On the Strategic Impact of an Event under Non-Common Priors

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The Common Prior Assumption (CPA):
used in most applied models with incomplete information.

The purpose of this paper:
to clarify the restrictions the CPA implicitly imposes
on strategic behavior,
allowing players to have heterogenous priors.

Specifically, focus on
impact of small probability events (think of crazy types)
through higher order beliefs ("contagion effect").

Ex.
Email game (Rubinstein (1989)),
Global games (Carlsson and van Damme (1993)).

Contagion (think of Email game):
a crazy type playing a^* at event E implies play of a^* everywhere as the unique rationalizable strategy of the incomp info game.
Say "a^* is contagious from E".

We show
any strict NE can be contagious from a small probability event E *under non-CP*,
while not all strict NE can be contagious *under CP*.

Under non-CP, a small probability event can have a large "impact".

We quantify this strategic impact by the concept of "belief potential"
(Morris, Rob, and Shin (*Econometrica* 1995)).

Implications on recent literature on CPA and higher order beliefs:

Predictions in a complete information may be sensitive to a small amount of incomplete information perturbation (as the Email game shows).

Does the CPA play a role in those incomplete info perturbations?

Relation to Weinstein and Yildiz (*Econometrica* 2007), Yildiz (2004, working paper), Lipman (*Econometrica* 2003).

Example Two players, A and B; two actions, L and R.

Complete information game $\mathbf{g}$:

	L	R
L	p, p	$0, 0$
R	$0, 0$	$1-p, 1-p$

Assume $1/2 < p < 1$.

$a^* = (R, R)$ is said to be *p-dominant*,

while (L, L) is $(1-p)$-dominant ($\Rightarrow$ risk-dominant).

Against $(1-q)[L] + q[R]$ where $q > p$,
R is the unique best response.

p larger $\Rightarrow$ a^* weaker.

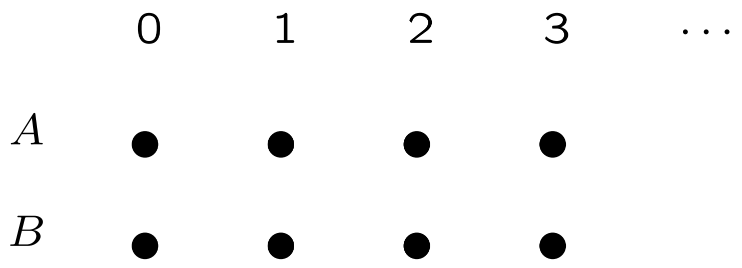
Question

Are there "perturbations" arbitrarily "close" to $\mathbf{g}$
in which a^* is the unique play?

1. Our paper:
Perturbations:
incomp info games with partition structure and non-common priors
($\mathbf{g}$: degenerate incomp info game);
Close to $\mathbf{g}$:
the payoffs are given by $\mathbf{g}$ with high ex ante probability.

1. Consider the following class of perturbations:

$\Omega = \{A, B\} \times \{0, 1, 2, \ldots\}$.



Priors:

$$P_i(i,k) = \frac{r}{r+1}\varepsilon(1-\varepsilon)^k, \qquad P_i(-i,k) = \frac{1}{r+1}\varepsilon(1-\varepsilon)^k \qquad (i = A, B), \qquad r \geq 1;$$

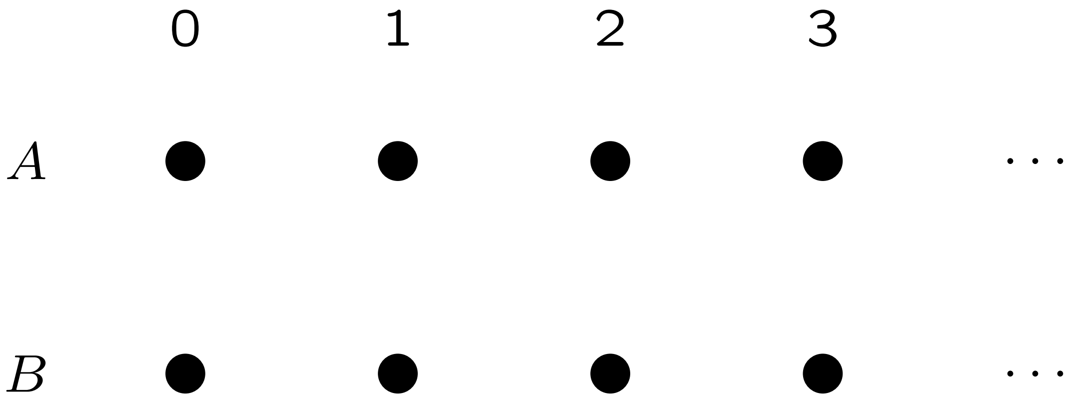
Information partitions:

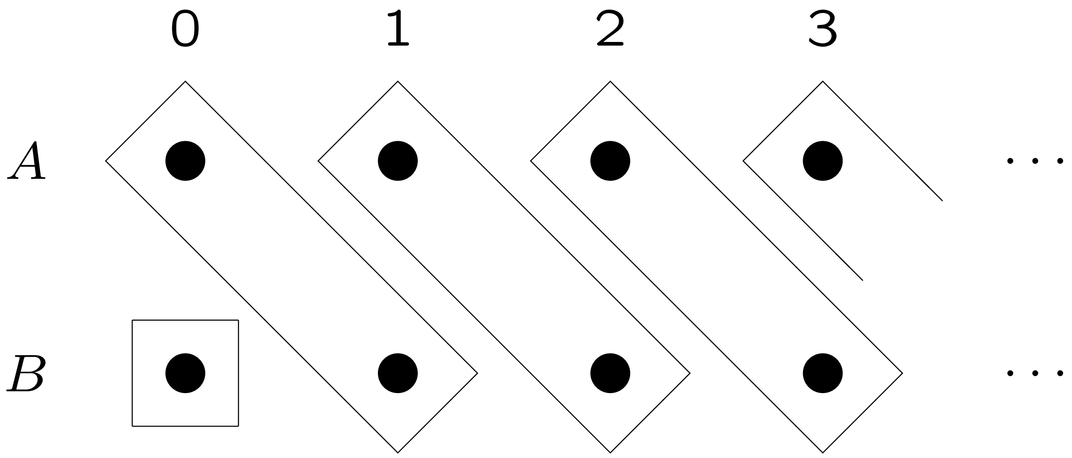
$\mathcal{Q}_A = \{\{(B,0)\}, \{(A,0),(B,1)\}, \{(A,1),(B,2)\}, \ldots\}$;
$\mathcal{Q}_B = \{\{(A,0)\}, \{(B,0),(A,1)\}, \{(B,1),(A,2)\}, \ldots\}$.

