On Efficient Partnership Dissolution under Ex Post Individual Rationality*

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Abstract

This paper studies ex post individually rational, efficient partnership dissolution in a setting with interdependent valuations. We derive a sufficient condition that ensures the existence of an efficient dissolution mechanism that satisfies Bayesian incentive compatibility, ex post budget balancedness, and ex post individual rationality. For equal-share partnerships, we show that our sufficient condition is satisfied for any symmetric type distribution whenever the interdependence in valuations is non-positive. This result improves former existence results, demonstrating that the stronger requirement of ex post individual rationality does not always rule out efficiency. We also show that if we allow for two-stage revelation mechanisms, in which agents report their realized payoffs from the allocation, as well as imposing penalties off the equilibrium path, efficient dissolution is always possible even when the interdependence is positive. We further discuss the possibility of efficient dissolution with ex post quitting rights. *Journal of Economic Literature* Classification Numbers: D02, D40, D44, D82, C72.

KEYWORDS: mechanism design; efficient trade; ex post individual rationality; Groves mechanism; interdependent valuation.

1 Introduction

Many business projects involve partnerships such as joint ventures and strategic alliances. A partnership comes to an end, for example when the project (e.g., development of a new product or technology) has been completed, or simply when the partners have conflicting opinions about future management of their business. Efficient dissolution of a partnership consists in allocating the partnership's asset (e.g., the developed product/technology or the company itself) to the partner with the highest valuation, in exchange for monetary compensations. Cramton *et al.* (1987, CGK henceforth) first consider the problem of efficient partnership dissolution in a symmetric model with independent private values. CGK show that while efficient dissolution is impossible when the initial ownership of the partnership is extreme as in the buyer-seller situation (Myerson and Satterthwaite (1983)), it is always (i.e., for all type distributions) possible when the partnership is equally shared among the agents.[1] In the present paper, focusing mainly on equal-share partnerships, we study the possibility of efficient dissolution in a symmetric *interdependent* valuation setting as in the subsequent contribution by Fieseler *et al.* (2003, FKM henceforth).[2] The distinguishing feature of our study is that, in contrast to CGK and FKM where individual rationality constraints are required to be fulfilled at the *interim* stage, we impose the stronger requirement of *ex post* individual rationality.

Interdependence in valuations naturally arises in many situations, e.g., where each agent is responsible for a different part of the project and thus receives a different piece of private information which also affects the others' valuations of the entire project. In an environment where the private and common value components are additively separable, FKM show that when the interdependence is positive (i.e., valuations are increasing in the other agents' signals), efficient dissolution is not always possible even for the equal-share case, while it becomes easier when the interdependence is negative (i.e., valuations are decreasing in the other agents' signals). In the case of negative interdependence, efficiency is easier to achieve as winning and losing are each a blessing: winning reveals that the other agents' signals are lower than one's own which contributes to raising the winner's valuation, and a symmetric argument applies to losing. In the case of positive interdependence, conversely, winning and losing are bad news, and winner's and loser's curses make efficiency more difficult to achieve.

[1]Insights obtained in analysis of partnership dissolution apply to more general "a-positional traders" situations (Galavotti (2009)) in which traders are not ex ante identified as a buyer or a seller, i.e., each trader may become a buyer or a seller depending on the realization of the valuations. Interesting studies of such situations include Eliaz and Spiegler (2007, 2009) and Gershkov and Schweinzer (2008) among others.

[2]Related papers, other than FKM, that consider partnership dissolution with interdependent valuations include Kittsteiner (2003), Morgan (2004), Jehiel and Pauzner (2006), Chien (2007), and Li (2009) among others.

Both CGK and FKM consider efficient mechanisms that satisfy *interim* individual rationality (IIR) as well as Bayesian (i.e., interim) incentive compatibility (IC) and (ex post) budget balancedness (BB). Our point of departure in the present paper is to require the outcomes of a mechanism to satisfy the property that no agent regrets his participation *ex post*, and thus we look for efficient mechanisms that satisfy *ex post* individual rationality (EPIR) along with IC and BB. Given the result of FKM, we restrict our attention to the case where interdependence in valuations is non-positive (i.e., valuations are private or negatively interdependent).[3] For this case, we show that efficient dissolution of an equal-share partnership is always possible even with EPIR. This demonstrates, for the case of equal-share partnerships, that whenever efficient dissolution is always possible with IIR, one can safely replace IIR with EPIR incurring no loss in efficiency as well as IC and BB. The proof is done by construction of a mechanism that satisfies the desired properties.[4]

EPIR mechanisms are also considered by Gresik (1991), Makowski and Mezzetti (1994), and Kosmopoulou (1999). Gresik (1991) considers EPIR and Bayesian IC bilateral trading mechanisms that maximize *ex ante* expected gains from trade. In a general setting with independent private valuations, Makowski and Mezzetti (1994) provide characterizations of ex post efficient, IIR, ex post BB, Bayesian IC mechanisms and ex post efficient, EPIR, ex ante BB, dominant strategy IC mechanisms, while Kosmopoulou (1999) shows a payoff equivalence result between these two classes of mechanisms in a restricted environment. Different from these papers, our approach concerns Bayesian IC mechanisms that satisfy *ex post* efficiency, EPIR, and *ex post* BB.

When, as we assume, types are interdependent, observing the realized payoff provides the agent with additional information about the types of the other agents. The mechanism designer could thus collect this information, thereby obtaining a broader set of mechanisms at his disposal. This is precisely the point made by Mezzetti (2004, 2007). We consider, in our context

[3]While valuations may be assumed to be positively interdependent in "standard" cases, e.g., when the information is about quality (anyone prefers high quality), they may well be negatively interdependent in other cases, e.g., when the agents have opposite characteristics in that they derive utility from mutually exclusive properties of the asset, i.e., "if information about the increased likelihood of property A (which yields relatively more utility for partner i) means that property B (which yields relatively more utility for partner j) becomes less likely" (FKM, Footnote 6). Such a situation arises for example when two partners who produce different goods possess different pieces of private information over the relative demand of the two goods of different groups of consumers. If the relative demand for one partner's product over the other's is high, this is good news for the former partner and bad news for the latter.

[4]In the *private* valuation case, our mechanism coincides with the one proposed by Fujinaka (2006) in a different context of envy-free allocation of an indivisible good. See also Athanassoglou *et al.* (2008), who study EPIR in two-agent partnership dissolution with private valuations.

of partnership dissolution, *two-stage mechanisms* in which agents first report their types and then the winning agent reports his realized payoff from the allocation, which allows the designer to cross-check the first-stage reports. We show in particular that by punishing first-stage lies with penalties (thus violating BB off the equilibrium path), the "shoot-the-liar" mechanism à la Mezzetti (2007) always efficiently dissolves the partnership satisfying EPIR for any ownership shares even when types are positively interdependent.

While we motivate our study of EPIR mechanisms by the requirement that the outcomes of a mechanism should be ex post regret-free in participation, one may consider a situation in which agents are allowed to quit or veto the mechanism ex post in any event. As noted by Matthews and Postlewaite (1989) and Forges (1999), with quitting rights *off* as well as on equilibrium the IC constraints are also modified.[5] We examine the modified IC constraints in our environment, and show that our (single-stage) mechanism always dissolves the partnership efficiently even with quitting rights when the degree of negative interdependence is large.

The paper is organized as follows. Section 2 describes our partnership dissolution problem. Section 3 derives our main sufficient condition for efficient dissolution with EPIR. Section 4 applies it to the symmetric and separable environment and obtains the possibility result for non-positively interdependent types. Section 5 studies two-stage mechanisms, while Section 6 discusses ex post quitting rights. Section 7 concludes.

2 Setup

In this section, we describe our problem of partnership dissolution, where we mostly follow the setup of FKM. There are one asset, and n risk-neutral agents indexed by $i \in N = \{1, \ldots, n\}$, where $n \geq 2$. Each agent i initially owns a share α_i of the asset ($0 \leq \alpha_i \leq 1$ and $\sum_{i \in N} \alpha_i = 1$). Each agent i has private information represented by type θ_i. We will denote $\theta = (\theta_1, \ldots, \theta_n)$ and $\theta_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n)$. Agents' types are statistically independent. The type θ_i is distributed according to a commonly known distribution F_i with support $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ and positive continuous density f_i. We denote $\Theta = \prod_{i \in N} \Theta_i$.

Agent i's valuation for the entire asset is given by a function $v_i(\theta_i, \theta_{-i})$, where the arguments are always ordered by the agents' indices: $v_i(\theta_i, \theta_{-i}) = v_i(\theta_1, \ldots, \theta_n)$. The function $v_i(\theta_i, \theta_{-i})$ is assumed to be strictly increasing in θ_i, and continuously differentiable. We further assume the single crossing property:

$$v_{i,i}(\theta) > v_{j,i}(\theta)$$

[5] See also Compte and Jehiel (2007, 2009), who show that with ex post quitting/veto rights, inefficiencies are inevitable in their bargaining model (even with correlations in types).

for all $i, j \neq i$ and $\theta \in \Theta$, where $v_{k,i} = \partial v_k / \partial \theta_i$. The ex post utility of agent i with valuation v_i, share s_i, and money m_i is given by $v_i s_i + m_i$.

In a direct revelation mechanism, or simply mechanism, each agent i simultaneously reports his own type θ_i, and then receives a share $s_i(\theta)$ of the asset and a monetary transfer $t_i(\theta)$. More precisely, a mechanism is a pair (s, t) of (measurable) functions $s\colon \Theta \to [0,1]^n$ such that $\sum_{i \in N} s_i(\theta) = 1$ (an *assignment rule*) and $t\colon \Theta \to \mathbb{R}^n$ (a *transfer rule*). Given a mechanism (s, t), the interim utility of agent i with type θ_i, when he reports $\hat{\theta}_i$ while the other agents report their types θ_{-i} truthfully, is given by

$$U_i(\theta_i, \hat{\theta}_i) = E_{\theta_{-i}}[v_i(\theta_i, \theta_{-i}) s_i(\hat{\theta}_i, \theta_{-i})] + E_{\theta_{-i}}[t_i(\hat{\theta}_i, \theta_{-i})],$$

where $E_{\theta_{-i}}[\cdot]$ is the expectation operator with respect to θ_{-i}. We denote $U_i(\theta_i) = U_i(\theta_i, \theta_i)$.

A mechanism (s, t) is *interim incentive compatible* (IC) if truth-telling constitutes a Bayesian Nash equilibrium in the incomplete information game induced by (s, t), i.e., for all $i \in N$,

$$U_i(\theta_i) \geq U_i(\theta_i, \hat{\theta}_i) \tag{IC}$$

for all $\theta_i, \hat{\theta}_i \in \Theta_i$. It is *ex post budget balanced* (BB) if the monetary transfers sum to zero for each realization of the types, i.e., $\sum_{i \in N} t_i(\theta) = 0$ for all $\theta \in \Theta$. It is *ex post efficient* (EF) if it allocates the asset to an agent with the highest valuation for each realization, i.e., for all $i \in N$ and all $\theta \in \Theta$, $s_i(\theta) > 0 \Rightarrow i \in \arg\max_j v_j(\theta)$. It is sufficient to consider the efficient assignment rule s^* defined by

$$s_i^*(\theta) = \begin{cases} 1 & \text{if } i = m(\theta), \\ 0 & \text{if } i \neq m(\theta), \end{cases} \tag{2.1}$$

where $m(\theta) = \max(\arg\max_j v_j(\theta))$.[6]

In the present study, we are interested in mechanisms that satisfy *no ex post regret of participation*, or *ex post individual rationality*, as a desideratum additional to the above three, while much work in the literature, including that of Myerson and Satterthwaite (1983), CGK, and FKM, is concerned with *interim* individual rationality. Let $u_i(\theta)$ be agent i's ex post utility under truth-telling:

$$u_i(\theta) = v_i(\theta) s_i(\theta) + t_i(\theta),$$

and $u_i^0(\theta)$ the outside option to agent i: $u_i^0(\theta) = \alpha_i v_i(\theta)$. The mechanism (s, t) is *ex post individually rational* (EPIR) if, for any realization of types,

[6]That is, when more than one agent have the highest valuation, we let $m(\theta)$ be the agent with the largest index. Our analysis is not affected by this particular choice of the tie-breaking rule.

no agent regrets his participating in the mechanism even after observing the realized values of his initial and final payoffs, i.e., for all $i \in N$,

$$u_i(\theta) \geq u_i^0(\theta) \tag{EPIR}$$

for all $\theta \in \Theta$; (s, t) is *interim individually rational* (IIR) if given his type, but before he learns the other agent's type, each agent prefers to participate in the mechanism, i.e., for all $i \in N$,

$$U_i(\theta_i) \geq E_{\theta_{-i}}\left[u_i^0(\theta_i, \theta_{-i})\right] \tag{IIR}$$

for all $\theta_i \in \Theta_i$. Clearly, EPIR implies IIR, but not vice versa.

We say that the partnership is *EPIR-dissolvable* (*IIR-dissolvable*, resp.) if there exists an IC, EF, and BB mechanism that is also EPIR (IIR, resp.). Our task in the present paper is to explore the possibility of EPIR-, rather than IIR-, dissolution. Note that IIR guarantees the agents non-negative net payoffs only *on average*, so that it may well happen that some agents' *actual* (i.e., *ex post*) net payoffs are negative. EPIR rules out this possibility, so that no agent will ever regret his participation in the mechanism.[7]

3 A Sufficient Condition for Existence

Let us recall the revenue equivalence property of efficient IC mechanisms in our environment (see, e.g., FKM or Fieseler *et al.* (2001, Theorem 1)).

