Booms and Slumps in a Game of Sequential Investment with the Changing Fundamentals

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Abstract

Many less developed countries have experienced prolonged periods of expansions and reversals in foreign investment inflows. This paper presents a simple game-theoretic model that can explain hysteretic patterns of serial correlation in investment behavior. We develop a sequential move game of coordinated investment played by short-run players under the changing economic environment and demonstrate that in a unique equilibrium of the game, the economy fluctuates over multiple static equilibria, generating hysteresis.

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1 Introduction

Many less developed countries have experienced economic fluctuations associated with expansions and sharp reversals in foreign investment inflows. In Latin American countries, lending booms in 1970s, which lasted until 1982 when those countries collapsed, were followed by prolonged slumps in economic activities in the “lost decade” of 1980s. The 1990s witnessed analogous patterns in Asian countries as well as in Latin American countries, around the currency crises in Mexico in 1994, and in Thailand, Korea, and Indonesia in 1997. Those dynamic patterns are characterized by serial correlation in investment behavior: that is to say, they exhibit hysteresis. The purpose of this paper is to present a theoretical framework that can explain economic fluctuations as a hysteretic phenomenon. To this end, we develop a simple game-theoretic model of sequential investment in which the underlying stage game typically has multiple equilibria due to strategic complementarities. The model generates a unique outcome in which the economy undergoes random switches between the multiple equilibria of the stage game.

We consider the following setting. A sequence of short-run investors decide in an exogenous order whether or not to invest in a country. When an investor invests in the country, the return is affected by the decisions of his successor as well as of his predecessor. There are strategic complementarities: investment leads to higher payoffs when the predecessor and/or the successor also choose to invest. The return to investment also depends on an exogenous variable (e.g., exchange rate, etc.) which summarizes the economic fundamentals changing stochastically over time. The state of the fundamentals is represented by a publicly observed payoff-relevant parameter \( \theta_t \) which follows a random walk. There is a chance that the state of fundamentals is bad enough (say, \( \theta_t < \theta \)) so that investment yields a negative return irrespective of the actions of other investors. Conversely, when the state of fundamentals is sound enough (say, \( \theta_t > \theta \)), investing becomes a dominant choice. This setting describes a sequential move game among short-run players where the stage game has two equilibria, the “boom” equilibrium where each player invests and the “slump” equilibrium where each does not, when \( \theta_t \) lies between \( \theta^* \) and \( \theta^{**} \) (\( \theta^* < \theta^{**} \)).

We show that in this sequential move game, a unique rationalizable strategy survives iterative strict dominance. This strategy is characterized by two thresholds \( \theta^* \) and \( \theta^{**} \) (\( \theta^* < \theta^{**} \)), which generate hysteresis (Figure 1). If the state of fundamentals falls below the lower threshold \( \theta^* \), the economy drops into the slump equilibrium. Unless the state rises over the higher threshold \( \theta^{**} \), the economy is stuck in the slump equilibrium. Similarly, once the economy is in the boom equilibrium, the economy will be in boom as long as the state is above the lower threshold.

An important feature is that the “hysteresis band”, \([\theta^*, \theta^{**}]\), is con-
Figure 1.

tained in the interval \((\theta, \theta_t)\), where the stage game has multiple equilibria. It is expectation, which is rationalizable here, that drives the economy to switch between equilibria. This is to be contrasted with the analysis by Cooper (1994), where it is assumed that the economy jumps from one equilibrium to another when the former disappears (i.e., in our terminology, when \(\theta_t\) falls below \(\theta\) or rises over \(\theta\)).

A large literature has discussed the potential of macroeconomic models with multiple equilibria (e.g., the “coordination failure” models by Cooper and John (1988) and Cooper (1994)) for understanding economic fluctuations. In models with multiple equilibria, economic fluctuations are associated with payoff irrelevant random variables such as sunspots, animal spirits, or waves of optimism and pessimism. Many studies demonstrate the existence of “sunspot equilibria” in which agents’ self-fulfilling expectations can cause economic fluctuations (e.g., Diamond and Fudenberg (1989), Howitt and MacAfee (1992), and Chatterjee, Cooper, and Ravikumar (1993), among others). Those models, however, leave unexplained why a particular sunspot equilibrium arises: such an equilibrium is only one possible outcome of the model, as there are many other equilibria, including one with no fluctuation. In our model, on the other hand, stochastic changes in the fundamentals force investors to coordinate their expectations on a particular outcome.