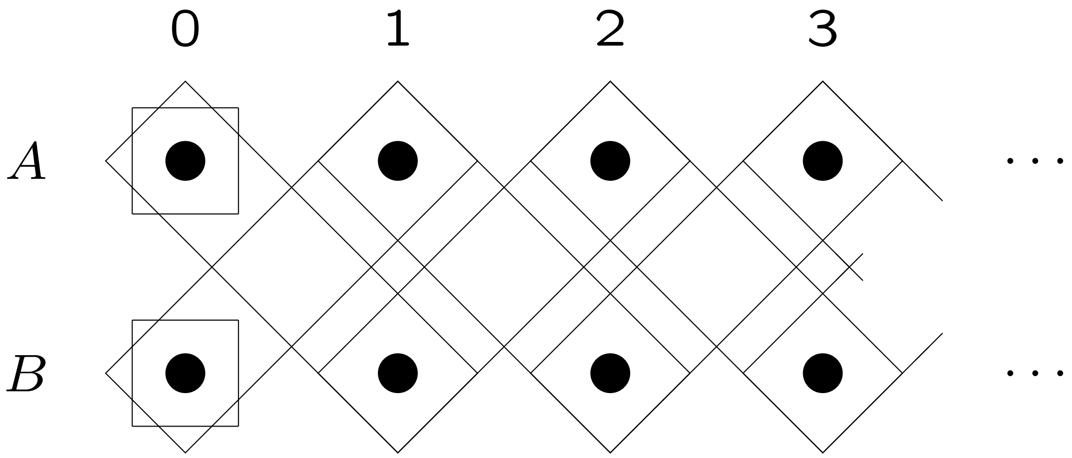
Payoffs:

$u_A(a,\omega) = g_A(a)$ for all $\omega \neq (B,0)$; $\quad u_B(a,\omega) = g_B(a)$ for all $\omega \neq (A,0)$.

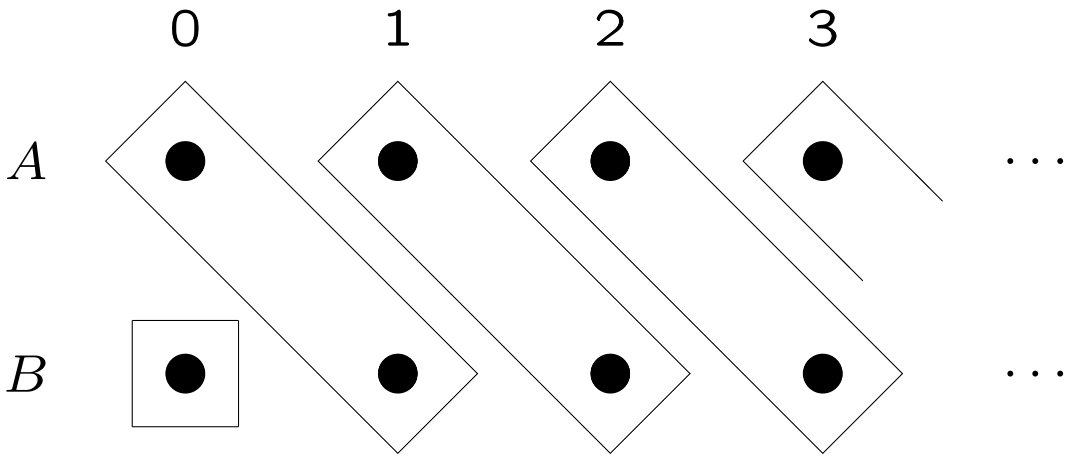
R is a dominant action for player A (B, resp.) at state $(B,0)$ ($(A,0)$, resp.).


0
1
2
3
A
B

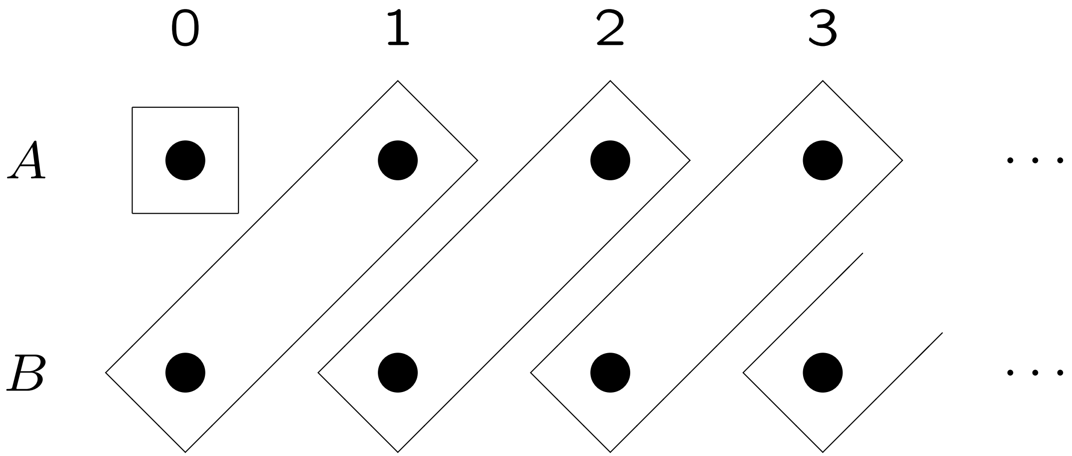

0
1
2
3
A
B
...
...


0
1
2
3
A
B
...
...