Revenue Equivalence. *Let s^* be the EF assignment rule. Then, (s^*, t) is IC if and only if for all $i \in N$,*

$$U_i(\theta_i) = U_i(\underline{\theta}_i) + \int_{\underline{\theta}_i}^{\theta_i} E_{\theta_{-i}}[v_{i,i}(x, \theta_{-i}) s_i^*(x, \theta_{-i})]\, dx \tag{3.1}$$

for all $\theta_i \in \Theta_i$.

As in FKM, for each i let $\theta_i^*(\theta_{-i}) \in \Theta_i$ be defined by

$$v_i(\theta_i^*(\theta_{-i}), \theta_{-i}) = \max_{j \neq i} v_j(\theta_i^*(\theta_{-i}), \theta_{-i})$$

if the equation has a solution, and arbitrarily if not. Let t^{G} denote the *generalized Groves mechanism* defined by

$$t_i^{\mathrm{G}}(\theta) = \begin{cases} 0 & \text{if } i = m(\theta), \\ v_i(\theta_i^*(\theta_{-i}), \theta_{-i}) & \text{if } i \neq m(\theta). \end{cases} \tag{3.2}$$

[7] Recall that Myerson and Satterthwaite (1983) prove an impossibility theorem and thus the weaker requirement of IIR makes their result stronger. CGK and FKM maintain the IIR requirement to highlight the crucial roles of ownership distributions (CGK) and interdependence in valuations (FKM).

Observe that (s^*, t^{G}) is IC, and in fact, ex post IC (truth-telling is an ex post equilibrium). Due to the Revenue Equivalence, (s^*, t) is IC if and only if t yields, up to constant, the same interim expected transfer as the generalized Groves mechanism t^{G}. Therefore, (s^*, t) is IC if and only if there exist functions $k_i \colon \Theta \to \mathbb{R}$, $i \in N$, such that

$$t_i(\theta) = t_i^{\mathrm{G}}(\theta) - k_i(\theta)$$

and

$$E_{\theta_{-i}}[k_i(\theta_i, \theta_{-i})] = E_{\theta_{-i}}[k_i(\theta_i', \theta_{-i})] \tag{3.3}$$

for all $\theta_i, \theta_i' \in \Theta_i$.

The other properties, BB and EPIR, are also rewritten in terms of the k_i functions as above. Denote by $b^{\mathrm{G}}(\theta)$ the *budget deficit* generated by the generalized Groves mechanism t^{G}:

$$b^{\mathrm{G}}(\theta) = \sum_{i \in N} t_i^{\mathrm{G}}(\theta).$$

Then, t satisfies BB if and only if

$$\sum_{i \in N} k_i(\theta) = b^{\mathrm{G}}(\theta) \tag{3.4}$$

for all $\theta \in \Theta$. Let $u_i^{\mathrm{G}}(\theta)$ denote the ex post utility of agent i under (s^*, t^{G}):

$$u_i^{\mathrm{G}}(\theta) = v_i(\theta) s_i^*(\theta) + t_i^{\mathrm{G}}(\theta).$$

Then, t satisfies EPIR if and only if for all $i \in N$, $u_i^{\mathrm{G}}(\theta) - k_i(\theta) \geq u_i^0(\theta)$ for all $\theta \in \Theta$, or equivalently,

$$\inf_{\theta \in \Theta} \{u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - k_i(\theta)\} \geq 0. \tag{3.5}$$

In summary, the partnership is EPIR-dissolvable if and only if there exist functions $k_1, \ldots, k_n$ that satisfy the conditions (3.3), (3.4), and (3.5).

We focus on a specific form of k_i functions. Specifically, our approach is to set

$$k_i(\theta) = b_i(\theta) - E_{\theta_{-i}}[b_i(\theta)] + \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] + C_i$$

for some functions b_i that satisfy

$$\sum_{i \in N} b_i(\theta) = b^{\mathrm{G}}(\theta)$$

and constants C_i with $\sum_{i \in N} C_i = 0$. It is immediate to verify that these k_i functions satisfy the IC condition (3.3) and the BB condition (3.4). The resulting transfer rule $t = t^{\mathrm{G}} - k$ is then written as

$$t_i(\theta) = t_i^{\mathrm{G}}(\theta) - b_i(\theta) + E_{\theta_{-i}}[b_i(\theta)] - \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] - C_i. \tag{3.6}$$

This can be given the following interpretation. The starting point is the Groves transfer rule t_i^{G}, which generates a budget deficit b^{G}. Functions b_i are considered as defining a *burden sharing rule* of the budget deficit b^{G}, where $b_i(\theta)$ is the burden borne by agent i. The term $E_{\theta_{-i}}[b_i(\theta)]$ is added to give the agent the right incentives to report the truth, while the other two terms, which are independent of θ_i, are to keep the budget balance unaffected. Notice that when $n \geq 3$, subtracting the term $\{1/(n-1)\} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)]$ is not the only way to balance the budget.

It remains to determine a condition under which the EPIR condition (3.5) is satisfied. The following result offers a sufficient condition for EPIR-dissolution in terms of burden sharing functions b_i.

Theorem 1. *If there exist functions $b_1, \ldots, b_n$ such that $\sum_{i \in N} b_i(\theta) = \sum_{i \in N} t_i^{\mathrm{G}}(\theta)$ for all θ and*

$$\sum_{i \in N} \inf_{\theta \in \Theta} \left\{ u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - b_i(\theta) + E_{\theta_{-i}}[b_i(\theta)] - \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] \right\} \geq 0, \tag{3.7}$$

then the partnership is EPIR-dissolvable.

When $n = 2$, the condition (3.7) is also necessary for EPIR-dissolution (note that $\{1/(n-1)\} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] = E_{\theta_i}[b_{-i}(\theta)]$ in this case).

Proof. Suppose that the condition (3.7) is satisfied with functions b_i where $\sum_{i \in N} b_i(\theta) = \sum_{i \in N} t_i^{\mathrm{G}}(\theta)$, and let the transfer rule t be as in (3.6). By construction, (s^*, t) satisfies EF and IC. It satisfies BB if and only if $\sum_{i \in N} C_i = 0$.

Now, for each $i \in N$, define

$$C_i^* = \inf_{\theta \in \Theta} \left\{ u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - b_i(\theta) + E_{\theta_{-i}}[b_i(\theta)] - \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] \right\}.$$

EPIR is thus satisfied if and only if $C_i^* \leq C_i$. Therefore, BB and EPIR are simultaneously satisfied if and only if $\sum_{i \in N} C_i^* \geq 0$, which completes the proof. ∎

Remark 3.1. Theorem 1 in fact holds for a fairly general setup beyond partnership dissolution, under the assumptions of quasilinear utilities and independently distributed, one-dimensional types along with the monotonicity and the sorting conditions as in Bergemann and Välimäki (2002, Propositions 3 and 4), where the transfer function t^{G} is to be adjusted accordingly (Bergemann and Välimäki (2002, Section 5)). The monotonicity condition is more general than the single crossing condition we assume, while the sorting condition is implied by our assumption of v_i strictly increasing in θ_i.

Remark 3.2. This class of transfer rules defined by (3.6) contains the *expected externality* (or AGV) *mechanism* t^{E} as a special case. To see this, set $b_i(\theta) = t_i^{\mathrm{G}}(\theta)$ (and $C_i = 0$), and then we have

$$t_i^{\mathrm{E}}(\theta) = E_{\theta_{-i}}[t_i^{\mathrm{G}}(\theta)] - \frac{1}{n-1}\sum_{j\neq i} E_{\theta_{-j}}[t_j^{\mathrm{G}}(\theta)]. \tag{3.8}$$

Another natural transfer rule is induced by what we call the *equal burden sharing*:

$$b_i(\theta) = \frac{1}{n} b^{\mathrm{G}}(\theta). \tag{3.9}$$

This rule will be discussed in Section 4 (Example 4.1).

4 Symmetric n-Agent Partnerships

In this section, we restrict our attention to the symmetric and separable environment as in FKM: we assume that for all $i \in N$, $[\underline{\theta}_i, \overline{\theta}_i] = [\underline{\theta}, \overline{\theta}]$, $F_i = F$, and

$$v_i(\theta) = g(\theta_i) + \sum_{j\neq i} h(\theta_j),$$

where g and h are continuously differentiable and satisfy $g' > 0$ and $g' > h'$. Under this assumption, $v_i(\theta) \geq v_j(\theta)$ if and only if $\theta_i \geq \theta_j$, so that $m(\theta) = \max(\arg\max_j v_j(\theta)) = \max(\arg\max_j \theta_j)$. Then, the generalized Groves mechanism becomes

$$t_i^{\mathrm{G}}(\theta) = \begin{cases} 0 & \text{if } i = m(\theta), \\ g\big(\theta_{m(\theta)}\big) + \sum_{j\neq i} h(\theta_j) & \text{if } i \neq m(\theta), \end{cases} \tag{4.1}$$

and thus its budget deficit is

$$\begin{aligned} b^{\mathrm{G}}(\theta) &= (n-1)g\big(\theta_{m(\theta)}\big) + \sum_{i\neq m(\theta)}\sum_{j\neq i} h(\theta_j) \\ &= (n-1)g\big(\theta_{m(\theta)}\big) + h\big(\theta_{m(\theta)}\big) + (n-2)\sum_{j\in N} h(\theta_j). \end{aligned} \tag{4.2}$$

In this environment, FKM obtain the following results for partnership dissolution with IIR.

Fact 1 (FKM)**.** (1) *If $h' > 0$, then the equal-share partnership is not IIR-dissolvable for some distribution function F.*

(2) *If $h' \leq 0$, then the equal-share partnership is IIR-dissolvable for any distribution function F.*

A trivial corollary to Fact 1 is that if $h' > 0$, then the equal-share partnership is not EPIR-dissolvable for some distribution function F. In the following section, we consider whether the equal-share partnership is always (i.e., for all distribution functions) EPIR-dissolvable in the case of non-positively interdependent valuations, i.e., when $h' \leq 0$. While valuations may be assumed to be positively interdependent in "standard" cases (e.g., when the information is about the quality of the asset), they may well be negatively interdependent in other cases, when the information is about properties which different agents evaluate differently. As such an example, consider two firms 1 and 2 having a joint project of collecting data (such as address, age, and so on) of consumers, where each firm has access to only a part of the whole data set. Assume that it is known that the products of the two firms have opposite characteristics, so that the product of firm 1 is preferred by a certain category of consumers, say younger consumers, whereas that of firm 2 is preferred by the opposite category, say older consumers. Then, information suggesting that the data set contains a larger number of younger consumers makes the data *more* valuable for firm 1 but *less* valuable for firm 2.

Now, we apply Theorem 1 to the problem of EPIR-dissolution of n-agent partnerships with equal ownership shares, where $\alpha_i = 1/\mathrm{n}$ so that

$$u_i^0(\theta) = \frac{1}{n} v_i(\theta).$$

Our main question here is whether the equal-share partnership is EPIR-dissolvable for any type distribution F. Given Fact 1, we restrict our attention to the case of non-positive interdependence, i.e., $h' \leq 0$. For this case, we show that the answer to our question is the affirmative.

Theorem 2. *Assume $h' \leq 0$. Then, the equal-share partnership is EPIR-dissolvable for any distribution function F.*

Proof. See Appendix A.1. ∎

The proof consists in finding burden sharing functions b_i as in Theorem 1. Denote $\theta^1 = \theta_{m(\theta)}$ and $\theta^2 = \max_{j \neq m(\theta)} \theta_j$. We set the functions b_i to be

$$b_i(\theta) = \frac{1}{n} b^{\mathrm{G}}(\theta) + d_i(\theta), \tag{4.3}$$

where

$$d_i(\theta) = \begin{cases} -\dfrac{n-1}{n} \displaystyle\int_{\theta^2}^{\theta^1} F(x)\, dg(x) & \text{if } i = m(\theta), \\ \dfrac{1}{n} \displaystyle\int_{\theta^2}^{\theta^1} F(x)\, dg(x) & \text{if } i \neq m(\theta). \end{cases}$$

We verify in Appendix A.1 that the sufficient condition in Theorem 1 is satisfied with this choice of b_i for any type distribution F, provided that $h' \leq 0$.

Example 4.1 (Symmetric Two-Agent Partnerships)**.** To illustrate this result, let us focus on the two-agent case, where $n = 2$ and $\alpha_1 = \alpha_2 = 1/2$ so that $u_i^0(\theta) = \frac{1}{2}v_i(\theta)$.[8] Two-agent equal-share partnerships are important empirically[9] as well as in the theoretical literature.[10] In this case, the generalized Groves mechanism and its budget deficit are given by

$$t_i^{\mathrm{G}}(\theta) = \begin{cases} 0 & \text{if } i = m(\theta), \\ v(\theta^1) & \text{if } i \neq m(\theta) \end{cases}$$

and $b^{\mathrm{G}}(\theta) = v(\theta^1)$, respectively, where we denote

$$v(x) = g(x) + h(x)$$

for $x \in [\underline{\theta}, \overline{\theta}]$. The implementing mechanism t^* (obtained by (3.6)) is then given by

$$t_i^*(\theta) = \begin{cases} -\dfrac{1}{2}\left[v(\theta^1) - \displaystyle\int_{\theta^2}^{\theta^1} F(x)\, dv(x)\right] & \text{if } i = m(\theta), \\ \dfrac{1}{2}\left[v(\theta^1) - \displaystyle\int_{\theta^2}^{\theta^1} F(x)\, dv(x)\right] & \text{if } i \neq m(\theta). \end{cases} \tag{4.4}$$

Actually, for the case of $n = 2$ the mechanism t^* is also obtained by setting the function b_i in Theorem 1 to be the equal burden sharing

$$b_i(\theta) = \frac{1}{2}b^{\mathrm{G}}(\theta),$$

since in this case, the function d_i in (4.3), which has the form $d_i(\theta) = -\big(A(\theta_i) - A(\theta_{-i})\big)$, satisfies

$$-d_i(\theta) + E_{\theta_{-i}}[d_i(\theta)] - E_{\theta_i}[d_{-i}(\theta)] = 0$$

for any θ, where we write $-i$ for the agent $j \neq i$.

