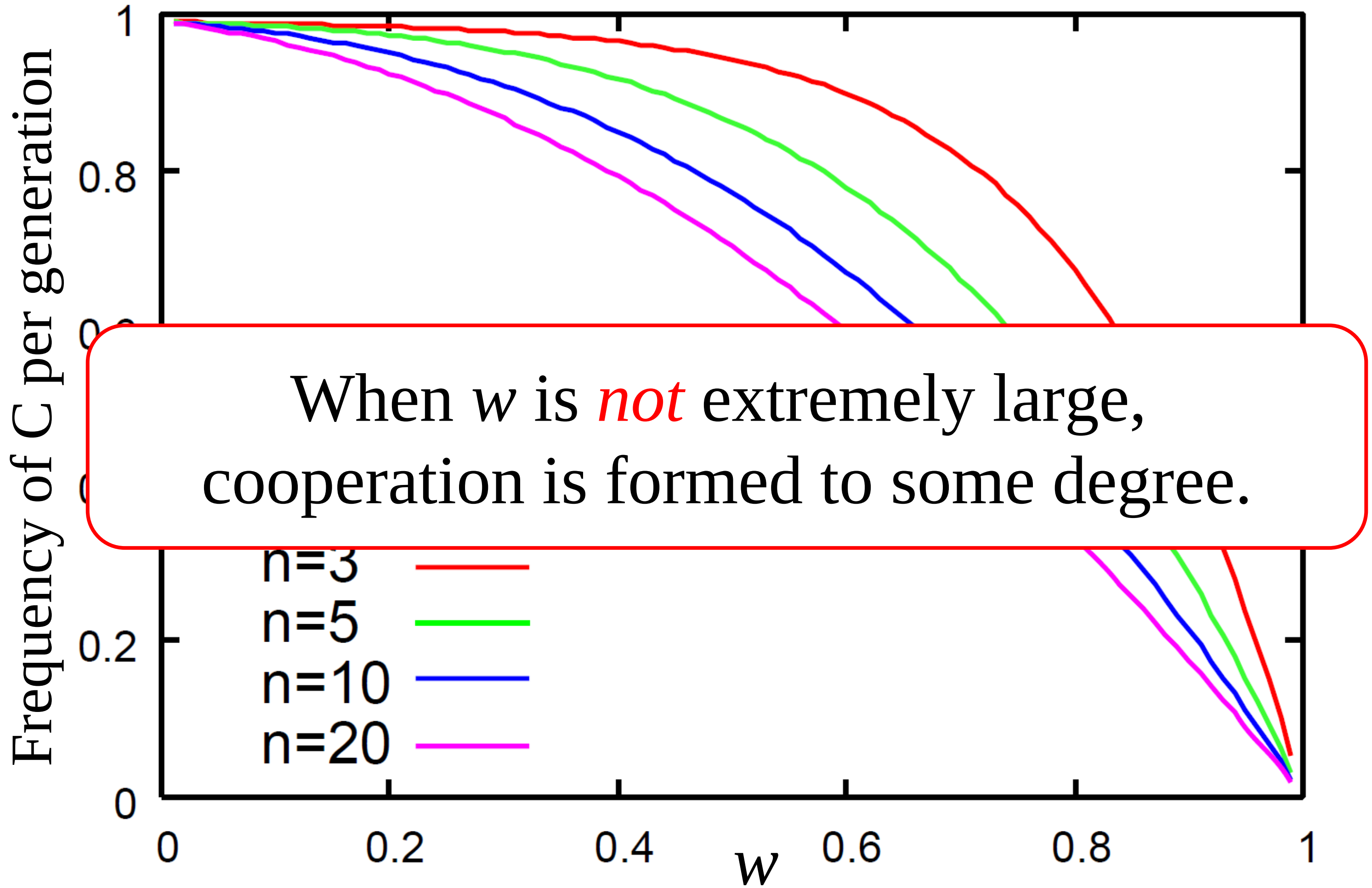
# 3-person game

- If an individual fails to cooperate, …




- *ALLC* cannot avoid being drawn into the chain of the defections. → *ALLC* cannot invade!

# Can *strict DIS* form cooperation?




*The frequency of cooperation does not depend on the benefit, *b*, and the cost, *c*, of cooperation.