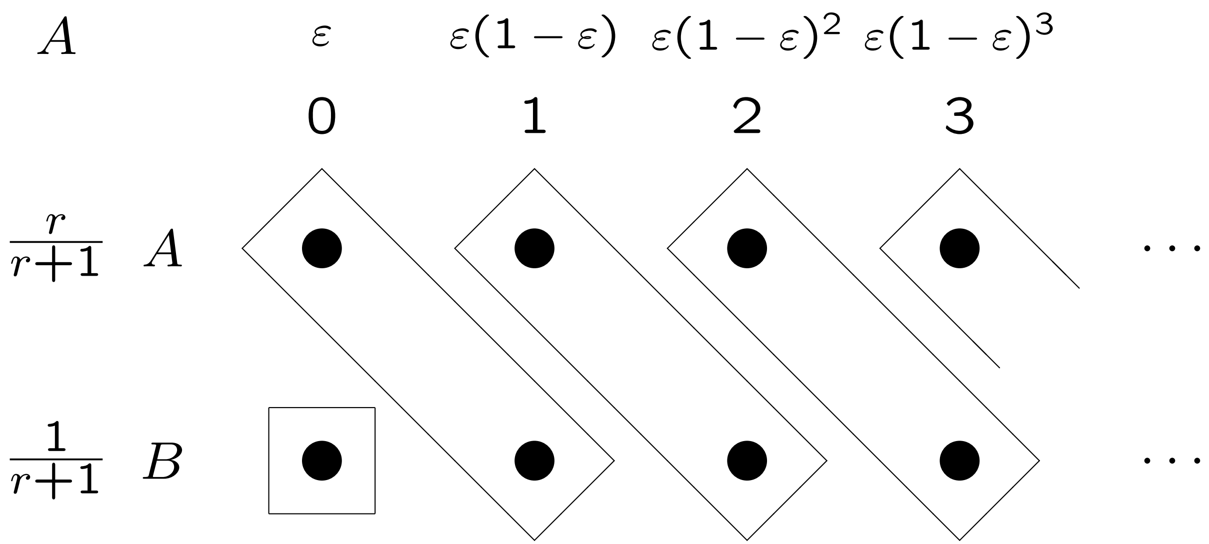
A



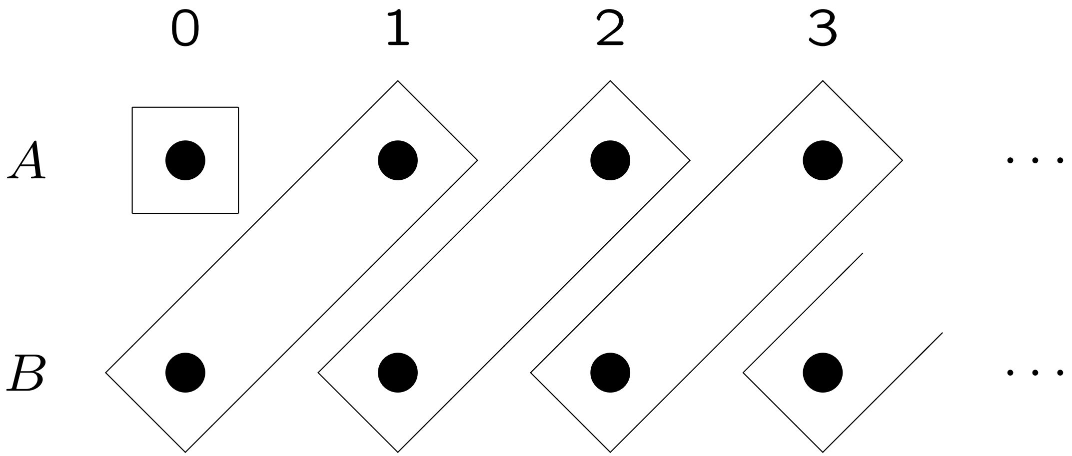
B

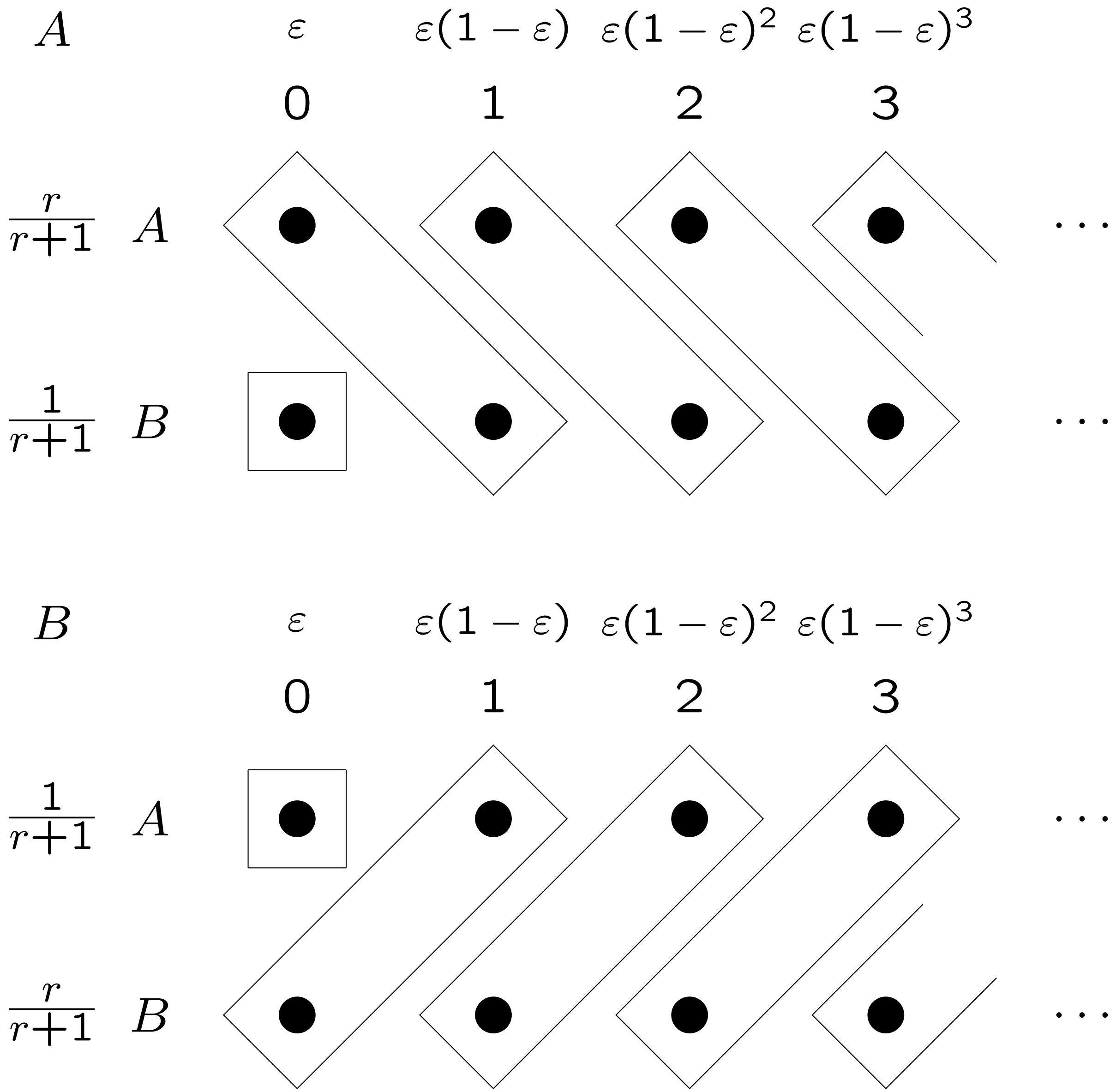


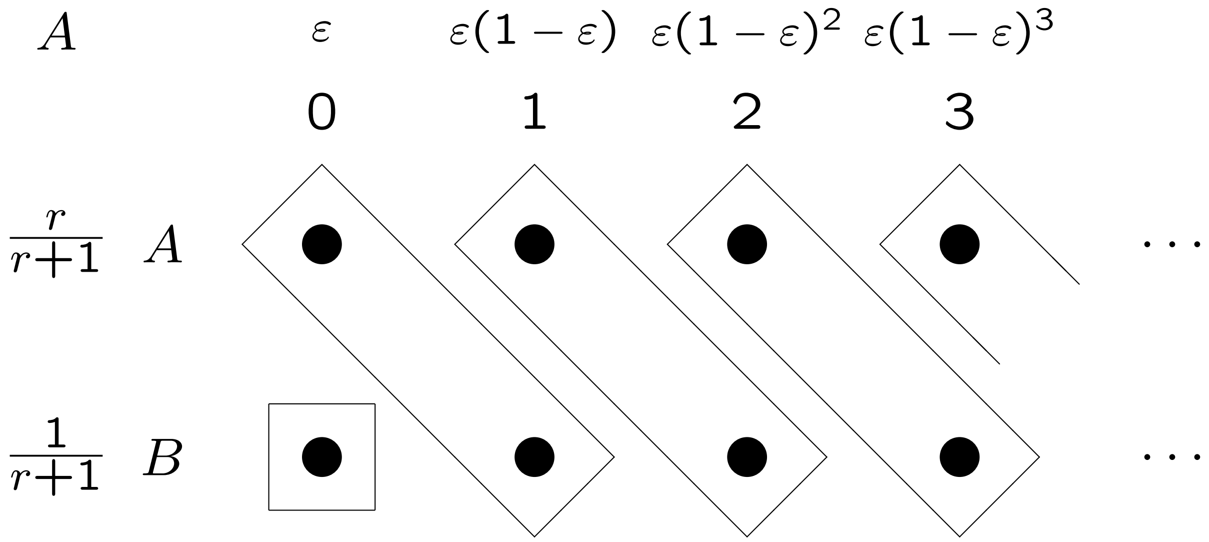
A


ε
$\varepsilon(1-\varepsilon)$
$\varepsilon(1-\varepsilon)^2$
$\varepsilon(1-\varepsilon)^3$
0
1
2
3
$\frac{r}{r+1}$ A
$\frac{1}{r+1}$ B
