Mathematical Economics
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Homework 5

Due on July 10

1. Let $(X, \precsim)$ be a lattice, and $f\colon X \to \mathbb{R}$ and $g\colon X \to \mathbb{R}$ supermodular. Prove the following.

(1) For $\alpha, \beta \geq 0$, the function $\alpha f + \beta g$ is supermodular.

(2) Suppose that

$$f(x) + g(y) \leq f(x \vee y) + g(x \wedge y)$$
$$g(x) + f(y) \leq g(x \vee y) + f(x \wedge y)$$

for all $x, y \in X$. Then the function $h(x) = \max\{f(x), g(x)\}$ is supermodular.

(3) Suppose that $f - g$ is non-decreasing, i.e., $f(x') - g(x') \leq f(x'') - g(x'')$ whenever $x' \precsim x''$. Then the function $h(x) = \max\{f(x), g(x)\}$ is supermodular.

2. Prove the equivalence among 2–4 in Proposition 5.1.