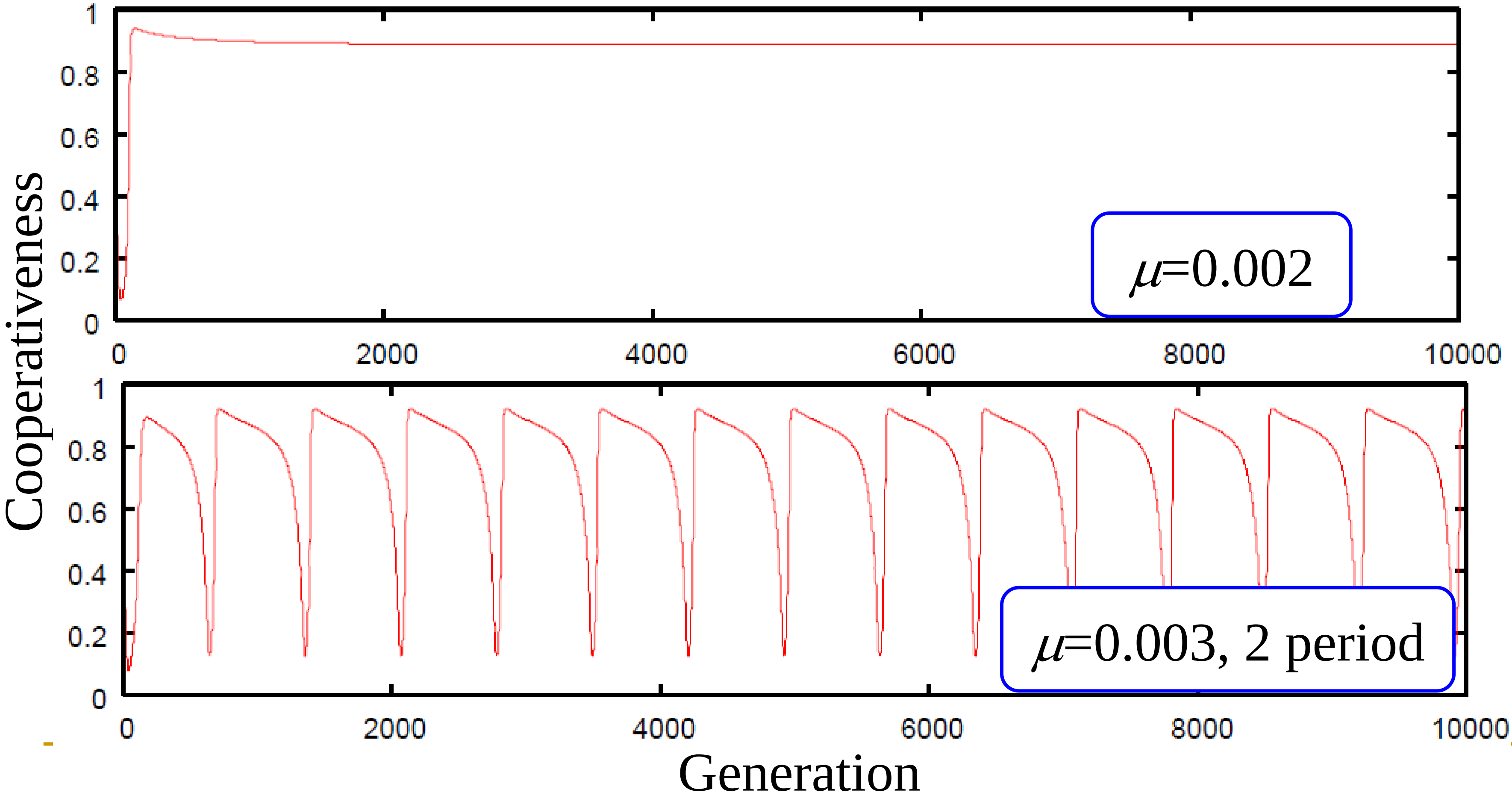
# Evolutionary dynamics

- Numerical simulation of replicator-mutator equation:
  - 4-person game ($n$ = 4).
  - The cost-to-benefit ratio of cooperation, $c/b$ = 1/12.
  - The probability that each of the subsequent rounds occurs, $w$ = 0.9.
  - Noise rate, $\varepsilon$ = 0.01.