Figure 1 illustrates the EPIR mechanism graphically (assuming $v > 0$ and $v' > 0$). The shaded area in the figure depicts the bracketed term in (4.4), which may be interpreted as the *price* of the asset (per unit) which we denote $p^*(\theta^1, \theta^2)$ as a function of θ^1 and θ^2:

$$p^*(\theta^1, \theta^2) = v(\theta^1) - \int_{\theta^2}^{\theta^1} F(x)\, dv(x), \tag{4.5}$$

[8]When $n = 2$, if a deterministic mechanism (i.e., such that for all $\theta \in \Theta$, $s_i(\theta) = 1$ for some $i \in N$) satisfies EPIR and BB, then it also satisfies EF, whereas it is not the case when $n \geq 3$.

[9]Hauswald and Hege (2006) report that, among their sample of U.S. joint ventures between 1985 and 2000, about 80% have two partners, and 68% of all two-partner partnerships exhibit equal-share ownership.

[10]Theoretical studies that focus on two-agent equal-share partnerships include Kittsteiner (2003), Morgan (2004), and de Frutos and Kittsteiner (2008).

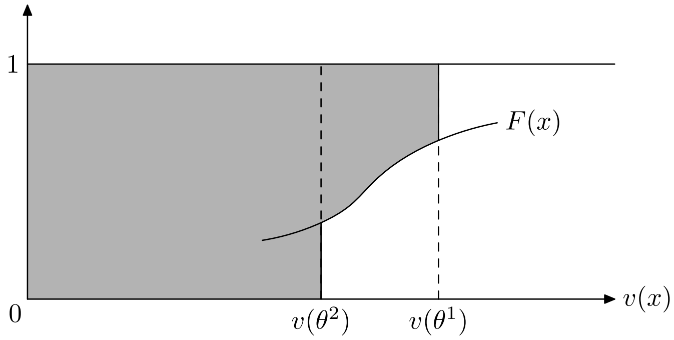

Figure 1: Pricing rule $p^*(\theta^1, \theta^2)$

where the "winner" (or ex post buyer) pays $(1/2)p^*(\theta^1, \theta^2)$ to the "loser" (or ex post seller) for the 1/2 units of the asset the loser owns. Then EPIR reduces to the condition that for any θ the price must lie between the winner's and the loser's ex post utilities, $v_{m(\theta)}(\theta)$ and $v_{-m(\theta)}(\theta)$. The figure immediately shows that $v(\theta^2) \leq p^*(\theta^1, \theta^2) \leq v(\theta^1)$. Since if $h' \leq 0$, then

$$v(\theta^2) = g(\theta^2) + h(\theta^2) \geq g(\theta^2) + h(\theta^1) = v_{-m(\theta)}(\theta),$$
$$v(\theta^1) = g(\theta^1) + h(\theta^1) \leq g(\theta^1) + h(\theta^2) = v_{m(\theta)}(\theta),$$

it follows that $v_{-m(\theta)}(\theta) \leq p^*(\theta^1, \theta^2) \leq v_{m(\theta)}(\theta)$, which implies EPIR.

5 Two-Stage Mechanisms

In requiring EPIR, we are concerned with the ex post stage in which each agent observes his payoff that results from the outcome of the mechanism. When, as we assume, types are interdependent, observing the realized payoff provides the agent with additional information about the types of the other agents. The mechanism designer could thus collect this information, thereby obtaining a broader set of mechanisms at his disposal. This is precisely the point made by Mezzetti (2004, 2007). In this section, we consider *two-stage mechanisms* à la Mezzetti (2004, 2007) in our context of partnership dissolution.

In a two-stage mechanism (s^*, τ) with the efficient allocation rule s^*, agents first report their types $\hat{\theta}_i$, based on which the outcome decision is made: the asset of the partnership is allocated to the agent $m(\hat{\theta})$ $(= \max(\arg\max_j v_j(\hat{\theta})))$. In the second stage, the winning agent $m(\hat{\theta})$ reports his realized decision-outcome value $\hat{v}_{m(\hat{\theta})}$. Then, transfers are made: each agent i receives the monetary transfer $\tau_i(\hat{\theta}, \hat{v}_{m(\hat{\theta})})$, which may depend on the report in the second stage as well as those in the first.

Specifically, we study the two classes of two-stage mechanisms introduced by Mezzetti (2004, 2007), namely, the two-stage Groves mechanism (Mezzetti (2004)) and the "shoot-the-liar" mechanism (Mezzetti (2007)),[11] and ask whether these mechanisms help to achieve EPIR-dissolution in the case of positive interdependence. In Subsection 5.1, we show that, in the symmetric and separable environment as considered in the previous section, if types are positively interdependent (i.e., $h' > 0$), then no mechanism in the class of two-stage Groves mechanisms in expectations IIR- (*a fortiori*, EPIR-)dissolves the equal-share partnership for all type distributions. In Subsection 5.2, we consider the general setting as introduced in Section 2 and show that under certain conditions similar to those in Mezzetti (2007), which are satisfied in particular when $h' > 0$ in the symmetric and separable environment, by punishing first-stage lies with penalties the shoot-the-liar mechanism EPIR-dissolves the partnership for any ownership distribution.

5.1 Two-Stage Groves Mechanism

The *two-stage Groves mechanism* $(s^*, \tau^{\mathcal{G}})$ consists of the efficient allocation rule s^* and the transfer rule $\tau^{\mathcal{G}}$ defined by

$$\tau_i^{\mathcal{G}}\left(\hat{\theta}, \hat{v}_{m(\hat{\theta})}\right) = \begin{cases} 0 & \text{if } i = m(\hat{\theta}), \\ \hat{v}_{m(\hat{\theta})} & \text{if } i \neq m(\hat{\theta}). \end{cases}$$

This mechanism achieves efficiency in ex post IC (Mezzetti (2004) shows this for a much more general setting): In the second stage, it is (weakly) optimal for the winning agent to report truthfully, $\hat{v}_{m(\hat{\theta})} = v_{m(\hat{\theta})}$, since the transfer to him does not depend on his report. Given this, in the first stage game, truth-telling constitutes an ex post equilibrium. The budget deficit, $b^{\mathcal{G}}$, and the ex post utilities, $u_i^{\mathcal{G}}$, generated (on the equilibrium path) by the two-stage mechanism are given respectively by

$$b^{\mathcal{G}}(\theta) = (n-1)v_{m(\theta)}(\theta)$$

and

$$u_i^{\mathcal{G}}(\theta) = v_{m(\theta)}(\theta).$$

Analogous to Makowski and Mezzetti (1994), we call (s^*, τ) a *two-stage Groves mechanism in expectations* if for each $i \in N$, τ_i is written as $\tau_i(\hat{\theta}, \hat{v}_{m(\hat{\theta})}) = \tau_i^{\mathcal{G}}(\hat{\theta}, \hat{v}_{m(\hat{\theta})}) - k_i(\hat{\theta})$ with some function k_i such that $E_{\theta_{-i}}[k_i(\theta_i, \theta_{-i})] = E_{\theta_{-i}}[k_i(\theta_i', \theta_{-i})]$ for all $\theta_i, \theta_i' \in \Theta_i$. Given truth-telling in the second reporting stage, it is clear that a two-stage Groves mechanism in expectations satisfies IC, i.e., truth-telling constitutes a Bayesian Nash equilibrium in the induced first stage game. Note also that our Theorem 1 remains valid with $\tau_i^{\mathcal{G}}$ and $u_i^{\mathcal{G}}$ in place of t_i^{G} and u_i^{G}.

[11] See also the working paper version Mezzetti (2002).

We again assume the symmetric and separable environment: $[\underline{\theta}_i, \overline{\theta}_i] = [\underline{\theta}, \overline{\theta}]$, $F_i = F$, and $v_i(\theta) = g(\theta_i) + \sum_{j \neq i} h(\theta_j)$ for all $i \in N$. For the case of $h' > 0$, we show that allowing for two-stage Groves mechanisms in expectations does not suffice to obtain the possibility of EPIR-dissolution of the equal-share partnership. In fact, we prove a stronger result, that *IIR*-dissolution for all F is impossible.

Theorem 3. *Assume $h' > 0$. Then, there exists a distribution function F such that the equal-share partnership is not IIR-dissolvable by any two-stage Groves mechanism in expectations.*

Proof. See Appendix A.2. ∎

Let
$$V_i^{\mathcal{G}}(\theta_i) = E_{\theta_{-i}}\left[u_i^{\mathcal{G}}(\theta) - u_i^0(\theta)\right],$$
where u_i^0 is the outside option of agent i. The necessary and sufficient condition for a two-stage Groves mechanism in expectations to achieve IIR-dissolution is that
$$E_\theta\left[b^{\mathcal{G}}(\theta)\right] \leq \sum_{i \in N} \min_{\theta_i \in \Theta_i} V_i^{\mathcal{G}}(\theta_i) \tag{5.1}$$
(Mezzetti (2002); see also Makowski and Mezzetti (1994, Theorem 3.1) and FKM (2003, Theorem 1)). The proof consists in constructing a distribution function F that violates this condition (5.1).

Intuitively speaking, the two-stage Groves mechanism (or those in expectations) achieves efficiency even with interdependent valuations since it makes the ex post utilities as if they came from a private valuation model, so that the standard VCG insights apply. One might therefore conjecture that EPIR-dissolution of the equal-share partnership, which is possible for private values, is also made possible by two-stage Groves mechanisms even when $h' > 0$. The impossibility of IIR-dissolution (and hence of EPIR dissolution) holds, however, since the two-stage Groves mechanism does not affect the outside options, and consequently the impossibility result by FKM continues to hold even with two-stage Groves mechanisms in expectations.

While with a single reporting stage an efficient IC mechanism must be a (generalized) Groves mechanism in expectations (as we considered in Section 3), there are other two-stage mechanisms, besides the two-stage Groves mechanisms, that implement efficiency, as emphasized in Mezzetti (2004) and indeed seen in the next subsection.

5.2 Shoot-the-Liar Mechanism

This subsection considers the "shoot-the-liar" mechanism, proposed by Mezzetti (2007) for the problem of full surplus extraction. We show that even when types are positively interdependent, for any ownership distribution EPIR-dissolution is possible with this class of mechanism. The result will be proven with the general setting introduced in Section 2.

For ownership shares $(\alpha_i)_{i\in N}$ and a constant $P > 0$, we consider the version of *shoot-the-liar mechanism* (s^*, τ^P) that consists of the efficient allocation rule s^* and the transfer rule τ^P defined by

$$\tau_i^P\left(\hat{\theta}, \hat{v}_{m(\hat{\theta})}\right) = \begin{cases} -(1-\alpha_i)v_i(\hat{\theta}) & \text{if } i = m(\hat{\theta}), \\ \alpha_i v_{m(\hat{\theta})}(\hat{\theta}) & \text{if } i \neq m(\hat{\theta}) \text{ and } \hat{v}_{m(\hat{\theta})} = v_{m(\hat{\theta})}(\hat{\theta}), \\ -P & \text{if } i \neq m(\hat{\theta}) \text{ and } \hat{v}_{m(\hat{\theta})} \neq v_{m(\hat{\theta})}(\hat{\theta}). \end{cases} \quad (5.2)$$

As in the two-stage Groves mechanism, the winning agent $m(\hat{\theta})$ reports his ex post valuation of the asset in the second stage. Since the transfer for this agent does not depend on his reported valuation, he has no incentive to deviate from truth-telling. Given this, if all the agents truthfully report their types in the first stage (i.e., $\hat{\theta}_i = \theta_i$), then the value $v_{m(\hat{\theta})}(\hat{\theta})$ estimated based on the reported type profile $\hat{\theta}$ will be equal to the report $\hat{v}_{m(\hat{\theta})}$ in the second stage. In this case, the ex post payoff (net of the transfer) is $\alpha_i v_{m(\theta)}(\theta)$ $(= \alpha_i \max_j v_j(\theta))$ for all agents i, so that EPIR is satisfied. By construction, BB is satisfied as well.

If, conversely, $\hat{v}_{m(\hat{\theta})} \neq v_{m(\hat{\theta})}(\hat{\theta})$, it implies that some agent has made a false report, and the designer then imposes penalty P on all the losing agents. Thus, truth-telling will constitute an equilibrium in the first reporting stage if existence of false reports can be detected with positive probability and the penalty level P is set sufficiently large. We impose the following assumption similar to Assumption 1 in Mezzetti (2007).

