## Summer 2014 Mathematics II Mock Exam

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- **1.** Write down the definition of convergence of a sequence in  $\mathbb{R}^N$ .
- 2. Problem like Problem 4 in Homework 1.
- 3.
- (1) Write down the definition of an open set in  $\mathbb{R}^N$ .
- (2) Write down the definition of a closed set in  $\mathbb{R}^N$ .
- 4. For  $A \subset \mathbb{R}^N$ , prove that the following two conditions are equivalent.
  - (i) A is a closed set.
  - (ii) For any convergent sequence  $\{x^m\}_{m=1}^{\infty}$  of elements in A with  $x^m \to \bar{x}$  as  $m \to \infty$ , we have  $\bar{x} \in A$ .

5.

- (1) Write down the definition of a compact set in  $\mathbb{R}^N$ .
- (2) Write down the definition of a continuous function from  $\mathbb{R}^N$  to  $\mathbb{R}^K$ .
- (3) Let  $X \subset \mathbb{R}^N$  be a nonempty compact set, and  $f: X \to \mathbb{R}$  be a continuous function. Prove that f has a maximizer.
- 6. Problem like Problem 2 in Homework 8.
- 7. Problem like Problem 1 in Homework 9.