  → at which no pure strategy is an ESS.
  - Mutation rate, $\mu$ = 0.002, 0.003, 0.006, 0.008, 0.009, 0.010, or 0.011 (to compare Nowak & Sigmund 1993).

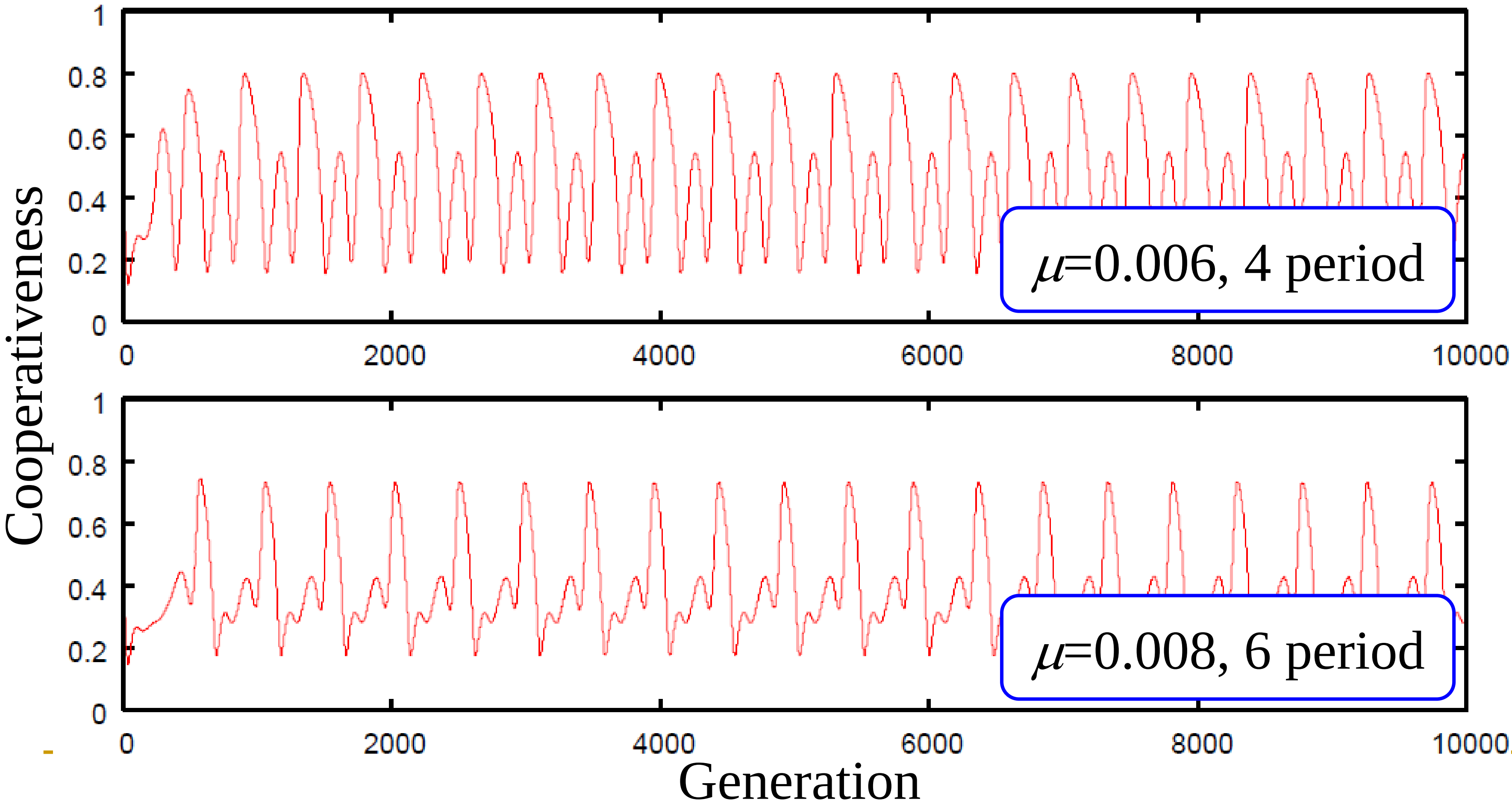
# Bifurcation of the evolutionary dynamics as mutation rate increases

- Mutation rate, $\mu$ = 0.002, 0.003.