Assumption 5.1. There exist $M_1, M_2 \geq 0$ such that for all $i \in N$ and all $\theta_i, \hat{\theta}_i \in \Theta_i$ with $\hat{\theta}_i \neq \theta_i$,

$$\begin{aligned} &E_{\theta_{-i}}\left[\mathbf{1}_{\{i=m(\hat{\theta}_i,\theta_{-i})\}}\left(v_i(\overline{\theta}_i, \theta_{-i}) - v_i(\hat{\theta}_i, \theta_{-i})\right)\right] \\ &\qquad \leq M_1 \sum_{j\neq i} E_{\theta_{-i}}\left[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})\neq v_j(\hat{\theta}_i,\theta_{-i})\}}\right], \end{aligned} \quad (5.3)$$

and

$$\begin{aligned} &\sum_{j\neq i} E_{\theta_{-i}}\left[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})= v_j(\hat{\theta}_i,\theta_{-i})\}}\right] \\ &\qquad \leq M_2 \sum_{j\neq i} E_{\theta_{-i}}\left[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})\neq v_j(\hat{\theta}_i,\theta_{-i})\}}\right]. \end{aligned} \quad (5.4)$$

Condition (5.3) implies in particular that, whenever $\hat{\theta}_i \neq \overline{\theta}_i$ and $E_{\theta_{-i}}\left[\mathbf{1}_{\{i=m(\hat{\theta}_i,\theta_{-i})\}}\right] > 0$,

$$\sum_{j\neq i} E_{\theta_{-i}}\left[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})\neq v_j(\hat{\theta}_i,\theta_{-i})\}}\right] > 0,$$

which ensures that any profitable false report in the first stage be detected with positive probability. Note that the assumption is violated in the case of private valuations. Conditions (5.3)–(5.4) together will guarantee a finite bound of the necessary level of penalty P.[12]

The symmetric and separable environment with $h' > 0$ (or $h' < 0$) satisfies Assumption 5.1 for all distributions F with $M_1 = \max_{x\in[\underline{\theta},\overline{\theta}]}\big(g'(x)/f(x)\big)$ and $M_2 = 0$. In fact, we have in this case

$$\begin{aligned}\frac{E_{\theta_{-i}}\left[\mathbf{1}_{\{i=m(\hat{\theta}_i,\theta_{-i})\}}\big(v_i(\overline{\theta}_i,\theta_{-i}) - v_i(\hat{\theta}_i,\theta_{-i})\big)\right]}{\sum_{j\neq i} E_{\theta_{-i}}\left[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})\neq v_j(\hat{\theta}_i,\theta_{-i})\}}\right]} &= \frac{F(\hat{\theta}_i)^{n-1}(g(\overline{\theta}) - g(\hat{\theta}_i))}{(1-F(\hat{\theta}_i)^{n-1})/(n-1)} \\ &\le \max_{x\in[\underline{\theta},\overline{\theta}]} \frac{g'(x)}{f(x)}\end{aligned}$$

and

$$\sum_{j\neq i} E_{\theta_{-i}}\left[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})=v_j(\hat{\theta}_i,\theta_{-i})\}}\right] = 0$$

for all i and all θ_i and $\hat{\theta}_i$ with $\hat{\theta}_i \neq \theta_i, \overline{\theta}$.

Under Assumption 5.1, we have the following possibility result.

Theorem 4. *For all valuation functions $(v_i)_{i\in N}$ and all distribution functions $(F_i)_{i\in N}$ that satisfy Assumption 5.1, there exists $P < \infty$ such that for any ownership shares $(\alpha_i)_{i\in N}$, the shoot-the-liar mechanism (s^*, τ^P) EPIR-dissolves the partnership.*

Proof. It is sufficient to show that truth-telling constitutes a Bayesian Nash equilibrium in the first reporting stage (assuming truth-telling in the second stage). Consider an agent i of type θ_i. If he reports $\hat{\theta}_i \neq \theta_i$ while the other agents report truthfully, then

$$\begin{aligned}U_i(\theta_i,\hat{\theta}_i) = E_{\theta_{-i}}\Big[&\mathbf{1}_{\{i=m(\hat{\theta}_i,\theta_{-i})\}}\Big(v_i(\theta_i,\theta_{-i}) - (1-\alpha_i)v_i(\hat{\theta}_i,\theta_{-i})\Big) \\ &+ \textstyle\sum_{j\neq i}\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})=v_j(\hat{\theta}_i,\theta_{-i})\}}\alpha_i v_j(\hat{\theta}_i,\theta_{-i}) \\ &- \textstyle\sum_{j\neq i}\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})\neq v_j(\hat{\theta}_i,\theta_{-i})\}} P\Big].\end{aligned}$$

On the other hand, if this agent reports truthfully,

$$\begin{aligned}U_i(\theta_i) &= E_{\theta_{-i}}\left[\mathbf{1}_{\{i=m(\theta)\}}\alpha_i v_i(\theta) + \mathbf{1}_{\{i\neq m(\theta)\}}\alpha_i v_{m(\theta)}(\theta)\right] \\ &\ge E_{\theta_{-i}}\left[\alpha_i v_i(\theta)\right].\end{aligned}$$

[12]Our assumption is slightly stronger than that of Mezzetti (2007, Assumption 1). Condition (5.4) is not needed for the full surplus extraction result of Mezzetti (2007), while it is imposed here for our goal of EPIR-dissolution, where transfer has to be made also for losing agents. Condition (5.3) is necessary also in the model of Mezzetti (2007) for the implementing penalty P to be finite. We present an example in Appendix A.3 which shows that an unbounded penalty is required when condition (5.3) or (5.4) does not hold.

Therefore,

$$
\begin{aligned}
&U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) \\
&\quad \geq E_{\theta_{-i}}\Big[\textstyle\sum_{j\neq i}\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})\neq v_j(\hat{\theta}_i,\theta_{-i})\}}\big(P + \alpha_i v_i(\theta_i, \theta_{-i})\big) \\
&\qquad - \mathbf{1}_{\{i=m(\hat{\theta}_i,\theta_{-i})\}}(1-\alpha_i)\Big(v_i(\theta_i, \theta_{-i}) - v_i(\hat{\theta}_i, \theta_{-i})\Big) \\
&\qquad - \textstyle\sum_{j\neq i}\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})=v_j(\hat{\theta}_i,\theta_{-i})\}}\alpha_i\Big(v_j(\hat{\theta}_i, \theta_{-i}) - v_i(\theta_i, \theta_{-i})\Big)\Big] \\
&\quad \geq \textstyle\sum_{j\neq i}E_{\theta_{-i}}\Big[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})\neq v_j(\hat{\theta}_i,\theta_{-i})\}}\Big]\Big(P - \max_{j,\theta}|v_j(\theta)|\Big) \\
&\qquad - E_{\theta_{-i}}\Big[\mathbf{1}_{\{i=m(\hat{\theta}_i,\theta_{-i})\}}\Big(v_i(\overline{\theta}_i, \theta_{-i}) - v_i(\hat{\theta}_i, \theta_{-i})\Big)\Big] \\
&\qquad - \textstyle\sum_{j\neq i}E_{\theta_{-i}}\Big[\mathbf{1}_{\{j=m(\hat{\theta}_i,\theta_{-i}),\, v_j(\theta_i,\theta_{-i})=v_j(\hat{\theta}_i,\theta_{-i})\}}\Big]2\max_{j,\theta}|v_j(\theta)|.
\end{aligned}
$$

Setting penalty P large enough that $P \geq M_1 + (2M_2+1)\max_{j,\theta}|v_j(\theta)|$ where M_1 and M_2 are as in Assumption 5.1, we have $U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) \geq 0$, as desired. ∎

Clearly, the shoot-the-liar mechanism violates BB off the equilibrium path. The mechanism designer is thus required to play an active role here, as opposed to the premise behind the BB assumption in the previous sections that a mechanism represents a decentralized institution and the designer is just a mediator who helps the agents to coordinate. This result is meant to demonstrate how far the full power of a two-stage mechanism theoretically enables us to reach.

Remark 5.1. As already noted by Mezzetti (2004, 2007), a setting in which agents' valuations are interdependent while types are statistically independent is similar to a setting in which agents' types are correlated, in that correlation in types also can be exploited to achieve first-best outcomes (Crémer and McLean (1985, 1988), McAfee and Reny (1992)). Kosmopoulou and Williams (1998) show, both in private value and in interdependent value settings, that if the transfers are bounded, then inefficiency results in models with independent types continue to hold when correlation is sufficiently small. One can show as in Kosmopolou and Williams (1998, Theorem 4) that the transfers must be bounded in a BB and EPIR mechanism also in our environment.

6 Ex Post Quitting Right

While we motivated our study of EPIR mechanisms by our desire that a mechanism be free from ex post regret of participation, one may imagine a situation in which agents actually reserve the right to quit the mechanism after observing the outcome. Matthews and Postlewaite (1989), Forges (1999), and Compte and Jehiel (2007, 2009) consider models with (ex post) *quitting rights* or *veto*, in which agents may enjoy their outside option *on and off* the equilibrium paths. Note, in contrast, that EPIR is imposed only on the equilibrium path (i.e., at the truth-telling outcome). In this section, we examine the performance of the mechanism (s^*, t^*) when we allow for quitting rights.

Introducing ex post quitting rights implies requiring EPIR. It also modifies the IC constraints, as each agent may assert the quitting right after he makes a false report, thus affecting the incentives to deviate. To formulate the modified IC constraints, let

$$u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) = v_i(\theta_i, \theta_{-i}) s_i(\hat{\theta}_i, \theta_{-i}) + t_i(\hat{\theta}_i, \theta_{-i})$$

and

$$U_i^*(\theta_i, \hat{\theta}_i) = E_{\theta_{-i}} \left[\max \left\{ u_i(\theta_i, \hat{\theta}_i, \theta_{-i}), \frac{1}{2} v_i(\theta_i, \theta_{-i}) \right\} \right],$$

and denote $U_i^*(\theta_i) = U_i^*(\theta_i, \theta_i)$. The max operator in $U_i^*(\theta_i, \hat{\theta}_i)$, the expected utility of agent i with type θ_i when he reports $\hat{\theta}_i$, represents the assumption that the agent can take the outside option $(1/2)v_i(\theta_i, \theta_{-i})$ whenever it is larger than his ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i})$. A mechanism (s, t) satisfies *interim incentive compatibility with ex post quitting rights*, or *interim incentive compatibility starred* (IC*), if for all $i \in N$,

$$U_i^*(\theta_i) \geq U_i^*(\theta_i, \hat{\theta}_i) \tag{IC*}$$

for all $\hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$. We say that the partnership is *dissolvable with quitting rights* if there exists a mechanism that satisfies EF, BB, IC*, and EPIR.

Clearly, IC* is a stronger condition than IC. The following result shows that efficient dissolution of the equal-share partnership is always possible even with IC* when the degree of negative dependence is large enough that $g' + h' \leq 0$.

Proposition 5. *Consider the two-agent equal-share partnership. The mechanism* (s^*, t^*) *defined by* (2.1) *and* (4.4) *satisfies IC* for any distribution* F *if and only if* $g' + h' \leq 0$.

Proof. See Appendix A.4. ▮

To understand the intuition behind the result, consider the borderline case where $g' + h' \equiv 0$. In this case, we may assume without loss of generality that $g(x) + h(x) = 0$ for all $x \in [\underline{\theta}, \overline{\theta}]$, so that $t_i^*(\theta_i, \theta_{-i}) = 0$ for all $\theta_i, \theta_{-i} \in [\underline{\theta}, \overline{\theta}]$, i.e., the mechanism is such that the agent with a higher report receives the entire asset with no monetary transfer. Now consider agent i with type θ_i, and suppose that his report $\hat{\theta}_i$ overstates his type θ_i (i.e., $\hat{\theta}_i > \theta_i$). Define

$$\Delta(\theta_{-i}) = \frac{1}{2} v(\theta_i, \theta_{-i}) - u_i(\theta_i, \hat{\theta}_i, \theta_{-i}),$$

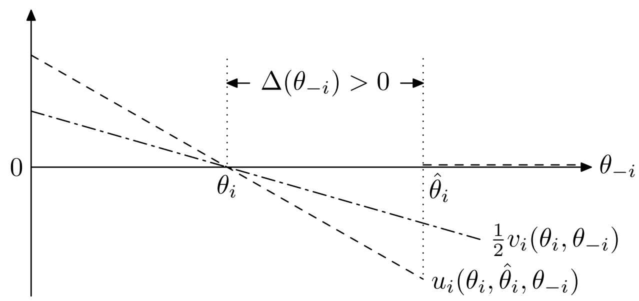

Figure 2: Case of $g' + h' \equiv 0$

which is the "ex post regret" of agent i when agent $-i$ truthfully reports θ_{-i} (here we assume $\theta_{-i} \neq \theta_i, \hat{\theta}_i$). Agent i has a (strict) incentive to exercise the quitting right ex post if and only if $\Delta(\theta_{-i}) > 0$. In the current case, $\Delta(\theta_{-i}) > 0$ if and only if $\theta_{-i} \in (\theta_i, \hat{\theta}_i)$ as in Figure 2, which depicts the graphs of the ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i})$ and the outside option $(1/2)v_i(\theta_i, \theta_{-i})$ as functions of θ_{-i}. If $\theta_i < \theta_{-i} < \hat{\theta}_i$, agent i receives the asset and obtains ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) = v_i(\theta_i, \theta_{-i}) = g(\theta_i) + h(\theta_{-i}) < g(\theta_i) + h(\theta_i) = 0$, which is smaller than his outside option $(1/2)v_i(\theta_i, \theta_{-i})$. But the outside option $(1/2)v_i(\theta_i, \theta_{-i})$ (< 0) that agent i will take when $\theta_{-i} \in (\theta_i, \hat{\theta}_i)$ is smaller than the ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i})$ $(= 0)$ that he would have obtained if he reported his type truthfully. If $\theta_{-i} < \theta_i$ or $\theta_{-i} > \hat{\theta}_i$, the outcome is no different from the one under truth telling. After all, the agent has no incentive to overstate his type even with ex post quitting rights (a symmetric argument applies to understatements). In fact, we have

$$U_i^*(\theta_i, \hat{\theta}_i) - U_i^*(\theta_i) = \frac{1}{2}\big(U_i(\theta_i, \hat{\theta}_i) - U_i(\theta_i)\big),$$

which is negative by IC. When $g' + h' < 0$, the set of θ_{-i}'s such that $\Delta(\theta_{-i}) > 0$ becomes smaller, which in effect makes false reports less profitable than in the case of $g' + h' \equiv 0$. In this case, it holds that $U_i^*(\theta_i, \hat{\theta}_i) - U_i^*(\theta_i) < (1/2)(U_i(\theta_i, \hat{\theta}_i) - U_i(\theta_i))$.