$\cdots$
$\cdots$

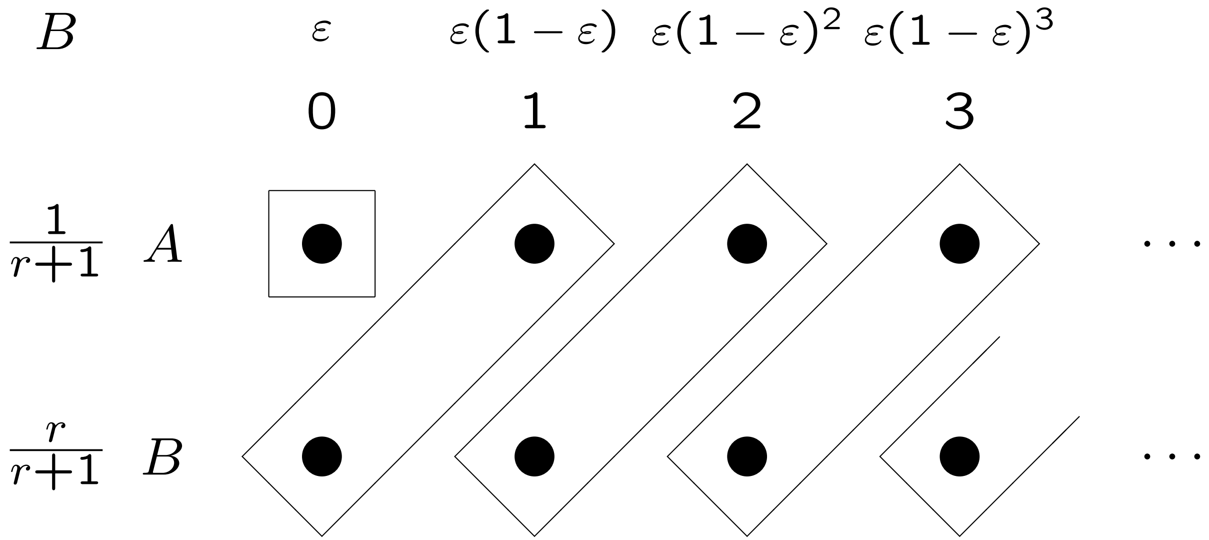
B

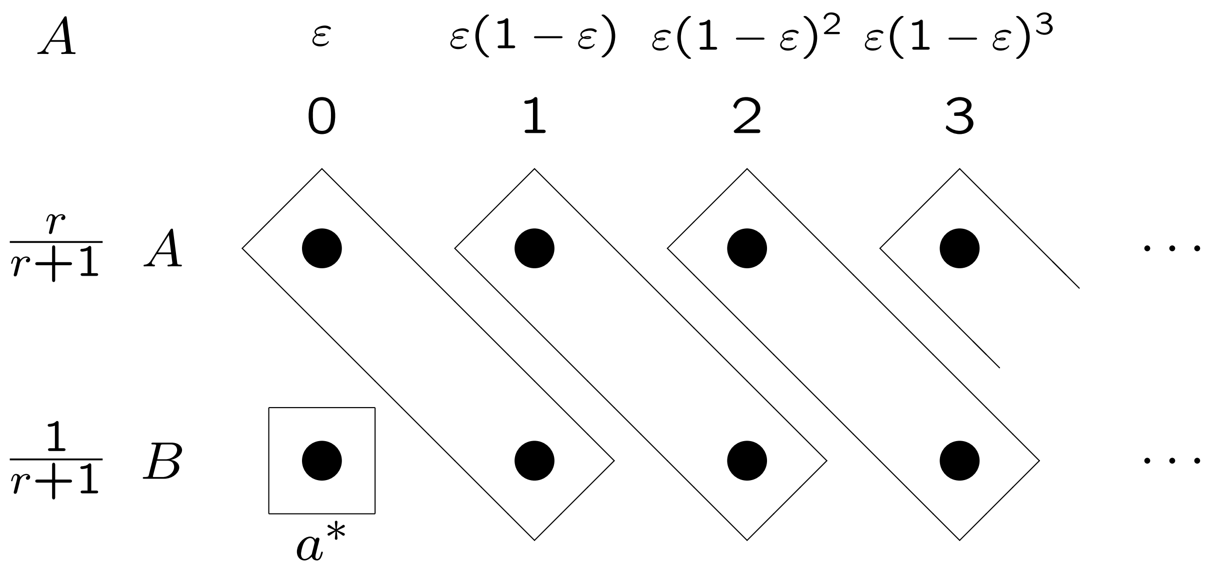

0
1
2
3
A
B
$\cdots$
$\cdots$


A
ε
ε(1 − ε)
ε(1 − ε)^2
ε(1 − ε)^3
0
1
2
3
r/(r+1)
A
1/(r+1)
B
B
1/(r+1)
A
r/(r+1)
B



$$\max\left\{\frac{P_A(\omega)}{P_B(\omega)}, \frac{P_B(\omega)}{P_A(\omega)}\right\} = r$$