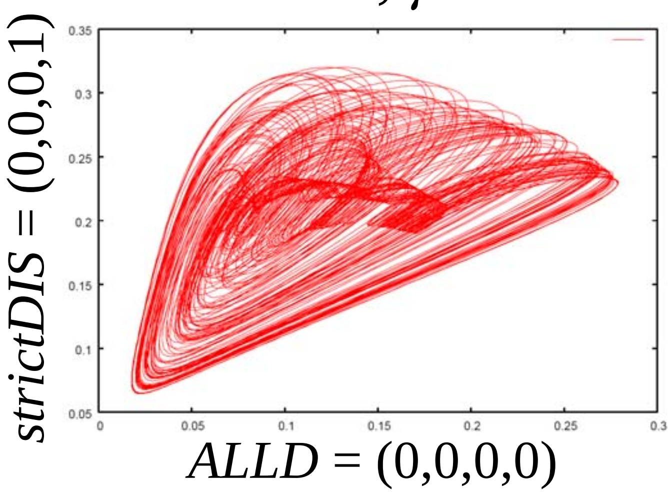
# Bifurcation of the evolutionary dynamics as mutation rate increases

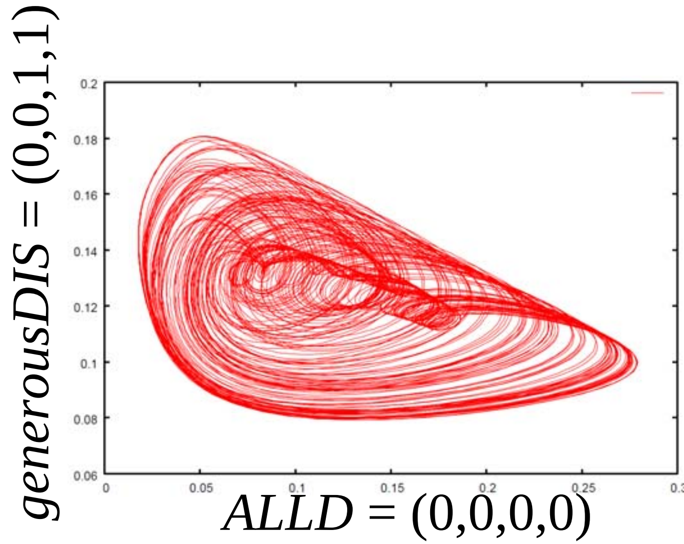
- Mutation rate, $\mu$ = 0.006, 0.008.