When $g' + h' > 0$, on the other hand, the set of θ_{-i}'s such that $\Delta(\theta_{-i}) > 0$ exceeds the interval $(\theta_i, \hat{\theta}_i)$. Indeed, consider θ_{-i} slightly smaller than θ. Then, agent i has to make a considerable amount of monetary transfer, according to $t_i^*(\hat{\theta}_i, \theta_i)$, compared to his valuation $v_i(\theta_i, \theta_{-i})$, in which case he exercises the quitting right, thereby enjoying a discrete marginal gain. If such θ_{-i}'s are assigned sufficiently larger probability densities than those in $(\theta_i, \hat{\theta}_i)$, the marginal gain that results from quitting can give a significant impact on $U_i^*(\theta_i, \hat{\theta}_i)$, violating IC*.

Example 6.1. Suppose that $g(x) = x$ and $h(x) = -\gamma x$, so that the valuation function v_i is given by

$$v_i(\theta) = \theta_i - \gamma\theta_{-i},$$

where $\gamma \geq 0$, and thus $v(x) = v_i(x, x) = (1 - \gamma)x$. By Proposition 5, the necessary and sufficient condition for our mechanism (s^*, t^*) to satisfy IC* for *all* type distributions is that $1 - \gamma \leq 0$, or $\gamma \geq 1$.

Now, we fix a type distribution F, and examine the condition for γ under which IC* is satisfied for this given distribution F. Specifically, let $[\underline{\theta}, \overline{\theta}] = [0, 1]$, and F be the uniform distribution on $[0, 1]$: $F(x) = x$. Then the transfer function t^* is written as

$$t_i^*(\theta) = \begin{cases} -\dfrac{1}{2}(1-\gamma)\left[\theta_i - \dfrac{1}{2}\left\{(\theta_i)^2 - (\theta_{-i})^2\right\}\right] & \text{if } \theta_i > \theta_{-i}, \\ \dfrac{1}{2}(1-\gamma)\left[\theta_{-i} - \dfrac{1}{2}\left\{(\theta_{-i})^2 - (\theta_i)^2\right\}\right] & \text{if } \theta_i < \theta_{-i}. \end{cases} \tag{6.1}$$

In this case, we can show that the necessary and sufficient condition for our mechanism (s^*, t^*) to satisfy IC* is that $1 - 2\gamma \leq 0$, or $\gamma \geq 1/2$. The proof is given in Appendix A.5.

Remark 6.1. In the *buyer-seller* setting with private values, Matthews and Postlewaite (1989, Theorem 3) show that any deterministic and monotonic mechanism that is IC, BB, and EPIR satisfies IC*. Our result implies that this does not hold in the case of equal-share partnership, as our mechanism (s^*, t^*) is deterministic and monotonic and satisfies IC, BB, and EPIR when $h' \leq 0$ but not IC* when $h' > -g'$. In the buyer-seller case, the buyer (seller, resp.) has no incentive to quit when trade occurs based on an under-statement (over-statement, resp.) of his type, in which case the IC* condition has no bite, while in ours, agents are not ex ante a buyer or a seller, so that IC* is relevant for both under- and over-statements.

7 Conclusion

In this paper, we have studied, in the context of partnership dissolution, the possibility of designing Bayesian IC and ex post BB trading mechanisms that yield efficient and EPIR outcomes. In a setting with interdependent valuations, we derived a simple sufficient condition for the existence of such a mechanism in terms of "burden sharing rule" used to cover the budget deficit induced by the (generalized) Groves mechanism. As an application to dissolution of partnerships with equal ownership shares in a symmetric and separable environment, we showed that EPIR efficient dissolution is possible for all symmetric type distributions whenever the interdependence in valuations is non-positive. This demonstrates, for the equal-ownership case, that the positive results under IIR due to CGK and FKM remain valid even if the stronger requirement of EPIR is imposed. We further showed that if we allow the designer to use two-stage mechanisms and impose penalties off the equilibrium path, then the "shoot-the-liar" mechanism always EPIR-dissolves the partnership with any ownership shares even when types are positively interdependent. We also discussed the possibility of efficient dissolution with ex post veto/quitting rights.

Appendix

A.1 Proof of Theorem 2

Recall that in the symmetric n-agent environment in consideration,

$$u_i^{\mathrm{G}}(\theta) = v_i(\theta)s_i^*(\theta) + t_i^{\mathrm{G}}(\theta) = \begin{cases} g(\theta_i) + \sum_{j\neq i} h(\theta_j) & \text{if } i = m(\theta), \\ g(\theta^1) + \sum_{j\neq i} h(\theta_j) & \text{if } i \neq m(\theta). \end{cases}$$

Proof of Theorem 2. We set

$$b_i(\theta) = \frac{1}{n}b^{\mathrm{G}}(\theta) + \begin{cases} -\dfrac{n-1}{n}\displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dg(x) & \text{if } i = m(\theta), \\ \dfrac{1}{n}\displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dg(x) & \text{if } i \neq m(\theta), \end{cases}$$

where

$$b^{\mathrm{G}}(\theta) = (n-1)g(\theta^1) + h(\theta^1) + (n-2)\sum_{j\in N} h(\theta_j),$$

and show that it satisfies the condition (3.7) in Theorem 1. For each $i \in N$, let

$$C_i(\theta) = u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - b_i(\theta) + E_{\theta_{-i}}[b_i(\theta)] - \frac{1}{n-1}\sum_{j\neq i} E_{\theta_{-j}}[b_j(\theta)].$$

It suffices to show that $C_i(\theta) \geq 0$ for all $\theta \in \Theta$.

First,

$$u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - b_i(\theta) = \Gamma_i(\theta) - \frac{1}{n}h(\theta^1) - \frac{n-2}{n}h(\theta_i) + \frac{1}{n}\sum_{j\neq i} h(\theta_j),$$

where

$$\Gamma_i(\theta) = \begin{cases} \dfrac{n-1}{n}\displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dg(x) & \text{if } i = m(\theta), \\ \dfrac{1}{n}\big(g(\theta^1) - g(\theta_i)\big) - \dfrac{1}{n}\displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dg(x) & \text{if } i \neq m(\theta). \end{cases}$$

Note that $\Gamma_i(\theta) \geq 0$ since, if $i \neq m(\theta)$,

$$
\begin{aligned}
\big(g(\theta^1) - g(\theta_i)\big) - \int_{\theta^2}^{\theta^1} F(x)\,dg(x) &\geq \big(g(\theta^1) - g(\theta^2)\big) - \int_{\theta^2}^{\theta^1} F(x)\,dg(x) \\
&= \int_{\theta^2}^{\theta^1} \big(1 - F(x)\big)\,dg(x) \geq 0,
\end{aligned}
$$

and that $\Gamma_i(\theta) = 0$ when $\theta_1 = \cdots = \theta_n$.

Second,

$$
\begin{aligned}
E_{\theta_{-i}}[b_i(\theta)] = {} & \frac{n-1}{n}\left(g(\theta_i)F^{n-1}(\theta_i) + \int_{\theta_i}^{\overline{\theta}} g(x)\,dF^{n-1}(x)\right) \\
& + \frac{1}{n}\left(h(\theta_i)F^{n-1}(\theta_i) + \int_{\theta_i}^{\overline{\theta}} h(x)\,dF^{n-1}(x)\right) \\
& + \frac{n-2}{n}\left(h(\theta_i) + (n-1)\int_{\underline{\theta}}^{\overline{\theta}} h(x)\,dF(x)\right) \\
& + \int_{\underline{\theta}}^{\theta_i}\left(-\frac{n-1}{n}\right)\int_{y}^{\theta_i} F(x)\,dg(x)\,dF^{n-1}(y) \\
& + \int_{\theta_i}^{\overline{\theta}} \frac{1}{n}\int_{\theta_i}^{y} F(x)\,dg(x)\,F^{n-2}(\theta_i)\,(n-1)\,dF(y) \\
& + \int_{\theta_i}^{\overline{\theta}}\int_{\theta_i}^{y_1} \frac{1}{n}\int_{y_2}^{y_1} F(x)\,dg(x)\,dF^{n-2}(y_2)\,(n-1)\,dF(y_1) \\
= {} & \frac{n-1}{n}\left(g(\overline{\theta}) - \int_{\theta_i}^{\overline{\theta}} F^{n-1}(x)\,dg(x)\right) \\
& + \frac{1}{n}\left(\int_{\underline{\theta}}^{\theta_i} F(x)\,dh(x) + \int_{\underline{\theta}}^{\overline{\theta}} h(x)\,dF^{n-1}(x)\right) \\
& + \frac{n-2}{n}\left(h(\theta_i) + (n-1)\int_{\underline{\theta}}^{\overline{\theta}} h(x)\,dF(x)\right) \\
& - \frac{n-1}{n}\int_{\underline{\theta}}^{\theta_i} F^n(x)\,dg(x) \\
& + \frac{n-1}{n}\int_{\theta_i}^{\overline{\theta}} \big(F(x) - F^2(x)\big)F^{n-2}(\theta_i)\,dg(x) \\
& + \frac{n-1}{n}\int_{\theta_i}^{\overline{\theta}} \big(F(x) - F^2(x)\big)\big(F^{n-2}(x) - F^{n-2}(\theta_i)\big)\,dg(x)
\end{aligned}
$$

$$= \frac{n-2}{n} h(\theta_i) + \frac{1}{n} \int_{\underline{\theta}}^{\theta_i} F(x)\, dh(x) + \frac{n-1}{n} \int_{\underline{\theta}}^{\overline{\theta}} g(x)\, dF^n(x)$$
$$+ \frac{1}{n} \int_{\underline{\theta}}^{\overline{\theta}} h(x)\, dF^{n-1}(x) + \frac{(n-1)(n-2)}{n} \int_{\underline{\theta}}^{\overline{\theta}} h(x)\, dF(x).$$

Thus, we have

$$\begin{aligned}
C_i(\theta) &= \Gamma_i(\theta) - \frac{1}{n} h(\theta^1) + \frac{1}{n} \sum_{j \neq i} h(\theta_j) + \frac{1}{n} \int_{\underline{\theta}}^{\theta_i} F(x)\, dh(x) \\
&\quad - \frac{1}{n-1} \sum_{j \neq i} \left(\frac{n-2}{n} h(\theta_j) + \frac{1}{n} \int_{\underline{\theta}}^{\theta_j} F(x)\, dh(x) \right) \\
&= \Gamma_i(\theta) - \frac{1}{n(n-1)} \sum_{j \neq i} \big(h(\theta^1) - h(\theta_j) \big) \\
&\quad + \frac{1}{n(n-1)} \sum_{j \neq i} \int_{\theta_j}^{\theta_i} F(x)\, dh(x) \\
&\geq \Gamma_i(\theta) + \frac{1}{n(n-1)} \sum_{j \neq i} \int_{\theta_j}^{\theta^1} \big(1 - F(x) \big)\, d(-h)(x) \geq 0,
\end{aligned}$$

with equality when $\theta_1 = \cdots = \theta_n$. ▮

The implementing mechanism is written as follows:

$$\begin{aligned}
t_i^*(\theta) = &- \frac{n-1}{n} \left[g(\theta^1) - \int_{\theta^2}^{\theta^1} F(x)\, dg(x) \right] \\
&- \frac{1}{n} h(\theta^1) + \frac{1}{n-1} \sum_{j \neq i} h(\theta_j) + \frac{1}{n(n-1)} \sum_{j \neq i} \int_{\theta_j}^{\theta_i} F^{n-1}(x)\, dh(x) \\
&- \sum_{j \neq i} h(\theta_j)
\end{aligned} \tag{A.1}$$

if $i = m(\theta)$, and

$$\begin{aligned}
t_i^*(\theta) = &\frac{1}{n} \left[g(\theta^1) - \int_{\theta^2}^{\theta^1} F(x)\, dg(x) \right] \\
&- \frac{1}{n} h(\theta^1) + \frac{1}{n-1} \sum_{j \neq i} h(\theta_j) + \frac{1}{n(n-1)} \sum_{j \neq i} \int_{\theta_j}^{\theta_i} F^{n-1}(x)\, dh(x)
\end{aligned} \tag{A.2}$$

if $i \neq m(\theta)$. When $n = 2$, the transfer rule (A.1)–(A.2) reduces to (4.4).