Hysteresis has also been observed in the context of international trade. A classical example is the slow response of the U.S. imports to the exchange rate in the 1980s. In this field, a number of studies have attributed hysteresis in investment behavior to sunk costs in entry and exit. Baldwin (1988) demonstrates hysteresis in a model in which investors perfectly anticipate the exchange rate path. Baldwin and Krugman (1989) extend the model by assuming that the exchange rate is independently and identically distributed across periods. Dixit (1989a, 1989b) applies the real options approach where the exchange rate follows a Brownian motion, showing that stochastic shocks combined with sunk costs can generate hysteresis. Most of the models presented in this “sunk cost hysteresis” literature do not take into account strategic interactions, while our model relies on strategic complementarities.

\(^1\)Cooper (1999) provides a comprehensive literature review.

\(^2\)For further surveys, see Dixit (1992) and Göcke (2002).

between investors, thereby providing an alternative framework for explaining investment hysteresis.

Our equilibrium uniqueness result is based on a contagion argument similar to that by Carlsson and van Damme (1993). They study static incomplete information games called global games, in which each player observes a noisy signal of the true payoffs, and show that contagion effects eliminate the multiplicity of equilibria. The present paper is also related to papers by Burdzy, Frankel, and Pauzner (2001) and Matsui (1998). Each paper establishes uniqueness results in a dynamic model with random payoffs that is different from ours. The relation to those results is discussed in Section 4.

The paper is organized as follows. The model is described in Section 2. Section 3 analyzes the model and presents the main result. Section 4 discusses a generalization of the model and reviews the related literature.

2 Model

Time is discrete, and the horizon is infinite. There is a sequence of investors, who are planning to invest in a country. At time $t$, the $t$th investor must decide whether to invest ($I$) or not ($N$). Not investing is a safe choice which gives payoff 0 independently of the actions of other investors. The $t$th investor’s payoff from investing depends on the actions taken by the $(t-1)$th and the $(t+1)$th investors. There are strategic complementarities: each investor has stronger incentive to invest if his predecessor have invested and/or if he expects his successor to do so as well. The return to investment also depends on the economic fundamentals during two periods $t$ and $t+1$. The state of fundamentals is represented by a random parameter $\theta$. The return to investment is increasing in $\theta$, so that higher values of $\theta$ correspond to stronger fundamentals. If the $t$th investor chooses action $a_t$ and the $(t-1)$th investor chose action $a_{t-1}$ in the previous period, the realized payoff that the $t$th investor (the $(t-1)$th investor, respectively) receives in period $t$ is denoted by $\pi(a_t, a_{t-1}, \theta_t)$ ($\pi(a_{t-1}, a_t, \theta_t)$, respectively), where $\theta_t$ is the realized value of the random parameter at $t$. In the first period ($t = 1$), action $a_0$ is exogenously given. One shot payoff $\pi(a, b, \theta)$ is given by

$$\pi(a, b, \theta) = \begin{cases} 
\theta & \text{if } a = I \text{ and } b = I, \\
\theta - 1 & \text{if } a = I \text{ and } b = N, \\
0 & \text{if } a = N.
\end{cases}$$

(1)

This setup describes a sequential move game among short-run players. At each time $t$, the $t$th and the $(t-1)$th players play the stage game given in Figure 2. Each player plays the game for two periods. The $t$th player chooses his action at time $t$ and must commit to the action for two periods,
I
\[\theta_t, \theta_t \]

- \[\theta_t - 1, 0\]

N
- \[0, \theta_t - 1\]
- \[0, 0\]

**Figure 2.**

i.e., he must play the same action at time \(t + 1\) as the one he chose at time \(t\).