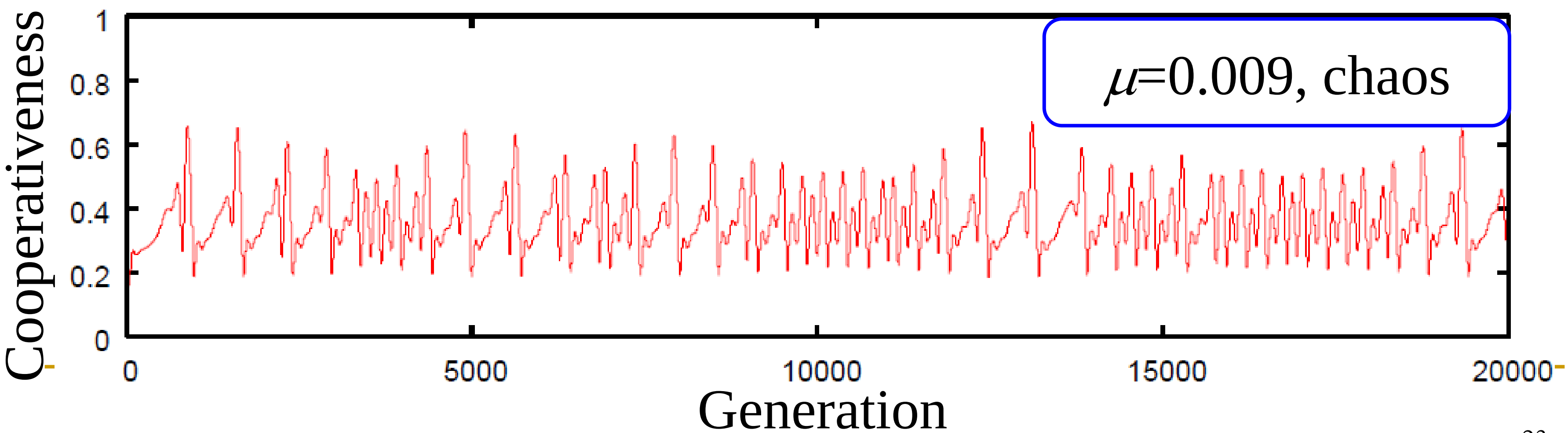


# Bifurcation of the evolutionary dynamics as mutation rate increases

- Mutation rate, $\mu$ = 0.009.

