Temptation, Certainty Effect, and Diminishing Self-Control

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Background

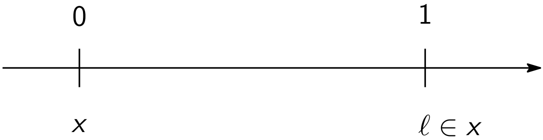
- Gul and Pesendorfer (2001) provide a theory of choice under temptation

Lunch Choice: Vegetable vs Hamburger

- temptation from hamburger: $\{v\} \succ \{v, h\}$
- self-control: $\{v\} \succ \{v, h\} \succ \{h\}$
- no self-control: $\{v\} \succ \{v, h\} \sim \{h\}$

Background

- C: set of outcomes
- $\Delta(C)$: set of lotteries over C
- $\mathcal{K}(\Delta(C))$: set of compact subsets (menus) of $\Delta(C)$
- $\succsim$ over $\mathcal{K}(\Delta(C))$



Background

Theorem (Gul and Pesendorfer, 2001)

$\succsim$ satisfies a set of axioms

$\Leftrightarrow$

There exist two expected utility functions $u, v : \Delta(C) \rightarrow \mathbb{R}$ such that $\succsim$ is represented by

$$U(x) = \max_{\ell \in x} \left\{ u(\ell) - \left(\max_{\ell' \in x} v(\ell') - v(\ell) \right) \right\}$$

Difference from Multiple Selves Approach

- No self-control:

$$v \succ^0 h,\ \ h \succ^1 v \ \Rightarrow\ \{v\} \succ \{v, h\} \sim \{h\}$$

Motivation

Axiom (Independence)

$$x \succ y \;\Rightarrow\; \lambda x + (1-\lambda)z \succ \lambda y + (1-\lambda)z$$

Definition (Mixture of Menus)

$$\lambda x + (1-\lambda)x' = \{\lambda \ell + (1-\lambda)\ell' \mid \ell \in x, \ell' \in x'\}$$

Motivation

- GP allow lotteries over outcomes
- "Certainty effect" implies what is certain is more tempting

Example (Certainty Effect)

$$\{(0,110)\} \succ \{(0,110),(100,0)\} \sim \{(100,0)\}$$

$$\left\{\left(0,110\tfrac{1}{2}0\right)\right\} \succ \left\{\left(0,110\tfrac{1}{2}0\right),\left(100\tfrac{1}{2}0,0\right)\right\} \succ \left\{\left(100\tfrac{1}{2}0,0\right)\right\}$$

- violates Independence

Motivation

- Ex post choice in GP model satisfies WARP:

$$\begin{aligned}
\mathcal{C}(x) &= \arg\max_{\ell\in x}\left\{u(\ell)-\left(\max_{\ell'\in x} v(\ell')-v(\ell)\right)\right\} \\
&= \arg\max_{\ell\in x}\left\{u(\ell)+v(\ell)-\max_{\ell'\in x} v(\ell')\right\} \\
&= \arg\max_{\ell\in x}\{u(\ell)+v(\ell)\}
\end{aligned}$$

- Dekel, Lipman and Rustichini (2006) and Noor(2006): Choice under temptation may violate WARP

Example (Menu-Dependent Self-Control)

$$\mathcal{C}(\{n,s\})=\{n\} \text{ and } \mathcal{C}(\{n,s,\ell\})=\{s\}$$

Functional Form

Convex Self-Control Representation

(i) Two expected utility functions: $u, v : \Delta(C) \to \mathbb{R}_+$

(ii) A continuous, strictly increasing, and weakly convex function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ with $\varphi(0) = 0$

such that

$$U(x) = \max_{\ell \in x} \left\{ u(\ell) - \varphi \left(\max_{\ell' \in x} v(\ell') - v(\ell) \right) \right\}$$

Functional Form

Ex post choice:

$$\mathcal{C}(x) = \arg\max_{\ell \in x} \left\{ u(\ell) - \varphi\left(\max_{\ell' \in x} v(\ell') - v(\ell) \right) \right\}$$

- ex post preference is concave in ℓ
- consistent with Allais Paradox (certainty effect) and menu-dependent self-control

Axioms

Axiom (Order)

$\succsim$ is complete and transitive

Axiom (Continuity)

$\{y \in \mathcal{Z} | x \succsim y\}$ and $\{y \in \mathcal{Z} | y \succsim x\}$ are closed

Axiom (Set Betweenness)