Note that if \(\theta_t < 0\) or \(\theta_t > 1\), the stage game has a strict dominance solution, \((I, I)\) for \(\theta_t > 1\) and \((N, N)\) for \(\theta_t < 0\). When the state of fundamentals is sound enough, investing in the country is a dominant choice. Conversely, when the state of fundamentals is bad enough, not investing becomes a dominant choice. If \(0 < \theta_t < 1\), both \((I, I)\) and \((N, N)\) are strict equilibria.

Payoff parameter \(\theta\) follows a random walk with two reflecting walls at \(\alpha\) and \(\beta\) where \(\alpha < \beta\). The evolution of \(\{\theta_t\}\) is defined by

\[
P(\theta_{t+1} = \theta_t + \sigma \mid \theta_t) = \begin{cases} 1 & \text{if } \theta_t \leq \alpha, \\ \frac{1}{2} & \text{if } \alpha < \theta_t < \beta, \\ 1 & \text{if } \theta_t \geq \beta, \end{cases}
\]

where the step size \(\sigma > 0\) may be arbitrarily small. At time \(t\), the \(t\)th player chooses his action \(a_t\) based on his observation of the \((t-1)\)th player’s action \(a_{t-1}\) taken in the previous period and the realized value \(\theta_t\) of the fundamentals at \(t\). A strategy of the \(t\)th player is a function \(s_t: \{I, N\} \times \mathbb{R} \rightarrow \{I, N\}\), where \(s_t(a_{t-1}, \theta_t)\) is the action that the \(t\)th player chooses, having observed action \(a_{t-1}\) and state \(\theta_t\). For the \((t-1)\)th player’s action \(a_{t-1}\), the \((t+1)\)th player’s strategy \(s_{t+1}\), and the state \(\theta_t\) in the \(t\)th period, the \(t\)th player’s expected payoff of choosing \(a_t\) is expressed as

\[
\frac{1}{1 + \delta} \left( \pi(a_t, a_{t-1}, \theta_t) + \delta E[\pi(a_t, s_{t+1}(a_t, \theta_{t+1}), \theta_{t+1}) \mid \theta_t] \right),
\]

where \(\delta > 0\) is a common discount factor.

We pose the following assumptions on the random parameter \(\{\theta_t\}\).

**Assumption 2.1.** The range of the random parameter contains a region where \(N\) is a dominant action \((\theta < 0)\) and another region where \(I\) is a dominant action \((\theta > 1)\), i.e.,

\[\alpha < 0 - \sigma \quad \text{and} \quad \beta > 1 + \sigma.\]
This assumption plays an essential role for the iterated elimination argument in Section 3 to operate, and therefore, for the equilibrium uniqueness result.

**Assumption 2.2.** The step size of the fundamentals is not large so that

\[ \sigma < \frac{2 - \delta}{2 + 2\delta}. \]

This assumption is crucial for the hysteresis result. If the step size is too large, it is possible that the economy jumps between equilibria of the stage game in every few periods, i.e., hysteresis does not emerge.

### 3 Analysis

We analyze the game by using the argument of iterated conditional dominance (Figure 3). When \( \theta_t \) is large enough, action \( I \) is strictly dominant due to Assumption 2.1. Then the \( t \)th player will choose \( I \) regardless of the actions of the other players. Let \( \overline{\theta}^0(a_{t-1}) \) be the infimum of the state \( \theta_t \) such that the \( t \)th player will choose \( I \) when the \((t - 1)\)th player chose \( a_{t-1} \) and the \((t + 1)\)th player is to choose \( N \). Note that \( \overline{\theta}^0(I) < \overline{\theta}^0(N) \). In the second step, assume that the \( t \)th player believes that the \((t + 1)\)th player will choose \( I \) if the \( t \)th chooses \( a_t \) and the state is to the right of \( \overline{\theta}^0(a_t) \). Let \( \overline{\theta}^1(a_{t-1}) \) be the boundary such that the \( t \)th player chooses \( I \) when to the right of \( \overline{\theta}^1(a_{t-1}) \). Notice that \( \overline{\theta}^1 \) is weakly to the left of \( \overline{\theta}^0 \). We proceed in this argument to give \( \overline{\theta}^2, \overline{\theta}^3, \ldots \). Let \( \overline{\theta}^\infty \) be the limit of this sequence.