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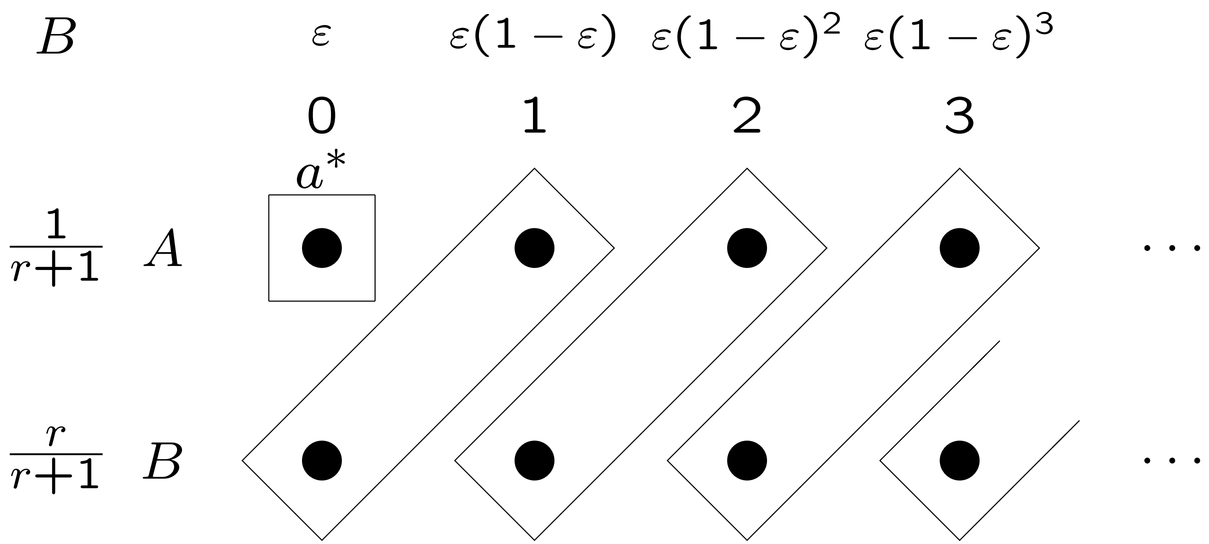
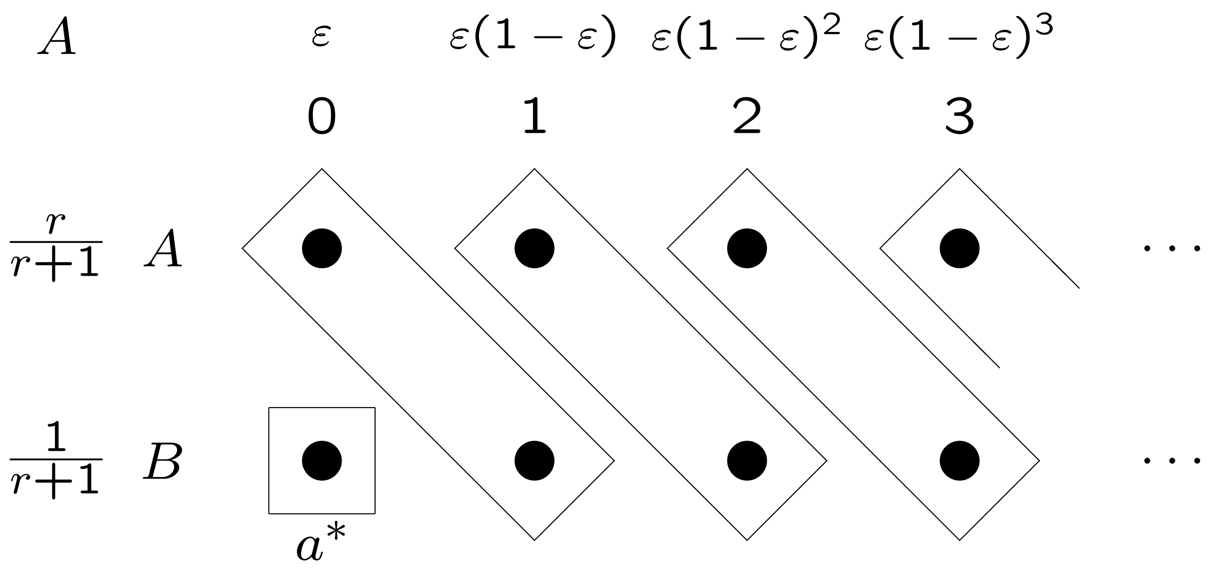
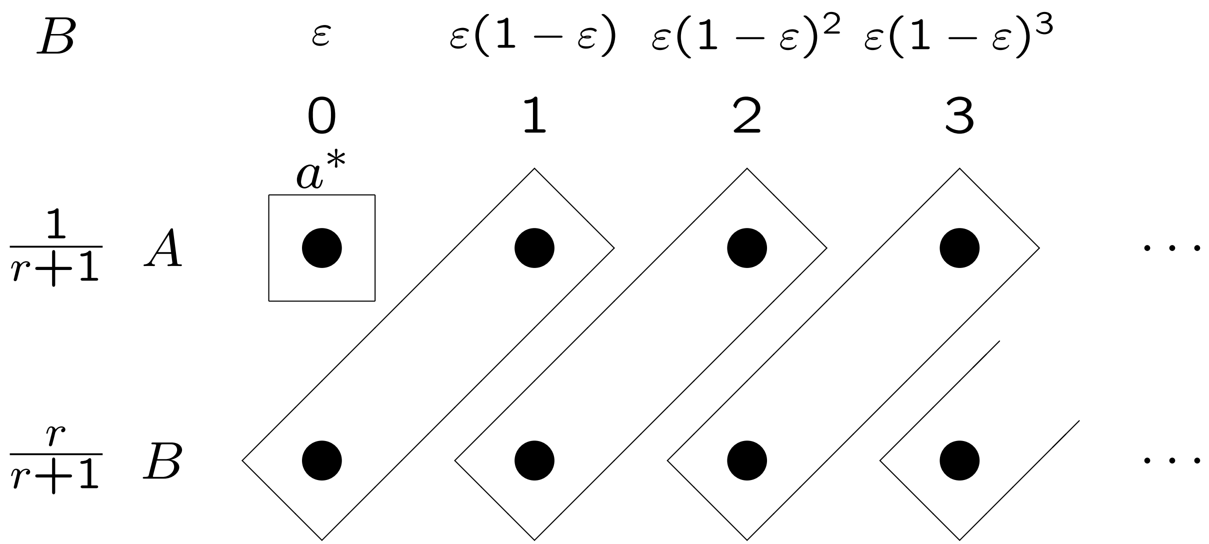