A.2 Proof of Theorem 3

The budget deficit generated (on the equilibrium path) by the two-stage Groves mechanism $(s^*, \tau^{\mathcal{G}})$ is

$$\begin{aligned} b^{\mathcal{G}}(\theta) &= (n-1)v_{m(\theta)}(\theta) \\ &= (n-1)g\big(\theta_{m(\theta)}\big) + (n-1)\sum_{j\neq m(\theta)} h(\theta_j) \\ &= (n-1)(g-h)\big(\theta_{m(\theta)}\big) + (n-1)\sum_{j\in N} h(\theta_j). \end{aligned}$$

The ex post utility and the outside option are

$$\begin{aligned} u_i^{\mathcal{G}}(\theta) &= v_{m(\theta)}(\theta) \\ &= (g-h)\big(\theta_{m(\theta)}\big) + \sum_{j\in N} h(\theta_j) \end{aligned}$$

and

$$u_i^0(\theta) = \frac{1}{n}(g-h)(\theta_i) + \frac{1}{n}\sum_{j\in N} h(\theta_j)$$

for all $i \in N$. Let

$$V_i^{\mathcal{G}}(\theta_i) = E_{\theta_{-i}}\left[u_i^{\mathcal{G}}(\theta) - u_i^0(\theta)\right].$$

The necessary and sufficient condition for a two-stage Groves mechanism in expectations to achieve IIR-dissolution is

$$E_\theta\left[b^{\mathcal{G}}(\theta)\right] \leq \sum_{i\in N} \min_{\theta_i\in\Theta_i} V_i^{\mathcal{G}}(\theta_i). \tag{A.3}$$

Proof of Theorem 3. We show that when $h' > 0$, there exists a distribution function F that violates the condition (A.3).

Observe that

$$\begin{aligned} V_i^{\mathcal{G}}(\theta_i) = (g-h)(\theta_i)\big(F^{n-1}(\theta_i) - \alpha_i\big) + \int_{\theta_i}^{\overline{\theta}} (g-h)(x)\, dF^{n-1}(x) \\ + (1-\alpha_i)h(\theta_i) + (1-\alpha_i)(n-1)\int_{\underline{\theta}}^{\overline{\theta}} h(x)\, dF(x), \end{aligned}$$

and hence,

$$\begin{aligned} \sum_{i\in N} V_i^{\mathcal{G}}(\tilde{\theta}_i) = \sum_{i\in N}\int_{\tilde{\theta}_i}^{\overline{\theta}} (g-h)(x)\, dF^{n-1}(x) \\ + \sum_{i\in N}(1-\alpha_i)h(\theta_i) + (n-1)^2\int_{\underline{\theta}}^{\overline{\theta}} h(x)\, dF(x), \end{aligned}$$

while

$$E_\theta\left[b^{\mathcal{G}}(\theta)\right] = (n-1)\int_{\underline{\theta}}^{\overline{\theta}}(g-h)(x)\,dF^n(x) + n(n-1)\int_{\underline{\theta}}^{\overline{\theta}} h(x)\,dF(x)$$

$$= n\int_{\underline{\theta}}^{\overline{\theta}}(g-h)(x)F(x)\,dF^{n-1}(x) + n(n-1)\int_{\underline{\theta}}^{\overline{\theta}} h(x)\,dF(x).$$

Let $\tilde{\theta}_i$ be defined by $F^{n-1}(\tilde{\theta}_i) = \alpha_i$. Then, a necessary condition for (A.3) is

$$\sum_{i\in N}\left\{\int_{\tilde{\theta}_i}^{\overline{\theta}}(g-h)(x)\,dF^{n-1}(x) - \int_{\underline{\theta}}^{\overline{\theta}}(g-h)(x)F(x)\,dF^{n-1}(x)\right\}$$
$$+\sum_{i\in N}(1-\alpha_i)h(\tilde{\theta}_i) - (n-1)\int_{\underline{\theta}}^{\overline{\theta}} h(x)\,dF(x) \geq 0. \quad \text{(A.4)}$$

When $\alpha_i = 1/n$, inequality (A.4) reads

$$n\int_{\tilde{\theta}}^{\overline{\theta}}(g-h)(x)\,dF^{n-1}(x) - n\int_{\underline{\theta}}^{\overline{\theta}}(g-h)(x)F(x)\,dF^{n-1}(x)$$
$$+(n-1)h(\tilde{\theta}) - (n-1)\int_{\underline{\theta}}^{\overline{\theta}} h(x)\,dF(x) \geq 0, \quad \text{(A.5)}$$

where $\tilde{\theta} = F^{-1}\big((1/n)^{1/(n-1)}\big)$. We have

$$\begin{aligned}
&\text{(LHS of (A.5))}\\
&= \left\{n\Big[(g-h)(x)F^{n-1}(x)\Big]_{\tilde{\theta}}^{\overline{\theta}} - n\int_{\tilde{\theta}}^{\overline{\theta}} F^{n-1}(x)\,d(g-h)(x)\right\}\\
&\quad - \left\{(n-1)\Big[(g-h)(x)F^n(x)\Big]_{\underline{\theta}}^{\overline{\theta}} - (n-1)\int_{\underline{\theta}}^{\overline{\theta}} F^n(x)\,d(g-h)(x)\right\}\\
&\quad + (n-1)h(\tilde{\theta}) - \left\{(n-1)\Big[h(x)F(x)\Big]_{\underline{\theta}}^{\overline{\theta}} - (n-1)\int_{\underline{\theta}}^{\overline{\theta}} F(x)\,dh(x)\right\}\\
&= \Big\{(g-h)(\overline{\theta}) - n(g-h)(\tilde{\theta})F^{n-1}(\tilde{\theta})\Big\} - n\int_{\tilde{\theta}}^{\overline{\theta}} F^{n-1}(x)\,d(g-h)(x)\\
&\quad + (n-1)\int_{\tilde{\theta}}^{\overline{\theta}} F^n(x)\,d(g-h)(x) + (n-1)\int_{\underline{\theta}}^{\tilde{\theta}} F^n(x)\,d(g-h)(x)\\
&\quad - (n-1)\big(h(\overline{\theta}) - h(\tilde{\theta})\big) + (n-1)\int_{\tilde{\theta}}^{\overline{\theta}} F(x)\,dh(x)\\
&\quad + (n-1)\int_{\underline{\theta}}^{\tilde{\theta}} F(x)\,dh(x),
\end{aligned}$$

$$
\begin{aligned}
&= \int_{\tilde\theta}^{\overline\theta} \big\{1 - nF^{n-1}(x) + (n-1)F^n(x)\big\}\, d(g-h)(x) \\
&\quad + (n-1)\int_{\underline\theta}^{\tilde\theta} F^n(x)\, d(g-h)(x) \\
&\quad - (n-1)\int_{\tilde\theta}^{\overline\theta} \big(1 - F(x)\big)\, dh(x) + (n-1)\int_{\underline\theta}^{\tilde\theta} F(x)\, dh(x).
\end{aligned}
$$

Recall $F^{n-1}(\tilde\theta) = 1/n$.

Given g and h (such that $g' > h' > 0$), we want to find a distribution F such that inequality (A.5) is violated, that is,

$$
\begin{aligned}
&\int_{\tilde\theta}^{\overline\theta} \big\{1 - nF^{n-1}(x) + (n-1)F^n(x)\big\}\, d(g-h)(x) + (n-1)\int_{\underline\theta}^{\tilde\theta} F^n(x)\, d(g-h)(x) \\
&\quad - (n-1)\int_{\tilde\theta}^{\overline\theta} \big(1 - F(x)\big)\, dh(x) + (n-1)\int_{\underline\theta}^{\tilde\theta} F(x)\, dh(x) < 0. \quad \text{(A.6)}
\end{aligned}
$$

Let $a = \max_{x \in [\underline\theta, \overline\theta]} g'(x) > 0$ and $b = \min_{x \in [\underline\theta, \overline\theta]} h'(x) > 0$. Note that $a > b$. Define

$$
\varphi(z) = \big\{1 - nz^{n-1} + (n-1)z^n\big\}(a-b) - (n-1)(1-z)b.
$$

Since $\varphi(1) = 0$ and $\varphi'(1) = (n-1)b > 0$, there exists $\delta \in (1/n^{1/(n-1)}, 1)$ such that $\varphi(\delta) < 0$. Take such a δ.

Now define

$$
F^*(x) = \begin{cases} \delta & \text{if } x \in [\underline\theta, \overline\theta), \\ 1 & \text{if } x = \overline\theta, \end{cases}
$$

and thus set $\tilde\theta = \underline\theta$. Then,

$$
\begin{aligned}
&\text{(LHS of (A.6))} \\
&= \int_{\underline\theta}^{\overline\theta} \big\{1 - n\delta^{n-1} + (n-1)\delta^n\big\}\, d(g-h)(x) - (n-1)\int_{\underline\theta}^{\overline\theta} (1-\delta)\, dh(x) \\
&\le \int_{\underline\theta}^{\overline\theta} \big\{1 - n\delta^{n-1} + (n-1)\delta^n\big\}\, (a-b)\, dx - (n-1)\int_{\underline\theta}^{\overline\theta} (1-\delta) b\, dx \\
&= \varphi(\delta)\left(\overline\theta - \underline\theta\right) < 0,
\end{aligned}
$$

where $1 - nz^{n-1} + (n-1)z^n > 0$ for all $z \in (0,1)$. This implies that inequality (A.6) is satisfied for $F = F^*$. The result thus follows since F^* can be approximated arbitrarily close by continuously differentiable distribution, strictly increasing functions. ▮

A.3 An Example Violating Assumption 5.1

We present an example showing that Assumption 5.1 is necessary for EPIR-dissolution with a finite penalty P.

There are three agents whose types are independently and uniformly distributed on $[0, 1]$. Their valuation functions are defined by

$$\begin{aligned} v_1(\theta_1, \theta_2, \theta_3) &= 2\theta_1, \\ v_2(\theta_1, \theta_2, \theta_3) &= \frac{1}{2}\theta_1 + \theta_2 + \frac{1}{2}\theta_3, \\ v_3(\theta_1, \theta_2, \theta_3) &= \frac{1}{2}\theta_1 + \frac{1}{2}\theta_2 + \theta_3. \end{aligned}$$

Note that v_2 and v_3 are symmetric. We assume $\alpha_1 < 1$ and $\alpha_2, \alpha_3 > 0$.

First consider agent $i = 1$. When agent 1 reports $\hat{\theta}_1 \neq \theta_1$, he is the winner if and only if

$$2\theta_2 + \theta_3 < 3\hat{\theta}_1 \text{ and } \theta_2 + 2\theta_3 < 3\hat{\theta}_1.$$

Suppose $\hat{\theta}_1 \geq 2/3$. Then the probability that each agent $j = 2, 3$ wins is $3(1-\hat{\theta}_1)^2/2$. Since for each $j = 2, 3$ and for all θ_{-1}, $v_j(\theta_1, \theta_{-1}) \neq v_j(\hat{\theta}_1, \theta_{-1})$ whenever $\theta_1 \neq \hat{\theta}_i$, we have

$$\sum_{j \neq 1} E_{\theta_{-1}} \left[\mathbf{1}_{\{j = m(\hat{\theta}_1, \theta_{-1}),\, v_j(\theta_1, \theta_{-1}) \neq v_j(\hat{\theta}_1, \theta_{-1})\}} \right] = 3(1 - \hat{\theta}_1)^2.$$

On the other hand,

$$E_{\theta_{-1}} \left[\mathbf{1}_{\{1 = m(\hat{\theta}_1, \theta_{-1})\}} \left(v_1(\overline{\theta}_1, \theta_{-1}) - v_1(\hat{\theta}_1, \theta_{-1}) \right) \right] = 2\big\{1 - 3(1 - \hat{\theta}_1)^2\big\}(1 - \hat{\theta}_1).$$

Since $2\big\{1 - 3(1-\hat{\theta}_1)^2\big\}(1-\hat{\theta}_1)/\big\{3(1-\hat{\theta}_1)^2\big\} \to \infty$ as $\hat{\theta}_1 \to \overline{\theta}_1 = 1$, there is no M_1 that satisfies condition (5.3) for all $\theta_1, \hat{\theta}_1$. Indeed, if $\hat{\theta}_1 < \theta_1$, then, since $U_1(\theta_1) \leq E_{\theta_{-1}}\big[\mathbf{1}_{1 = m(\hat{\theta}_1, \theta_{-1})}\big]\alpha_1 \cdot 2\theta_1 + E_{\theta_{-1}}\big[\mathbf{1}_{1 \neq m(\hat{\theta}_1, \theta_{-1})}\big]\alpha_1 \cdot 2$, we have

$$\begin{aligned} &U_1(\theta_1) - U_1(\theta_1, \hat{\theta}_1) \\ &\quad \leq -2\big\{1 - 3(1 - \hat{\theta}_1)^2\big\}(1 - \alpha_1)(\theta_1 - \hat{\theta}_1) + 3(1 - \hat{\theta}_1)^2(P + 2\alpha_1). \end{aligned}$$

For any $P < \infty$, there exist θ_1 and $\hat{\theta}_1$ sufficiently close to 1 such that the right hand side is negative. Hence, for any finite constant P, the IC condition does not hold for some θ_1, $\hat{\theta}_1$.

Next consider agent $i = 2$. The probability that agent 3 wins converges to zero as $\hat{\theta}_2 \to \overline{\theta}_2 = 1$, while that of agent 1 converges to a positive probability 1/6. It follows that there is no M_2 that satisfies (5.4) for all $\theta_2, \hat{\theta}_2$. Indeed, if $\hat{\theta}_2 < \theta_2$, then we have

$$\begin{aligned} U_2(\theta_2) - U_2(\theta_2, \hat{\theta}_2) \leq & - E_{\theta_{-2}} \left[\mathbf{1}_{\{2=m(\hat{\theta}_2, \theta_{-2})\}} \right] (1-\alpha_2)(\theta_2 - \hat{\theta}_2) \\ & - E_{\theta_{-2}} \left[\mathbf{1}_{\{1=m(\hat{\theta}_2, \theta_{-2})\}} \right] 2\alpha_2 \\ & + E_{\theta_{-2}} \left[\mathbf{1}_{\{3=m(\hat{\theta}_2, \theta_{-2})\}} \right] (P + 2\alpha_2). \end{aligned}$$

For any $P < \infty$, there exist θ_2 and $\hat{\theta}_2$ sufficiently close to 1 such that the right hand side is negative. Again, for any P, the IC condition does not hold for some θ_2, $\hat{\theta}_2$.