Proceeding in the same way from the left where \( N \) is a dominant action (the existence of such a region is guaranteed by Assumption 2.1), we have a sequence \( \overline{\theta}^0, \overline{\theta}^1, \overline{\theta}^2, \ldots \) with the limit \( \overline{\theta}^\infty \).

\[ \begin{array}{cccccc}
\theta_t & | & N & | & \cdots & | & I \\
\theta_{t-1} & = & I & | & \overline{\theta}^0 & | & \overline{\theta}^\infty & | & \ldots & | & I \\
\theta_{t-1} & = & N & | & \overline{\theta}^0 & | & \overline{\theta}^\infty & | & \ldots & | & \overline{\theta}^1 & | & \overline{\theta}^0 & | & I \\
\end{array} \]

**Figure 3.**

By the above argument, all strategies but those such that \( s_t(a_{t-1}, \theta_t) = I \) for all \( \theta_t > \overline{\theta}^\infty(a_{t-1}) \) and \( s_t(a_{t-1}, \theta_t) = N \) for all \( \theta_t < \overline{\theta}^\infty(a_{t-1}) \) have been deleted for each player \( t \). We claim that \( \overline{\theta}^\infty = \overline{\theta}^\infty \) and therefore a unique strategy survives iterated strict dominance (Figure 4).
Theorem 1. The game has a unique equilibrium, which is given by

\[
s_t(a_{t-1}, \theta_t) = \begin{cases} 
I & \text{if } a_{t-1} = I, \theta_t > \theta^*, \\
N & \text{if } a_{t-1} = I, \theta_t < \theta^*, \\
I & \text{if } a_{t-1} = N, \theta_t > \theta^{**}, \\
N & \text{if } a_{t-1} = N, \theta_t < \theta^{**}
\end{cases}
\] (4)

for some $\theta^*$ and $\theta^{**}$ with $0 < \theta^* < \theta^{**} < 1$.

\[
s_t(a_{t-1}, \theta_t) = \begin{cases} 
I & \text{if } a_{t-1} = I, \theta_t > \theta^*, \\
N & \text{if } a_{t-1} = I, \theta_t < \theta^*, \\
I & \text{if } a_{t-1} = N, \theta_t > \theta^{**}, \\
N & \text{if } a_{t-1} = N, \theta_t < \theta^{**}
\end{cases}
\] (4)

Proof. We first show that

\[
\bar{\theta}^\infty(I) = \frac{\delta}{2 + 2\delta}.
\] (5)

Suppose that the $t$th player observes $(a_{t-1}, \theta_t) = (I, \bar{\theta}^\infty(I))$. Then, it must be true that

\[
\frac{1}{1 + \delta} \left\{ \pi(I, I, \bar{\theta}^\infty(I)) \\
+ \delta \left( \frac{1}{2} \pi(I, I, \bar{\theta}^\infty(I) + \sigma) + \frac{1}{2} \pi(I, N, \bar{\theta}^\infty(I) - \sigma) \right) \right\} \leq 0,
\]

otherwise the iterative process would have gone beyond $\bar{\theta}^\infty(I)$. We therefore have $\bar{\theta}^\infty(I) \leq \delta/(2 + 2\delta)$.

Similarly, for the player observing $(a_{t-1}, \theta_t) = (I, \bar{\theta}^\infty(I))$,

\[
\frac{1}{1 + \delta} \left\{ \pi(I, I, \bar{\theta}^\infty(I)) \\
+ \delta \left( \frac{1}{2} \pi(I, I, \bar{\theta}^\infty(I) + \sigma) + \frac{1}{2} \pi(I, N, \bar{\theta}^\infty(I) - \sigma) \right) \right\} \geq 0
\]

must hold, so that $\bar{\theta}^\infty(I) \geq \delta/(2 + 2\delta)$. It follows from $\bar{\theta}^\infty(I) \leq \bar{\theta}^\infty(I)$ that $\bar{\theta}^\infty(I) = \bar{\theta}^\infty(I) = \delta/(2 + 2\delta)$.