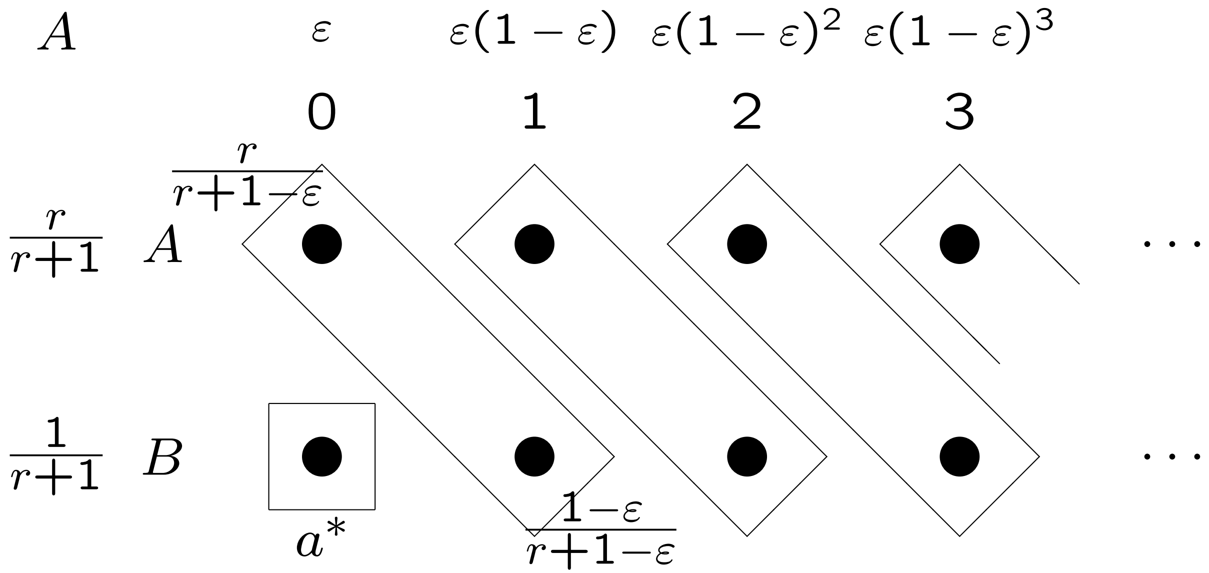


$$\max\left\{\frac{P_A(\omega)}{P_B(\omega)}, \frac{P_B(\omega)}{P_A(\omega)}\right\} = r$$