A.4 Proof of Proposition 5

Fix any agent $i \in N$ and his type $\theta_i \in [\underline{\theta}, \overline{\theta}]$, and suppose that agent i reports $\hat{\theta}_i$, while agent $-i$ truthfully reports his type θ_{-i}. Define the "ex post regret" under (s^*, t^*),

$$\Delta(\theta_{-i}) = \frac{1}{2} v_i(\theta_i, \theta_{-i}) - u_i(\theta_i, \hat{\theta}_i, \theta_{-i}), \tag{A.7}$$

as a function of agent $-i$'s type θ_{-i}. Then, we have

$$U_i^*(\theta_i, \hat{\theta}_i) = U_i(\theta_i, \hat{\theta}_i) + E_{\theta_{-i}} \left[\Delta(\theta_{-i}) \mathbf{1}_{\{\Delta(\theta_{-i}) \geq 0\}} \right].$$

Notice that

$$\Delta(\theta_{-i}) = \begin{cases} -\frac{1}{2} v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i}) & \text{if } \hat{\theta}_i > \theta_{-i}, \\ \frac{1}{2} v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i}) & \text{if } \hat{\theta}_i < \theta_{-i}. \end{cases} \tag{A.8}$$

Lemma A.1. *If $\hat{\theta}_i > \theta_i$, then $\Delta(\theta_{-i}) < 0$ for all $\theta_{-i} > \hat{\theta}_i$, while if $\hat{\theta}_i < \theta_i$, then $\Delta(\theta_{-i}) < 0$ for all $\theta_{-i} < \hat{\theta}_i$.*

Proof. Consider the former case where $\hat{\theta}_i > \theta_i$. Suppose that $\theta_{-i} > \hat{\theta}_i$, so that player $-i$ obtains the entire asset. Then, we have

$$u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) = t_i^*(\hat{\theta}_i, \theta_{-i}) \geq \frac{1}{2} v_i(\hat{\theta}_i, \theta_{-i}) > \frac{1}{2} v_i(\theta_i, \theta_{-i}),$$

where the first inequality follows from the fact that (s^*, t^*) satisfies EPIR, while the second inequality follows from the assumption that $v_i(\theta_i, \theta_{-i})$ is strictly increasing in θ_i.

Consider then the latter case where $\hat{\theta}_i < \theta_i$. Suppose that $\theta_{-i} < \hat{\theta}_i$, so that player i obtains the asset. Then we have

$$\begin{aligned} u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) &= v_i(\theta_i, \theta_{-i}) + t_i^*(\hat{\theta}_i, \theta_{-i}) \\ &\geq v_i(\theta_i, \theta_{-i}) - \frac{1}{2} v_i(\hat{\theta}_i, \theta_{-i}) > \frac{1}{2} v_i(\theta_i, \theta_{-i}), \end{aligned}$$

where the first inequality follows from EPIR, $v_i(\hat{\theta}_i, \theta_{-i}) + t_i^*(\hat{\theta}_i, \theta_{-i}) \geq (1/2)v_i(\hat{\theta}_i, \theta_{-i})$, while the second from the assumption that $v_i(\theta_i, \theta_{-i})$ is strictly increasing in θ_i. ▌

Lemma A.2. *For each $\hat{\theta}_i \neq \theta_i$, there exists $\beta(\hat{\theta}_i) \in [\underline{\theta}, \overline{\theta}]$ such that if $\hat{\theta}_i > \theta_i$, then $\beta(\hat{\theta}_i) < \hat{\theta}_i$ and*

$$\{\theta_{-i} \neq \hat{\theta}_i \mid \Delta(\theta_{-i}) > 0\} = \Big(\beta(\hat{\theta}_i), \hat{\theta}_i\Big),$$

while if $\hat{\theta}_i < \theta_i$, then $\beta(\hat{\theta}_i) > \hat{\theta}_i$ and

$$\{\theta_{-i} \neq \hat{\theta}_i \mid \Delta(\theta_{-i}) > 0\} = \Big(\hat{\theta}_i, \beta(\hat{\theta}_i)\Big).$$

Proof. Consider first the case where $\hat{\theta}_i > \theta_i$. By Lemma A.1, $\Delta(\theta_{-i}) < 0$ for all $\theta_{-i} > \hat{\theta}_i$, and thus we consider $\theta_{-i} < \hat{\theta}_i$, where player i obtains the entire asset. Since $\Delta(\theta_{-i})$ is continuous (in fact differentiable) on $[\underline{\theta}, \hat{\theta}_i)$, it is sufficient to show that $\Delta(\theta_{-i})$ is strictly increasing on $[\underline{\theta}, \hat{\theta}_i)$ and that $\lim_{\theta_{-i} \nearrow \hat{\theta}_i} \Delta(\theta_{-i}) > 0$. Indeed, recalling that

$$\begin{aligned}
\Delta(\theta_{-i}) &= -\frac{1}{2}v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i}) \\
&= -\frac{1}{2}v_i(\theta_i, \theta_{-i}) + \frac{1}{2}\left[v(\hat{\theta}_i) - \int_{\theta_{-i}}^{\hat{\theta}_i} F(x)\, dv(x)\right],
\end{aligned}$$

we have

$$\begin{aligned}
\Delta'(\theta_{-i}) &= -\frac{1}{2}h'(\theta_{-i}) + \frac{1}{2}F(\theta_{-i})\big(g'(\theta_{-i}) + h'(\theta_{-i})\big) \\
&= \frac{1}{2}F(\theta_{-i})g'(\theta_{-i}) - \frac{1}{2}\big(1 - F(\theta_{-i})\big)h'(\theta_{-i}) > 0 \qquad \text{(A.9)}
\end{aligned}$$

for all $\theta_{-i} \in (\underline{\theta}, \hat{\theta}_i)$, where the inequality follows from the assumption that $g' > 0$ and $h' \leq 0$. Since $\lim_{\theta_{-i} \nearrow \hat{\theta}_i} t_i^*(\hat{\theta}_i, \theta_{-i}) = -(1/2)v_i(\hat{\theta}_i, \hat{\theta}_i)$, we also have

$$\lim_{\theta_{-i} \nearrow \hat{\theta}_i} \Delta(\theta_{-i}) = -\frac{1}{2}v_i(\theta_i, \hat{\theta}_i) + \frac{1}{2}v_i(\hat{\theta}_i, \hat{\theta}_i) > 0,$$

where the inequality follows from the assumption that $v_i(\theta_i, \theta_{-i})$ is strictly increasing in θ_i. Thus, defining $\beta(\hat{\theta}_i)$ as follows gives the first expression in the lemma:

$$\beta(\hat{\theta}_i) = \begin{cases} \underline{\theta} & \text{if } \Delta(\theta_{-i}) > 0 \text{ for all } \theta_{-i} \in [\underline{\theta}, \hat{\theta}_i), \\ \theta_{-i} \text{ satisfying } \Delta(\theta_{-i}) = 0 & \text{otherwise.} \end{cases}$$

For the other case where $\hat{\theta}_i < \theta_i$, the similar argument shows that $\Delta(\theta_{-i})$ is strictly decreasing on $(\hat{\theta}_i, \overline{\theta}]$ and that $\lim_{\theta_{-i} \searrow \hat{\theta}_i} \Delta(\theta_{-i}) > 0$. Thus, define $\beta(\hat{\theta}_i)$ as follows:

$$\beta(\hat{\theta}_i) = \begin{cases} \overline{\theta} & \text{if } \Delta(\theta_{-i}) > 0 \text{ for all } \theta_{-i} \in (\hat{\theta}_i, \overline{\theta}], \\ \theta_{-i} \text{ satisfying } \Delta(\theta_{-i}) = 0 & \text{otherwise.} \end{cases}$$

This completes the proof. ▮

Lemma A.3. *Let $\beta(\hat{\theta}_i)$ be as in Lemma A.2. Then, for all $i \in N$ and all $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$,*

$$\begin{cases} \beta(\hat{\theta}_i) \geq \theta_i & \text{if } \hat{\theta}_i > \theta_i, \\ \beta(\hat{\theta}_i) \leq \theta_i & \text{if } \hat{\theta}_i < \theta_i \end{cases}$$

if and only if $g' + h' \leq 0$.

Proof. It suffices to examine the sign of $\Delta(\theta_{-i})$ at $\theta_{-i} = \theta_i$. If $\theta_i < \hat{\theta}_i$, then

$$\begin{aligned} \Delta(\theta_i) &= -\frac{1}{2} v_i(\theta_i, \theta_i) - t_i^*(\hat{\theta}_i, \theta_i) \\ &= -\frac{1}{2} v_i(\theta_i, \theta_i) + \frac{1}{2}\left[v(\hat{\theta}_i) - \int_{\theta_i}^{\hat{\theta}_i} F(x)\, dv(x) \right] \\ &= \frac{1}{2} \int_{\theta_i}^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x), \end{aligned} \tag{A.10}$$

while if $\hat{\theta}_i < \theta_i$, then

$$\begin{aligned} \Delta(\theta_i) &= \frac{1}{2} v_i(\theta_i, \theta_i) - t_i^*(\hat{\theta}_i, \theta_i) \\ &= \frac{1}{2} v_i(\theta_i, \theta_i) - \frac{1}{2}\left[v(\theta_i) - \int_{\hat{\theta}_i}^{\theta_i} F(x)\, dv(x) \right] \\ &= \frac{1}{2} \int_{\hat{\theta}_i}^{\theta_i} F(x)\, dv(x). \end{aligned}$$

It follows that $\Delta(\theta_i) \leq 0$ for all $\theta_i \neq \hat{\theta}_i$ in both cases if and only if $v' = g' + h' \leq 0$ (since $0 < F(x) < 1$ for all $x \in (\underline{\theta}, \overline{\theta})$). But, by Lemma A.2, for all $\theta_i < \hat{\theta}_i$ ($\theta_i > \hat{\theta}_i$, resp.), $\Delta(\theta_i) \leq 0$ if and only if $\beta(\hat{\theta}_i) \geq \theta_i$ ($\beta(\hat{\theta}_i) \leq \theta_i$, resp.). ▮

Proof of Proposition 5. "If" part: Suppose that $g' + h' \leq 0$. We want to show that for each $i \in N$, $U_i^*(\theta_i) \geq U_i^*(\theta_i, \hat{\theta}_i)$ for all $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$. We show this only for the case where $\theta_i < \hat{\theta}_i$. In this case, we have $\beta(\hat{\theta}_i) < \hat{\theta}_i$ as in Lemma A.2, and

$$U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) = \big(U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i)\big) - \int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta(y)\, dF(y). \tag{A.11}$$

Recalling (A.8), we have

$$\begin{aligned}\Delta(y) &= -\frac{1}{2}\big(g(\theta_i) + h(y)\big) + \frac{1}{2}\left(v(\hat{\theta}_i) - \int_y^{\hat{\theta}_i} F(x)\, dv(x)\right) \\ &= \frac{1}{2}\big(g(y) - g(\theta_i)\big) + \frac{1}{2}\int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x),\end{aligned}$$

and therefore,

$$\begin{aligned}&\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta(y)\, dF(y) \\ &\qquad = \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF(y) - \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\theta_i} \big(g(y) - g(\theta_i)\big)\, dF(y) \\ &\qquad\quad + \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x)\, dF(y) \\ &\qquad = \frac{1}{2}\big(U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i)\big) - \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\theta_i} \big(g(y) - g(\theta_i)\big)\, dF(y) \\ &\qquad\quad + \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x)\, dF(y),\end{aligned}$$

where in the last equality we used the formula

$$U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) = \int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF(y),$$

which follows from the Revenue Equivalence. Hence,

$$\begin{aligned}\text{(A.11)} = \frac{1}{2}\big(U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i)\big) + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} \big(g(y) - g(\theta_i)\big)\, dF(y) \\ + \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, d(-v)(x)\, dF(y). \qquad \text{(A.12)}\end{aligned}$$

Here, the first term is non-negative by IC, and so are the other two since g is increasing and $v' = g' + h' \leq 0$ by assumption. Thus, we have $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) \geq 0$ as desired.

"Only if" part: Suppose that $g' + h' > 0$. We want to find a type distribution and types $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$ for which $U_i^*(\theta_i) < U_i^*(\theta_i, \hat{\theta}_i)$. For ease of notation, we let $[\underline{\theta}, \overline{\theta}] = [0, 1]$.

We first give a heuristic argument to outline the formal proof that follows.

Let the type distribution F be given by

$$F(x) = \begin{cases} 0 & \text{if } 0 \leq x < \dfrac{1}{4}, \\ \dfrac{1}{2} & \text{if } \dfrac{1}{4} \leq x < \dfrac{3}{4}, \\ 1 & \text{if } \dfrac{3}{4} \leq x \leq 1, \end{cases}$$

which violates the full-support assumption, and set $\theta_i = 1/4$ and $\hat{\theta}_i = 1/2$. Then we have

$$U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) = \int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF(y) = 0,$$

while, since $\beta(\hat{\theta}_i) < \theta_i$ $(< \hat{\theta}_i)$ by Lemma A.3,

$$\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta(y)\, dF(y) = \Delta(\theta_i) F(\theta_i) > 0.$$

Thus, from (A.11) we have $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) < 0$.