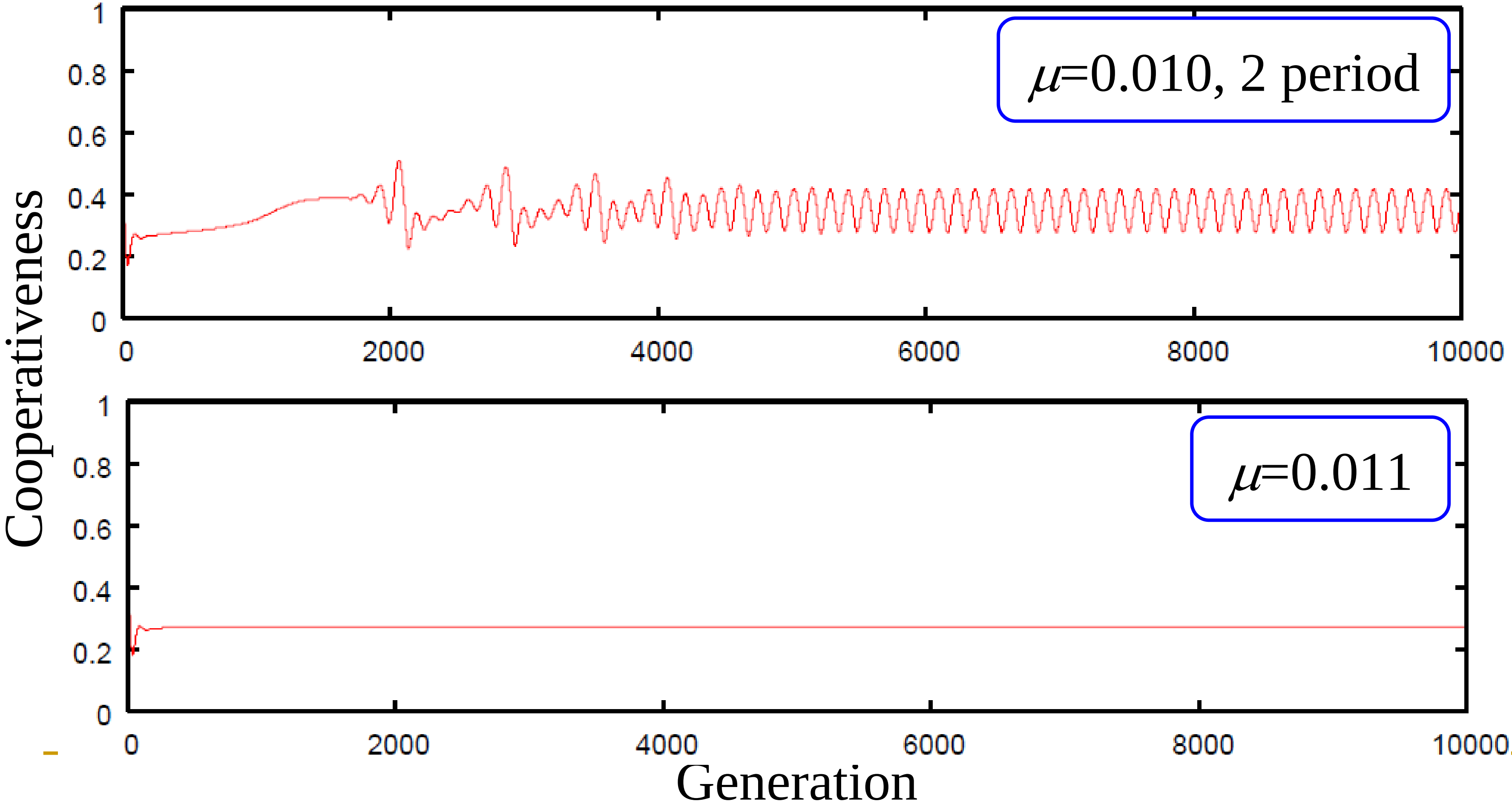


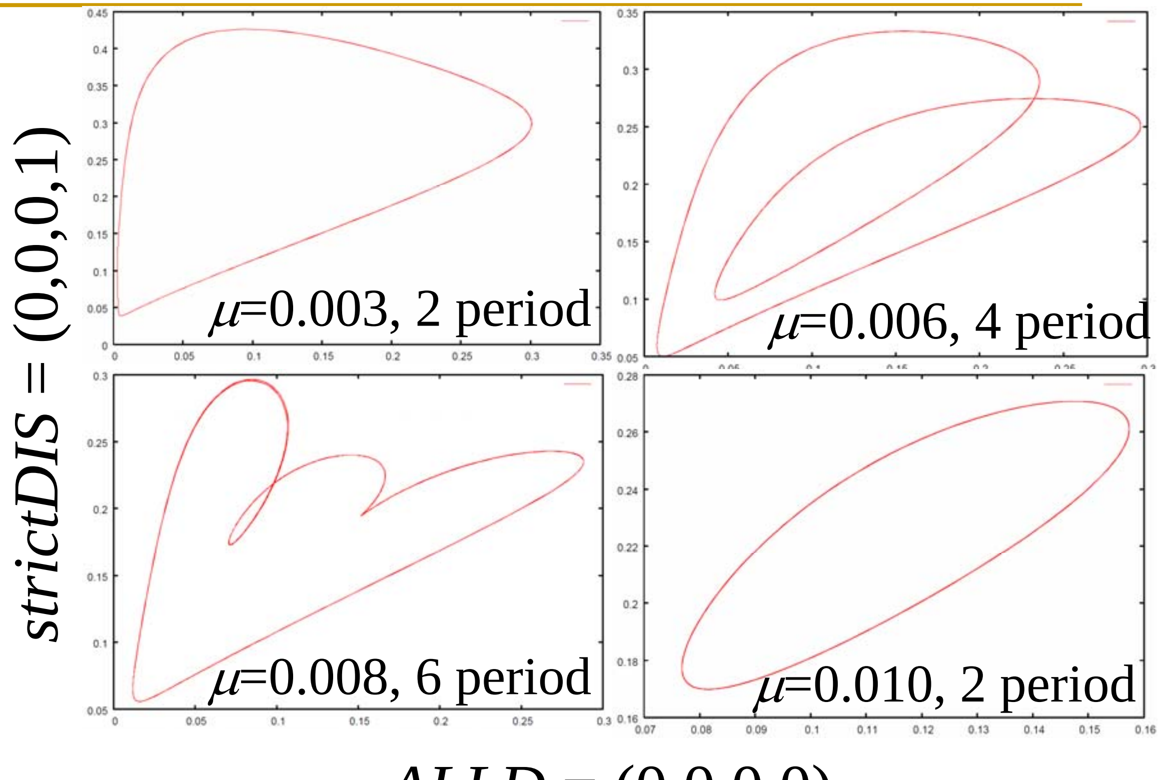



# Bifurcation of the evolutionary dynamics as mutation rate increases

- Mutation rate, $\mu$ = 0.010, 0.011.

