Mathematical Economics
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Homework 4

Due on July 3

1. Solve Problem 7.7. (Show that the game is supermodular.)

2. Solve Problem 7.8.

3. Let $X \subset \mathbb{R}^n$ be a compact convex set, and suppose that $(X, \leq)$ is a lattice with the usual vector order $\leq$ (which is in fact a complete lattice by the compactness of X). For functions $f\colon X \to X$ and $g\colon X \to X$, assume the following conditions:

(i) f is non-decreasing.

(ii) $f(x) \leq g(x)$ for all $x \in X$.

(iii) g is continuous.

Prove the following:

(1) f has a fixed point.

(2) g has a fixed point x^{**} such that $x^* \leq x^{**}$ for every fixed point x^* of f.