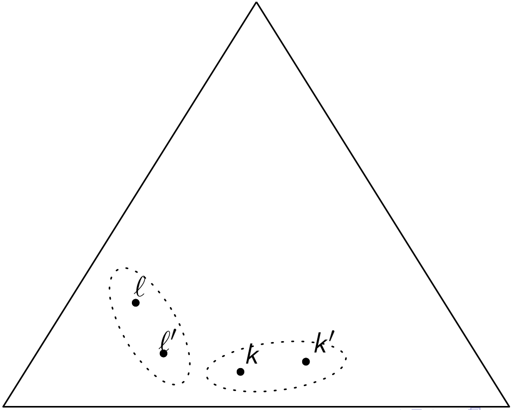
$x \succsim y \Rightarrow x \succsim x \cup y \succsim y$

$$\{v\} \succ \{h\} \Rightarrow \{v\} \succsim \{v, h\} \succsim \{h\}$$

Axioms

Axiom (Translation Invariance)

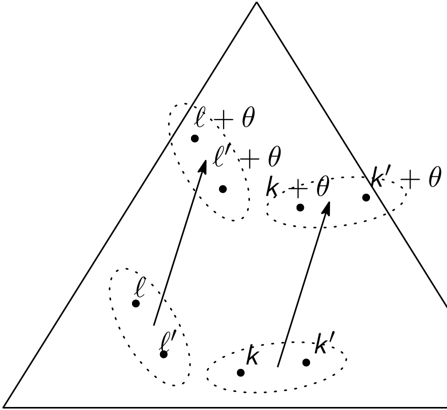
$$\{\ell, \ell'\} \succsim \{k, k'\} \;\Rightarrow\; \{\ell + \theta, \ell' + \theta\} \succsim \{k + \theta, k' + \theta\}$$



Axioms

Axiom (Translation Invariance)

$$\{\ell, \ell'\} \succsim \{k, k'\} \;\Rightarrow\; \{\ell+\theta, \ell'+\theta\} \succsim \{k+\theta, k'+\theta\}$$



Axioms

Axiom (Temptation Convexity)

(i) $\{\ell\} \succ \{\ell, \ell'\}$ and $\{\ell\} \succ \{\ell, \ell''\} \Rightarrow \{\ell\} \succ \{\ell, \ell'\lambda\ell''\}$

(ii) $\{\ell\} \sim \{\ell, \ell'\} \succ \{\ell'\}$ and $\{\ell\} \sim \{\ell, \ell''\} \succ \{\ell''\}$
$\Rightarrow \{\ell\} \sim \{\ell, \ell'\lambda\ell''\} \succ \{\ell'\lambda\ell''\}$

(iii) $\{\ell\} \succ \{\ell, \ell'\} \Rightarrow \{\ell\} \succ \{\ell, \ell\lambda\ell'\}$

(iv) $\{\ell\} \sim \{\ell, \ell'\} \succ \{\ell'\} \Rightarrow \{\ell\} \sim \{\ell, \ell\lambda\ell'\} \succ \{\ell\lambda\ell'\}$

Axioms

Axiom (Temptation Consistency)

Assume $\{\ell\} \succ \{\ell, \ell'\} \succ \{\ell'\}$ and $\{\ell\} \succ \{\ell, \ell''\}$

Either $\{\ell''\} \succ \{\ell', \ell''\}$ or $\{\ell'\} \sim \{\ell', \ell''\} \succ \{\ell''\}$

$$\Rightarrow \{\ell, \ell''\} \succsim \{\ell, \ell'\}$$

Axioms

Axiom (Mixing Preserves Self-Control)

(i) $\{\ell\} \succ \{\ell, \ell'\} \succ \{\ell'\} \Rightarrow \{\ell\lambda\ell''\} \succ \{\ell\lambda\ell'', \ell'\lambda\ell''\} \succ \{\ell'\lambda\ell''\}$

(ii) $\{\ell\} \succ \{\ell, \ell'\} \succ \{\ell'\}$ and $\{\ell\} \succ \{\ell, \ell''\} \succ \{\ell''\}$

$$\Rightarrow \{\ell, \ell'\lambda\ell''\} \succsim \{e(\{\ell, \ell'\})\lambda e(\{\ell, \ell''\})\}$$

Definition (Commitment Equivalent)

$e(x) \in \Delta(C)$ satisfies $\{e(x)\} \sim x$

Representation Theorem

Theorem

The following statements are equivalent:

(a) Preference $\succsim$ satisfies Order, Continuity, Set Betweenness, Translation Invariance, Temptation Convexity, Temptation Consistency, MPSC, Monotone Self-Control, and Properness

(b) Preference $\succsim$ is represented by

$$U(x) = \max_{\ell \in x} \left\{ u(\ell) - \varphi \left(\max_{\ell' \in x} v(\ell') - v(\ell) \right) \right\}$$

Proof Sketch

Step-1 $\succsim$ satisfies vNM independence on $\Delta(C)$ $\Rightarrow u$

Step-2

$$\{\ell^+\} \succsim x \succsim \{\ell^-\} \quad \Rightarrow \quad x \sim \{\alpha(x)\ell^+ + (1-\alpha(x))\ell^-\}$$
$$\Rightarrow \quad U(x) \equiv u(\alpha(x)\ell^+ + (1-\alpha(x))\ell^-)$$

Step-3 $\{\ell\} \succ \{\ell, \ell'\} \;\Leftrightarrow\; v(\ell') > v(\ell)$

Step-4 $\varphi(v(\ell') - v(\ell)) = u(\ell) - U(\{\ell, \ell'\})$ for $\{\ell\} \succ \{\ell, \ell'\} \succ \{\ell'\}$

Step-5 $U(x)$ can be rewritten as the desired form for binary x, finite x, and all x

Uniqueness

Theorem

Suppose (u, v, φ) and $(\tilde{u}, \tilde{v}, \tilde{\varphi})$ represent $\succsim$
Then:

(i) $\tilde{u} = \alpha_u u + \beta_u$ and $\tilde{v} = \alpha_v v + \beta_v$

(ii) $\tilde{\varphi}(\tilde{w}) = \alpha_u \varphi\left(\frac{\tilde{w}}{\alpha_v}\right)$ on

$$W(\tilde{u}, \tilde{v}, \tilde{\varphi}) = \{\tilde{w} = \tilde{v}(\ell') - \tilde{v}(\ell) \in \mathbb{R}_+ \mid \{\ell\} \succ \{\ell, \ell'\} \succ \{\ell'\}\}$$

For $\tilde{w} = \alpha_v w$

$$\frac{\tilde{w}\tilde{\varphi}''(\tilde{w})}{\tilde{\varphi}'(\tilde{w})} = \frac{w\varphi''(w)}{\varphi'(w)}$$

Uniqueness

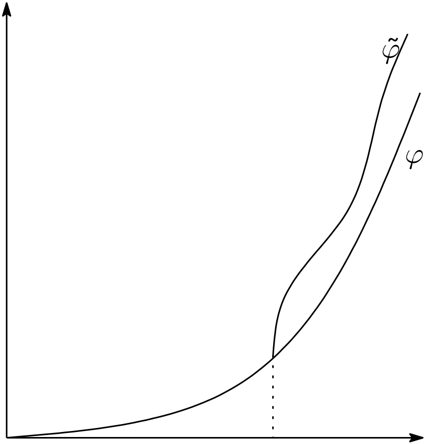
Theorem

Suppose (u, v, φ) represents $\succsim$

$$\text{continuous and strictly increasing } \tilde{\varphi} \begin{cases} = & \varphi \quad \text{on } W(u, v, \varphi) \\ \geq & \varphi \quad \text{otherwise} \end{cases}$$

$\Rightarrow (u, v, \tilde{\varphi})$ represents $\succsim$

Uniqueness



$W(u, v, \varphi)$

Related Literature

Noor, 2006: Menu-Dependent Self-Control Model

$$U(x) = \max_{\ell \in x} \left\{ u(\ell) - \kappa(x) \left(\max_{\ell' \in x} v(\ell') - v(\ell) \right) \right\}$$

Ex post choice: $\mathcal{C}(x) = \arg\max_{\ell \in x} \left\{ u(\ell) + \kappa(x) v(\ell) \right\}$

Related Literature

Fudenberg and Levine, 2006: Dual-Self Model

$$U(x) = \max_{\ell \in x} \left\{ u(\ell) - \gamma \left(\max_{\ell' \in x} v(\ell') - v(\ell) \right)^{\theta} \right\}$$

where $\gamma > 0, \theta > 1$

Special case of CSCM: $\varphi(w) = \gamma w^{\theta}$

Conclusion

- generalization of Gul and Pesendorfer (2001)
- axiomatic foundation for convex self-control model
- can explain certainty effect and menu-dependent self-control