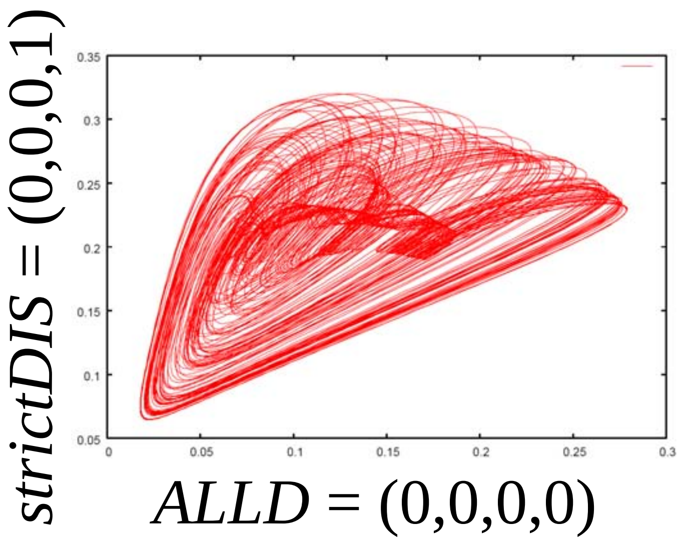


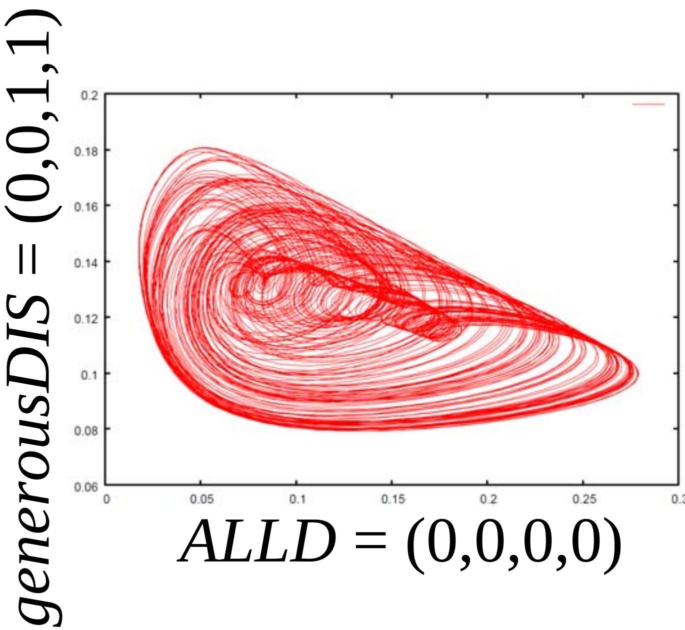
# Oscillation




- *ALLD*=(0,0,0,0) → *strictDIS* =(0,0,0,1) → *generousDIS* =(0,0,1,1) or (0,1,1,1), *ALLC* =(1,1,1,1) → *ALLD* =(0,0,0,0).
  - Unconditional defectors → strict reciprocators → generous reciprocators or unconditional cooperators → unconditional defectors.

# Chaotic oscillation




- Irregularly, *ALLD*=(0,0,0,0) → *strictDIS* =(0,0,0,1) → *generousDIS* =(0,0,1,1) or (0,1,1,1), (1,0,0,1), (0,1,0,1), *ALLC* =(1,1,1,1) → *ALLD*.

# Evolutionary dynamics

- Evolutionary dynamics change as mutation rate increases:
  - convergence to a fixed point → oscillation (2 period) → oscillation (4 period) → oscillation (6 period) → chaotic oscillation → oscillation (2 period) → convergence.
  - Oscillation:
    - *ALLD* → *strictDIS* → *generousDIS* or *ALLC* → *ALLD*.
  - Chaotic oscillation:
    - Irregularly, *ALLD* → *strictDIS* → *generousDIS*, (1,0,0,1), (0,1,0,1) or *ALLC* → *ALLD*.

# Evolutionary dynamics

- A similar transition of the evolutionary dynamics is observed in iterated 2-person prisoner's dilemma games (Nowak & Sigmund, 1993).

| | 2-person game | *n*-person game |
|---|---|---|
| Iterated PD (direct reciprocity) | Nowak & Sigmund (1993) | ? |

We conjecture that the transition of the dynamics is a common nature of the evolution of reciprocal cooperation.

# Conclusion

- Indirect reciprocity can be an ESS in *n*-person games under image scoring.
  - In 2-person games, indirect reciprocity can not be an ESS (Panchanathan & Boyd 2003, Ohtsuki & Iwasa 2005).
- Indirect reciprocal cooperation can be maintained as (sometimes chaotic) oscillation under image scoring.
  - As mutation rate increases, evolutionary dynamics change: convergence to a fixed point → oscillation → chaotic oscillation → oscillation → convergence.