Now let us consider the following sequence of distribution functions $(F_n)_{n=1,2,\ldots}$ with full support on $[0,1]$:

$$F_n(x) = \begin{cases} \dfrac{4n+3}{2n+3} x & \text{if } 0 \leq x < \dfrac{1}{4}, \\ \dfrac{2^{4n+1}}{2n+3}\Big(x - \dfrac{1}{2}\Big)^{2n+1} + \dfrac{1}{2n+3}\Big(x - \dfrac{1}{2}\Big) + \dfrac{1}{2} & \text{if } \dfrac{1}{4} \leq x < \dfrac{3}{4}, \\ \dfrac{4n+3}{2n+3}(x-1) + 1 & \text{if } \dfrac{3}{4} \leq x \leq 1. \end{cases}$$

The function F_n is continuously differentiable on $[0,1]$, and satisfies $F_n(0) = 0$, $F_n(1/2) = 1/2$, and $F_n(1) = 1$. Note that for $x \in [1/4, 3/4]$, $F_n(x) \to 1/2$ as $n \to \infty$. For each F_n, let Δ_n and β_n be as in (A.7) and Lemma A.2, respectively. Set $\theta_i = 1/4$ and $\hat{\theta}_i = 1/2$, where $\beta_n(\hat{\theta}_i) < \theta_i$ by Lemma A.3. We first have

$$\begin{aligned} U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) &= \int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF_n(y) \\ &< \big(g(\hat{\theta}_i) - g(\theta_i)\big)\big(F_n(\hat{\theta}_i) - F_n(\theta_i)\big) \to 0 \text{ as } n \to \infty. \end{aligned}$$

On the other hand, by (A.9), $\Delta_n'(x)$ is bounded from above uniformly for n and $x \in [0, \theta_i]$, and by (A.10), $\Delta_n(\theta_i)$ is bounded from zero uniformly for n, as

$$\begin{aligned} \Delta_n(\theta_i) &= \frac{1}{2} \int_{\theta_i}^{\hat{\theta}_i} \big(1 - F_n(x)\big)\, dv(x) \\ &\geq \frac{1}{2}\big(v(\hat{\theta}_i) - v(\theta_i)\big)\big(1 - F_n(\hat{\theta}_i)\big) = \frac{1}{4}\big(v(\hat{\theta}_i) - v(\theta_i)\big) > 0. \end{aligned}$$

It follows that we can take a $\delta > 0$ with $\theta_i - \delta \geq \underline{\theta}$ and a $D > 0$ such that for all n, $\Delta_n(x) > D$ for all $x \in [\theta_i - \delta, \theta_i]$. Hence, we have

$$\begin{aligned}\int_{\beta_n(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta_n(y)\, dF_n(y) &> \int_{\theta_i-\delta}^{\theta_i} \Delta_n(y)\, dF_n(y)\\ &> D\delta \frac{4n+3}{2n+3} > D\delta > 0\end{aligned}$$

for all n. Thus, from (A.11) we have $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) < 0$ for sufficiently large n. ∎

A.5 Proof for Example 6.1

We show the following.

Proposition A.4. *Let $[\underline{\theta}, \overline{\theta}] = [0, 1]$, and F be the uniform distribution on $[0, 1]$. Assume that $g(x) = x$ and $h(x) = -\gamma x$, where $\gamma \geq 0$. Then, (s^*, t^*) satisfies IC* if and only if $\gamma \geq 1/2$.*

Proof. We have already shown in Proposition 5 that (s^*, t^*) satisfies IC* if $g' + h' = 1 - \gamma \leq 0$. It is therefore sufficient to consider only the case where $1 - \gamma > 0$. In this case, by Lemma A.3, θ_i falls between $\beta(\hat{\theta}_i)$ and $\hat{\theta}_i$.

"If" part: Assume that $1/2 \leq \gamma$ (< 1), or $(0 <)$ $1 - \gamma \leq 1/2$. We want to show that $U_i^*(\theta_i) \geq U_i^*(\theta_i, \hat{\theta}_i)$ for all $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$. We show this only for the case where $\theta_i < \hat{\theta}_i$. In this case, we have $\beta(\hat{\theta}_i) < \hat{\theta}_i$ as in Lemma A.2. By (A.12),

$$\begin{aligned}&U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i)\\ &\quad= \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} (y - \theta_i)\, dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} (y - \theta_i)\, dy\\ &\qquad - \frac{1}{2}(1-\gamma)\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} (1 - x)\, dx\, dy\\ &\quad\geq \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} (y - \theta_i)\, dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} (y - \theta_i)\, dy - \frac{1}{4}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} dx\, dy\\ &\quad= \frac{1}{4}\big(\hat{\theta}_i - \theta_i\big)^2 + \frac{1}{4}\big(\theta_i - \beta(\hat{\theta}_i)\big)^2 - \frac{1}{8}\big(\hat{\theta}_i - \beta(\hat{\theta}_i)\big)^2\\ &\quad= \frac{1}{8}\big(\hat{\theta}_i + \beta(\hat{\theta}_i) - 2\theta_i\big)^2 \geq 0,\end{aligned}$$

as desired.

"Only if" part: Assume that $\gamma < 1/2$. We want to find $\theta_i, \hat{\theta}_i \in [0, 1]$ for which $U_i^*(\theta_i) < U_i^*(\theta_i, \hat{\theta}_i)$. Take a small number $\delta > 0$ such that $\delta < 1 - 2\gamma$. Note that $(1 + \delta)/2 < 1 - \gamma$. Then take a large number $A > 1$ so that $1/(2A - 2)^2 < \delta$, and let $B = 2A - 1$ (and hence $1/(B - 1)^2 < \delta$). Finally let $\varepsilon > 0$ be a positive number, which will be taken to be sufficiently small. Set $\theta_i = A\varepsilon$ and $\hat{\theta}_i = B\varepsilon$. When $\theta_{-i} = \varepsilon$ $(< \hat{\theta}_i)$,

$$\begin{aligned}\Delta(\varepsilon) &= -\frac{1}{2}v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i})\\ &= -\frac{1}{2}(A\varepsilon - \gamma\varepsilon) + \frac{1}{2}(1-\gamma)\left[B\varepsilon - \frac{1}{2}\left\{(B\varepsilon)^2 - \varepsilon^2\right\}\right]\\ &= \frac{\varepsilon}{2}\left[-(A-\gamma) + (1-\gamma)\left\{B - \frac{1}{2}(B^2-1)\varepsilon\right\}\right].\end{aligned}$$

We claim that for sufficiently small ε, $\Delta(\varepsilon) > 0$. Indeed, as $\varepsilon \to 0$, the bracketed term in the last line goes to $-(A-\gamma) + (1-\gamma)B = -(B-1)\gamma - (A-\gamma) > -(B-1)(1/2) - (A-1/2) = 0$. It therefore follows from Lemma A.2 that $\beta(B\varepsilon) < \varepsilon$ for sufficiently small ε.

By (A.12),

$$\begin{aligned}&U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i)\\ &= \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i}(y-\theta_i)\,dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)}(y-\theta_i)\,dy\\ &\quad - \frac{1}{2}(1-\gamma)\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i}\int_y^{\hat{\theta}_i}(1-x)\,dx\,dy\\ &< \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i}(y-\theta_i)\,dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)}(y-\theta_i)\,dy\\ &\quad - \frac{1}{2}\frac{1+\delta}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i}\int_y^{\hat{\theta}_i}(1-\hat{\theta}_i)\,dx\,dy\\ &= \frac{1}{8}\big(\hat{\theta}_i + \beta(\hat{\theta}_i) - 2\theta_i\big)^2 - \frac{1}{8}\big\{(1+\delta)(1-\hat{\theta}_i) - 1\big\}\big(\hat{\theta}_i - \beta(\hat{\theta}_i)\big)^2\\ &= \frac{1}{8}\big(-\varepsilon + \beta(B\varepsilon)\big)^2 - \frac{1}{8}\big\{(1+\delta)(1-B\varepsilon) - 1\big\}\big(B\varepsilon - \beta(B\varepsilon)\big)^2\\ &< \frac{1}{8}\varepsilon^2 - \frac{1}{8}\big\{(1+\delta)(1-B\varepsilon) - 1\big\}(B\varepsilon - \varepsilon)^2\\ &= \frac{1}{8}\varepsilon^2\left[1 - \big\{(1+\delta)(1-B\varepsilon) - 1\big\}(B-1)^2\right],\end{aligned}$$

where the second inequality follows from $0 \leq \beta(B\varepsilon) < \varepsilon$. We claim that for sufficiently small ε, $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) < 0$. Indeed, as $\varepsilon \to 0$, the bracketed term in the last line goes to $1 - \delta(B-1)^2$, which is negative by the choice of δ and B. ▮

References

Athanassoglou, S., S. J. Brams, and J. Sethuraman (2008). "Minimizing Regret when Dissolving a Partnership," mimeo.

Bergemann, D. and J. Välimäki (2002). "Information Acquisition and Efficient Mechanism Design," *Econometrica* **70**, 1007-1033.

Chien, H.-K. (2007). "Incentive Efficient Mechanisms for Partnership Dissolution," mimeo.

Compte, O. and P. Jehiel (2007). "On Quitting Rights in Mechanism Design," *American Economic Review Papers and Proceedings* **97**, 137-141.

Compte, O. and P. Jehiel (2009). "Veto Constraint in Mechanism Design: Inefficiency with Correlated Types," *American Economic Journal: Microeconomics* **1**, 182-206.

Cramton, P., R. Gibbons, and P. Klemperer (1987). "Dissolving a Partnership Efficiently," *Econometrica* **55**, 615-632.

Crémer, J. and R. P. McLean (1985). "Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist when Demands are Interdependent," *Econometrica* **53**, 345-361.

Crémer, J. and R. P. McLean (1988). "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica* **56**, 1247-1257.

de Frutos, M.-A. and T. Kittsteiner (2008). "Efficient Partnership Dissolution under Buy-Sell Clauses," *RAND Journal of Economics* **39**, 184-198.

Eliaz, K. and R. Spiegler (2007). "A Mechanism-Design Approach to Speculative Trade," *Econometrica* **75**, 875-884.

Eliaz, K. and R. Spiegler (2009). "Bargaining over Bets," *Games and Economic Behavior* **66**, 78-97.

Fieseler, K., T. Kittsteiner, and B. Moldovanu (2001). "Partnerships, Lemons, and Efficient Trade," SonderForschungsBereich 504 Working Paper 01-18, University of Mannheim.

Fieseler, K., T. Kittsteiner, and B. Moldovanu (2003). "Partnerships, Lemons, and Efficient Trade," *Journal of Economic Theory* **113**, 223-234.

Forges, F. (1999). "Ex Post Individually Rational Trading Mechanisms," in A. Alkan, C. Aliprantis, and N. Yannelis, eds., *Current Trends in Economics: Theory and Applications*, Springer-Verlag, Berlin.

Fujinaka, Y. (2006). "Efficiency, Envy-Freeness, and Bayesian Incentive Compatibility in Economies with One Indivisible Good and Money," Working Paper Series No.203, Kobe University.

Galavotti, S. (2009). "Bilateral Trade with Apositional Traders," *Research in Economics* **63**, 172-188.

Gershkov, A. and P. Schweinzer (2008). "When Queueing is Better than Push and Shove," forthcoming in *International Journal of Game Theory.*

Gresik, T. A. (1991). "Ex Ante Efficient, Ex Post Individually Rational Trade," *Journal of Economic Theory* **54**, 131-145.

Hauswald, R. and U. Hege (2006). "Ownership and Control in Joint Ventures," mimeo.

Jehiel, P. and A. Pauzner (2006). "Partnership Dissolution with Interdependent Values," *RAND Journal of Economics* **37**, 1-22.

Kittsteiner, T. (2003). "Partnerships and Double Auctions with Interdependent Valuations," *Games and Economic Behavior* **44**, 54-76.

Kosmopoulou, G. (1999). "Payoff Equivalence between Bayesian and Ex Post Individually Rational Dominant Strategy Mechanisms," *Economic Theory* **13**, 229-237.

Kosmopoulou, G. and S. R. Williams (1998). "The Robustness of the Independent Private Value Model in Bayesian Mechanism Design," *Economic Theory* **12**, 393-421.

Li, J. (2009). "Partnership Dissolution and Proprietary Information," mimeo.

Makowski, L. and C. Mezzetti (1994). "Bayesian and Weakly Robust First Best Mechanisms: Characterizations," *Journal of Economic Theory* **64**, 500-519.

Matthews, S. A. and A. Postlewaite (1989). "Pre-play Communication in Two-Person Sealed-Bid Double Auctions," *Journal of Economic Theory* **48**, 238-263.

McAfee, R. P. and P. J. Reny (1992). "Correlated Information and Mecanism Design," *Econometrica* **60**, 395-421.

Mezzetti, C. (2002). "Auction Design with Interdependent Valuations: The Generalized Revelation Principle, Efficiency, Full Surplus Extraction and Information Acquisition," mimeo.

MEZZETTI, C. (2004). "Mechanism Design with Interdependent Valuations: Efficiency," *Econometrica* **72**, 1617-1626.

MEZZETTI, C. (2007). "Mechanism Design with Interdependent Valuations: Surplus Extraction," *Economic Theory* **31**, 473-488.

MORGAN, J. (2004). "Dissolving a Partnership (Un)fairly," *Economic Theory* **23**, 909-923.

MYERSON, R. B. AND M. A. SATTERTHWAITE (1983). "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory* **29**, 265-281.