Second, we show that

\[
\bar{\theta}^\infty(N) = \bar{\theta}^\infty(N) = \frac{1}{1 + \delta}.
\] (6)
Suppose that the $t$th player observes $(a_{t-1}, \theta_t) = (N, 1/(1 + \delta) + \varepsilon)$, where $\varepsilon > 0$ is arbitrarily small. By the fact proved above and Assumption 2.2, the state of fundamentals in the next period, $\theta_{t+1}$, stays above $\theta^\infty(I)$, i.e.,

$$\theta^\infty(I) = \frac{\delta}{2 + 2\delta} < \theta_{t+1} = \frac{1}{1 + \delta} + \varepsilon \pm \sigma,$$

so that $s_{t+1}(I, 1/(1 + \delta) + \varepsilon + \sigma) = s_{t+1}(I, 1/(1 + \delta) + \varepsilon - \sigma) = I$. Therefore, his payoff to choosing $I$ is given by

$$\frac{1}{1 + \delta} \left\{ \pi(I, N, \frac{1}{1 + \delta} + \varepsilon) + \delta \left( \frac{1}{2} \pi(I, I, \frac{1}{1 + \delta} + \varepsilon + \sigma) + \frac{1}{2} \pi(I, I, \frac{1}{1 + \delta} + \varepsilon - \sigma) \right) \right\} = \varepsilon > 0.$$

Hence, we have $\theta^\infty(N) \leq 1/(1 + \delta)$.

Suppose next that the $t$th player observes $(a_{t-1}, \theta_t) = (N, 1/(1 + \delta) - \varepsilon)$, where $\varepsilon > 0$ is an arbitrarily small constant such that $\varepsilon < (2 - \delta)/(2 + 2\delta) - \sigma$. We can pick such an $\varepsilon$ due to Assumption 2.2. His payoff to choosing $I$ is then given by

$$\frac{1}{1 + \delta} \left\{ \pi(I, N, \frac{1}{1 + \delta} - \varepsilon) + \delta \left( \frac{1}{2} \pi(I, I, \frac{1}{1 + \delta} - \varepsilon + \sigma) + \frac{1}{2} \pi(I, I, \frac{1}{1 + \delta} - \varepsilon - \sigma) \right) \right\} = -\varepsilon < 0,$$

which implies that $\theta^\infty(N) \geq 1/(1 + \delta)$. It follows from $\theta^\infty(N) \leq \theta^\infty(N)$ that $\theta^\infty(N) = \theta^\infty(N) = 1/(1 + \delta)$.

Finally, setting $\theta^* = \delta/(2 + 2\delta)$ and $\theta^{**} = 1/(1 + \delta)$ completes the proof.

An important feature of the equilibrium is that it exhibits hysteresis. For a given sample path of $\{\theta_t\}$, the outcome is as depicted in Figure 5, where players take $I$ along the thick segments in the path, while $N$ is played along the thin segments. Two thresholds $\theta^*$ and $\theta^{**}$ generate hysteresis. If the state of fundamentals falls below $\theta^*$, the economy drops into the slump equilibrium. Unless the state rises over $\theta^{**}$, the economy is stuck in the slump equilibrium. Similarly, once the economy is in the boom equilibrium, the economy will be in boom as long as the state is above $\theta^*$. 

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Remark 1. The game has a unique equilibrium regardless of Assumption 2.2. If Assumption 2.2 is not satisfied, $\theta^{**}$ is given by

$$\theta^{**} = \begin{cases} \frac{\delta}{2 + 2\delta} + \sigma & \text{if } \frac{2 - \delta}{2 + 2\delta} \leq \sigma < \frac{1}{1 + \delta}, \\ \frac{2 + \delta}{2 + 2\delta} & \text{if } \sigma \geq \frac{1}{1 + \delta}, \end{cases}$$

while $\theta^*$ is unchanged. In this case, the equilibrium fails to exhibit hysteresis, as $\theta^*$ and $\theta^{**}$ lie within one step of the state of fundamentals.

4 Discussion

4.1 General Payoffs

The uniqueness result established in the previous section does not depend on the simplicity of the payoffs, such as their linearity in $\theta$. Define one-shot payoff $\pi(a, b, \theta)$ to be

$$\pi(a, b, \theta) = \begin{cases} u(b, \theta) & \text{if } a = I, \\ 0 & \text{if } a = N. \end{cases}$$

(7)

For the uniqueness result, it is sufficient to assume the following:

Assumption 4.1. $u$ is strictly increasing in $\theta$. 