$$P_i(\text{Crazy type event } E) = \varepsilon$$

$$(E = \{(A, 0), (B, 0)\})$$

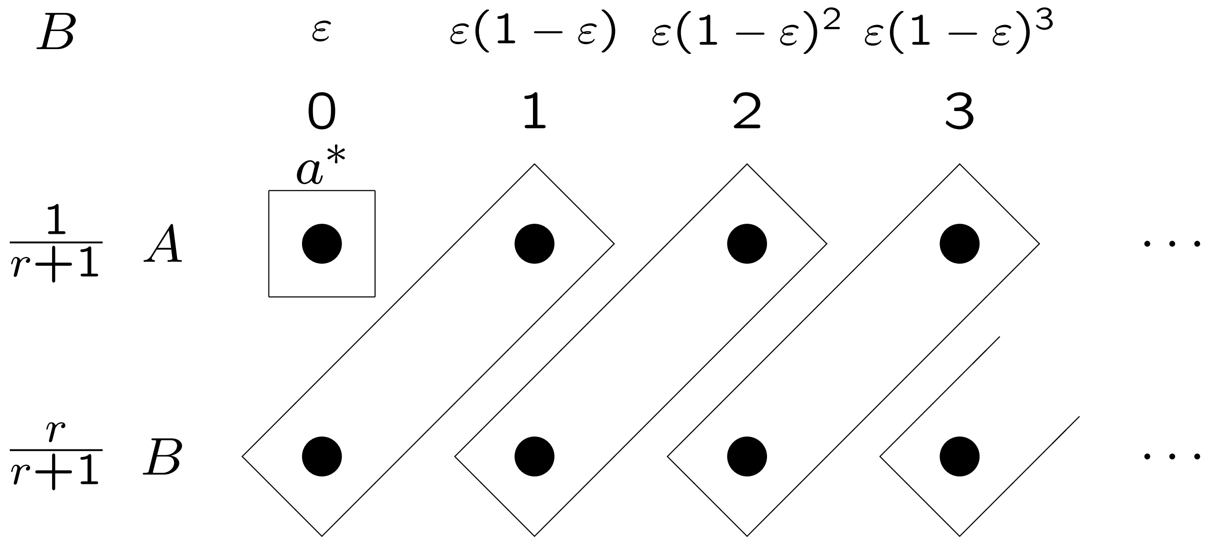


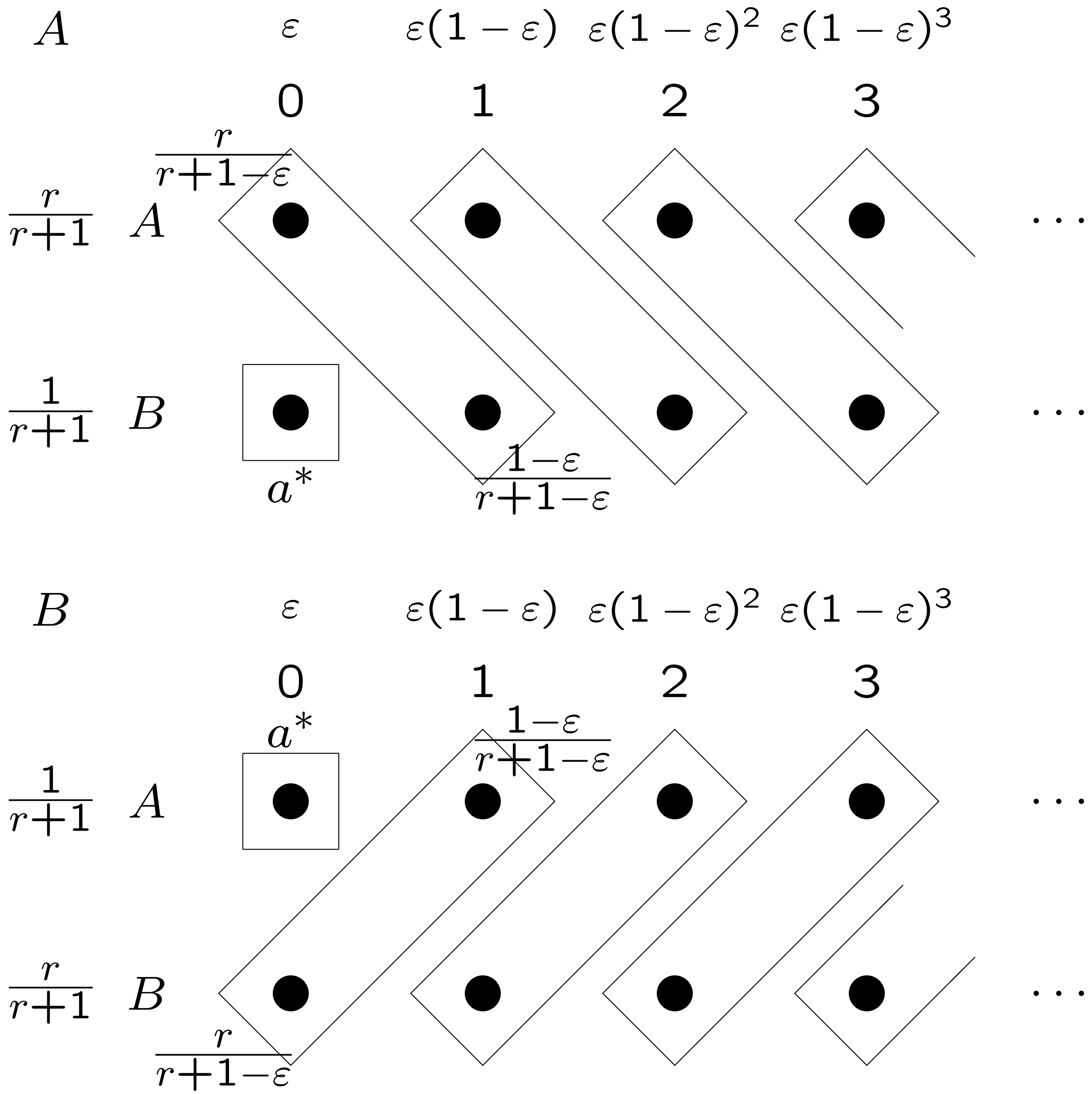


$$\max\left\{\frac{P_A(\omega)}{P_B(\omega)}, \frac{P_B(\omega)}{P_A(\omega)}\right\} = r$$

$$P_i(\text{Crazy type event } E) = \varepsilon$$

$$(E = \{(A,0),(B,0)\})$$





$$\max\left\{\frac{P_A(\omega)}{P_B(\omega)}, \frac{P_B(\omega)}{P_A(\omega)}\right\} = r$$

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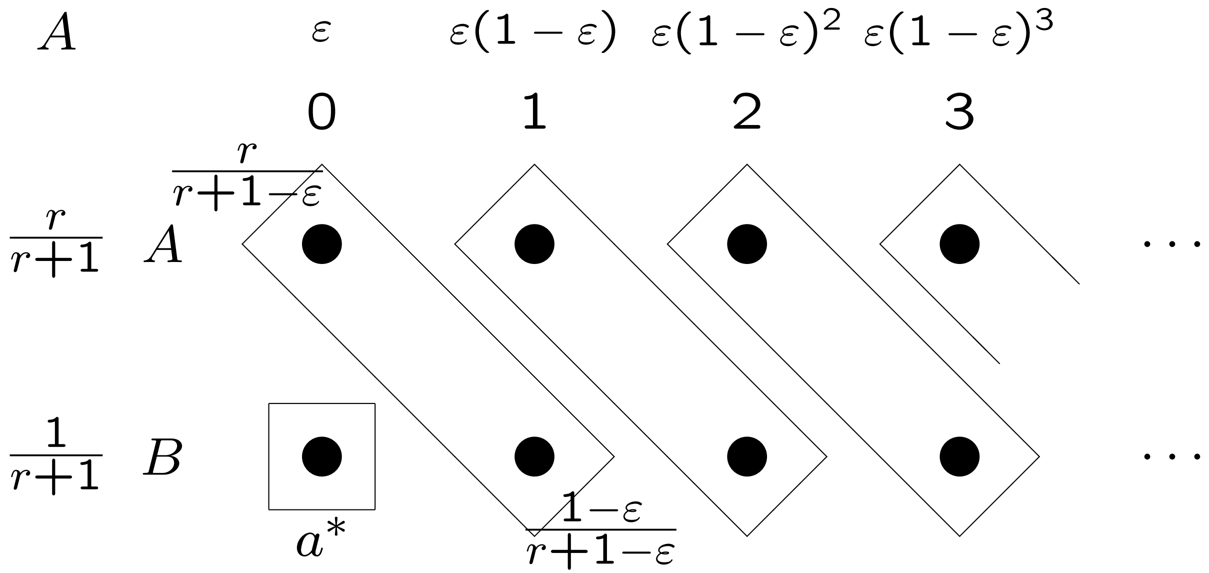

$$\max\left\{\frac{P_A(\omega)}{P_B(\omega)}, \frac{P_B(\omega)}{P_A(\omega)}\right\} = r$$