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Assumption 4.2. The game has strategic complementarities, i.e.,
\[ u(I, \theta) > u(N, \theta) \]
for all \( \theta \).

Assumption 4.3. There are dominance regions, i.e., for \( \theta_t \) large enough,
\[
\frac{1}{1 + \delta} \left( u(N, \theta_t) + \delta E \left[ u(N, \theta_{t+1}) \mid \theta_t \right] \right) > 0,
\]
and for \( \theta_t \) small enough,
\[
\frac{1}{1 + \delta} \left( u(I, \theta_t) + \delta E \left[ u(I, \theta_{t+1}) \mid \theta_t \right] \right) < 0.
\]

Theorem 2. Under Assumptions 4.1–4.3, the game has a unique equilibrium.

4.2 Relation to the Global Game Literature

Our uniqueness result is obtained by a contagion argument similar to that by Carlsson and van Damme (1993). They study one-shot 2 × 2 coordination games where payoffs are observed privately with noise by each player. The space of (ex ante) possible payoffs contains regions where each action is strictly dominant. Using iterated strict dominance, they show that as the noise vanishes, the risk-dominant equilibrium must be played for any particular realization of payoffs. Iterated strict dominance causes a contagion effect that starts from the dominance regions and determines players’ behavior at the whole space of possible payoffs.\(^3\)

The present paper is also related to Burdzy, Frankel, and Pauzner (2001, henceforth BFP), Frankel and Pauzner (2000), Frankel (2001), and Matsui (1998). BFP consider a continuum of infinitely-lived players who are repeatedly and randomly matched to play a 2 × 2 coordination game whose payoff matrix changes over time according to a random walk. There are frictions in action revisions: each player must commit to a particular action for a random time interval. BFP show that in the limit as the period length goes to zero, a unique equilibrium survives iterative dominance. The period length must be taken to be zero so that the possibility of simultaneous action choices becomes negligible and thus players change actions (almost) asynchronously.\(^4\) Frankel and Pauzner (2000) employ the Brownian motion version of BFP and show that similar results obtain in a model.

\(^3\)Frankel, Morris, and Pauzner (2003) extend the result to general games with strategic complementarities. Morris and Shin (1998) apply the global game approach to select a unique equilibrium in a model of coordinated currency attacks. See also Morris and Shin (2003) for a survey on general results and applications.

\(^4\)Matsui and Matsuyama (1995) study a similar dynamic environment but with fixed payoffs and examine the stability of equilibria of a 2 × 2 coordination game in the dynamics.
of development. Extending these techniques, Frankel (2001) demonstrates that payoff shocks eliminate multiplicity in the expectation-driven business cycles models.

Matsui (1998) also considers sequential move games with short-run investors in the context of currency crises. Each investor’s payoff is dependent both on a random parameter that changes in each period and on the action of his successor, but is independent of the past actions. Thus, unlike our model, each investor’s choice of action has no effect on future play. This situation corresponds to that in our model where δ is ‘equal to infinity’. He shows that the game has a unique equilibrium, in which every investor takes a switching strategy with one threshold. This unique threshold is given so that investment becomes a risk-dominant action if and only if the payoff parameter exceeds the threshold. Hysteresis therefore does not emerge in his model. Note that, as observed from Remark 1, two thresholds in our model, θ∗ and θ∗∗, converge to the same limit, 1/2, as δ goes to infinity.

\[ \delta \]

They show that the risk-dominant equilibrium is likely to be played: when the friction is sufficiently small, there exists a perfect foresight equilibrium path from the risk-dominated equilibrium to the risk-dominant equilibrium, but not vice versa. Oyama (2002) shows that for symmetric \( n \times n \) games, the 1/2-dominant equilibrium is selected in the Matsui-Matsuyama environment.

An independent work by Levin (2001) considers a broader class of sequential move games, which includes models of Matsui and the present paper, and obtains similar uniqueness results.

Here, δ should be interpreted as the weight of payoff dependence on the successor’s action, rather than the discount factor.
References