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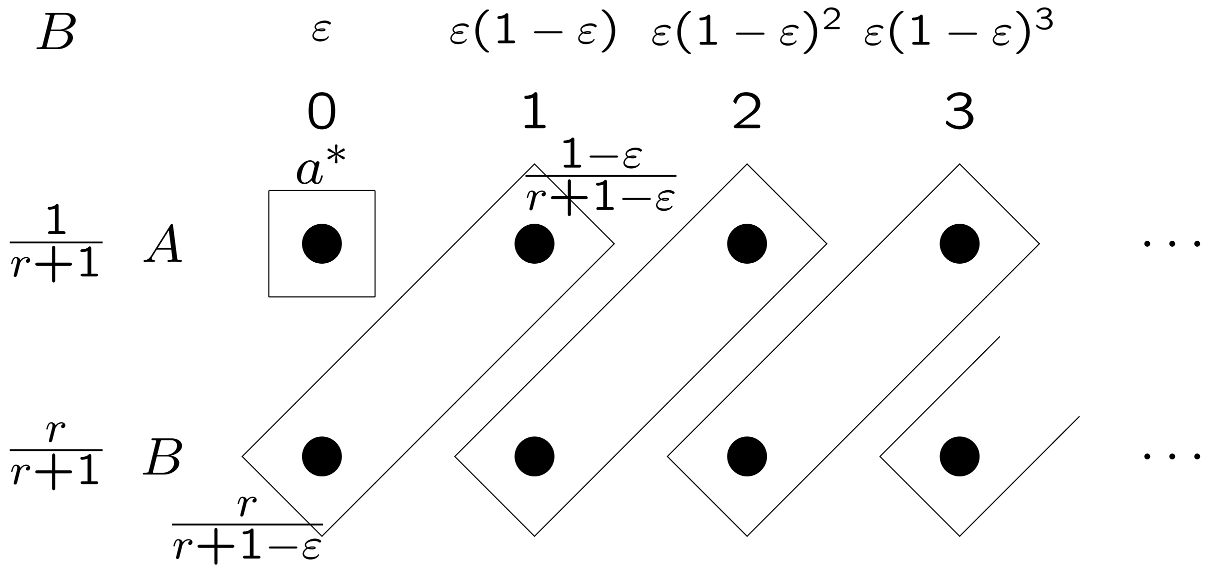

$$\frac{r}{r+1-\varepsilon} \to 1 \text{ as } r \to \infty$$

Observation

If $p < \frac{r}{r+1-\varepsilon}$, then playing $a^* = (R, R)$ is the unique rationalizable str.

"a^* is contagious from E".

P_i(crazy type event E) $= \varepsilon$ for each $i = A, B$;

$\max\left\{\frac{P_A(\omega)}{P_B(\omega)}, \frac{P_B(\omega)}{P_A(\omega)}\right\} = r$ for all $\omega \in \Omega$.

This number

$$\sigma(E) = \frac{r}{r + 1 - \varepsilon}$$

is caled the *belief potential* of event E

(Morris, Rob, and Shin (*Econometrica* 1995)).

Definitions

An information system $(\Omega, (P_i)_{i=1,2}, (\mathcal{Q}_i)_{i=1,2})$: given.

- *p*-Belief operator

$$B_i^p(E) = \{\omega \mid P_i(E|Q_i(\omega)) \geq p\},$$

- $(H_i^p)^1(E) = B_i^p(B_{-i}^p(E)) \cup E$,
  and for $k \geq 2$,

$$(H_i^p)^k(E) = H_i^p((H_i^p)^{k-1}(E)).$$

- E has *global impact* p if for some i,

$$(H_i^p)^\infty(E) = \Omega.$$

- The *belief potential of event* E, $\sigma(E)$, is

$$\sigma(E) = \text{sup}\{p \in [0,1] \mid E \text{ has global impact } p\}.$$

The belief potential of an event E, $\sigma(E)$,

- is the necessary strength of a NE to be contagious from E,

- measures the strategic impact of E.

In the Example, $\sigma(E) = \dfrac{r}{r+1-\varepsilon}$.

(max $\{P_A(\omega)/P_B(\omega), P_B(\omega)/P_A(\omega)\} = r$ for all ω; $P_i(E) = \varepsilon$.)

(1) Common prior case ($r = 1$):
$p < \frac{1}{2-\varepsilon} \to \frac{1}{2}$ as $\varepsilon \to 0$.
Only a risk-dominant equilibrium can be contagious for small ε.
$\cdots$ Under CP, the impact of a small prob event is not so large.

(2) Non-common prior case ($r > 1$):
$p < \dfrac{r}{r+1-\varepsilon} \to 1$ as $r \to \infty$.
Any strict Nash equilibrium can be contagious by letting r large.
$\cdots$ Under non-CP, the impact can be arbitrarily large.

Recall

Question:

Are there "perturbations" arbitrarily "close" to $\mathbf{g}$ in which a^* is the unique play?

1. Our paper:

Perturbations:

incomp info games with partition structure and non-common priors ($\mathbf{g}$: degenerate incomp info game);

Close to $\mathbf{g}$:

the payoffs are given by $\mathbf{g}$ with high ex ante probability.

Answer

Yes, non-common priors (with large r);

No, with common prior.

Theorem.

$r \geq 1$, $\varepsilon > 0$ given.

For any information system $(\Omega, (\mathcal{Q}_i)_{i=1,2}, (P_i)_{i=1,2})$

such that $\max_{i \neq j} \sup_{\omega \in \Omega} \dfrac{P_i(\omega)}{P_j(\omega)} = r$, and

for any $E \subset \Omega$ such that $P_i(E) \leq \varepsilon$ for each $i = 1, 2$,

$$\sigma(E) \leq \frac{r}{r + 1 - \varepsilon}.$$

(This upper bound is tight.)

Relation to Weinstein and Yildiz (2007)

Recall

Question:

Are there "perturbations" arbitrarily "close" to $\mathbf{g}$ in which a^* is the unique play?

2. Weinstein and Yildiz (2007):

$\mathbf{g}$: a point in the universal type space;

"Perturbations" being "close" to $\mathbf{g}$:

types convergent to $\mathbf{g}$ w.r.t. product topology.

2. Weinstein and Yildiz (2007)
Yes.
∃ sequence of types in the universal type space
- convergent to $\mathbf{g}$ w.r.t. product topology
- each of which plays a^* as the unique rationalizable str.

Moreover, these types can be taken from common prior models.
(By Lipman (2003).)

However, by our Theorem,
in order for a strict NE a^* to be contagious from an event E

- keeping the CPA,
  $P(E)$ has to be large;

- keeping $P_i(E)$ to be small,
  we have to drop the CPA.

Conclusion

- We measured the strategic impact of a small probability event:

  - arbitrarily large under non-CP;
  - bounded from above under CP.

- From an "ex ante viewpoint", models with CP and non-CP can be very different.

  Provide a new view on the structure of higher order beliefs different from Weinstein and Yildiz' (2007) and Lipman's (2003).

- Under non-CP, we can manipulate relevant posteriors as we want.

  $\Rightarrow$ In games with more than one strict Nash equilibria,
  we can construct a perturbation in which
  a given NE is unique rationalizable play.

  $\Rightarrow$ The other NE are not "robust" to incomplete information
  under non-CP.

"Robust Equilibria under Non-Common Priors."
In generic games,
a NE is robust to incomplete information under non-CP
$\Longleftrightarrow$ it is a unique rationalizable action profile
(unique action profile that survives iterated elimination of dominated actions).