# Agglomeration under forward-looking expectations: Potentials and global stability

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**Abstract**

This paper considers a class of migration dynamics with forward-looking agents in a multi-country solvable variant of the core-periphery model of Krugman [Krugman, P., 1991. Increasing returns and economic geography. Journal of Political Economy 99, 483–499]. We find that, under a symmetric externality assumption, our static model admits a potential function, which allows us to identify a stationary state that is uniquely absorbing and globally accessible under the perfect foresight dynamics whenever the degree of friction in relocation decisions is sufficiently small. In particular, when trade barriers are low enough, full agglomeration in the country with the highest barrier is the unique stable state for small frictions. New aspects in trade and tax policy that arise due to forward-looking behavior are discussed.
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*Key words:* economic geography, agglomeration, perfect foresight dynamics, history versus expectations, stability, potential game, equilibrium selection

## 1. Introduction

Spatial agglomeration of industry is an important feature of international as well as regional economies. Following the 'core-periphery model' due to Krugman (1991a), the New Economic Geography (NEG) literature has demonstrated how increasing returns give rise to agglomeration in general equilibrium models with monopolistic competition, factor mobility, and trade costs (Fujita et al., 1999; Baldwin et al., 2003; Fujita and Thisse, 2009). In most of these models, first, agents are *myopic*. They are assumed to base their migration decisions only on current utility differences between locations. Second, migration is modeled as a continuous process. It is thus implicitly assumed that there is *friction* in individual location revisions which limits the rate at which the whole economy adjusts. Sometimes agents are assumed to follow replicator dynamics (e.g., Fujita et al., 1999), which originates in evolutionary game theory. Finally, predictions on long-run spatial distributions of industry are made based on their *local stability* under the myopic dynamics. A fundamental conclusion in the literature is that when barriers to trade are low enough, agglomeration forces due to scale economies dominate market crowding effects due to trade barriers, so that core-periphery configurations where all industry is concentrated in a single location are locally stable, while when trade barriers are high enough, a fully dispersed configuration is a unique stable state.[1] In the former case, the model cannot select among multiple locally stable states: history (i.e., the initial distribution of industry) alone determines the long-run outcome of the economy.

In the presence of friction in location revisions, however, relocation should be considered as an investment decision, and agents are naturally concerned not only with current utilities but also with expected future utilities, which depend on future location patterns of the economy. In the case where agglomeration forces are strong, it is conceivable that expectations about future location patterns may become self-fulfilling, driving the economy from a core-periphery configuration to another. In this paper, we study the role of forward-looking expectations of rational migrants in determining long-run spatial distributions of industry. We employ a solvable variant of the core-periphery model, due to Martin and Rogers (1995) and Pflüger (2004), with an extension to many countries that are asymmetric with respect to their import barriers and market sizes, in which a continuum of entrepreneurs migrate internationally with their firms. We embed the model in the context of explicit dynamics and conduct a global stability analysis by appealing to techniques from the theory of population games, notably those utilizing a potential function. It is shown that, except for degenerate cases, there exists a unique spatial distribution that is both locally and globally stable under perfect foresight dynamics, whenever the degree of friction is sufficiently small. When trade barriers are low enough, one of the core-periphery configurations is such a stable state, for which we are able to give a complete characterization in terms of key parameters. In particular, if countries are

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[1] See Robert-Nicoud (2005) for a thorough analysis on comparative statics of the set of spatial equilibria with respect to the trade cost parameter in standard two-location NEG models.

symmetric with respect to market sizes (trade barriers, resp.), the core-periphery configuration with full agglomeration in the country with the highest barrier (the largest market, resp.) is the unique stable state for small frictions. This demonstrates that allowing for forward-looking migration behavior alters results in existing models with myopic agents, where all the core-periphery configurations are locally stable under myopic dynamics.

Our dynamic framework follows Matsuyama (1991) and Matsui and Matsuyama (1995) in formulating frictions in migration decisions.[2] Each entrepreneur, once he chooses to locate his firm at some country, is locked in that country for a random time interval, so that he cares about the future location pattern of the economy. Opportunities to revise locations follow Poisson processes which are independent across individuals. The dynamics thus exhibits inertia in that the location distribution of entrepreneurs (and hence of firms) in the economy changes continuously over time. Each individual forms his belief about the future path of the distribution and, when given a relocation opportunity, locates in a country that maximizes his expected discounted utility. A *perfect foresight path* is defined to be a feasible path of location distributions along which every revising agent optimizes against the future location pattern of the society. While the stationary states of this dynamics are precisely the spatial equilibria of the static model, there may also exist a perfect foresight path that escapes from a stationary state when the degree of friction, defined as the discounted average duration of a lock-in, is sufficiently small. We consider such a state unstable, and thus employ the following stability concepts: A state $x^*$ is said to be *absorbing* if any perfect foresight path converges to $x^*$ whenever the initial state is close enough to $x^*$; $x^*$ is said to be *globally accessible* if for any initial state, there exists a perfect foresight path converging to $x^*$. We are interested in a (unique, by definition) state, if any, that is both absorbing and globally accessible for sufficiently small degrees of friction.

The key observation for our global analysis of the perfect foresight dynamics is that, viewed as a societal game with a continuum of players, our static model admits a potential function (Monderer and Shapley, 1996; Sandholm, 2001),[3] a real-valued function defined on the space of location distributions such that the change in any agent's utility resulting from a relocation exactly equals the marginal change in the value of this function.[4] We can therefore apply the results on the stability under the perfect foresight dynamics in potential games due to Hofbauer and Sorger (1999): that there generically exists a state that is absorbing and globally accessible whenever the degree of friction is sufficiently small, and such a state is characterized as a unique global maximizer of the potential function.

Once we have the above results in hand, our task is to characterize the shape of the potential function. When the trade barriers are low enough, we show that the potential function is convex and hence attains its global maximum at a vertex of the state space, and full agglomeration in the country with the highest trade barrier or with the largest market is the unique state that is absorbing and globally accessible for a small friction. This "equilibrium selection" result is in sharp contrast with the case of myopic agents, where all the agglomeration outcomes are locally stable under a broad class of myopic dynamics. When the trade barriers are high enough, in contrast, the potential function is concave and maximized at a unique state in the interior (a fully dispersed configuration), and any perfect foresight path converges to this state regardless of the initial state or the degree of friction. Hence, in this case of high trade barriers, there is no room for history or expectations to play a role.

We then briefly discuss, for the case of low trade barriers, new aspects of trade and tax policy that arise when one incorporates forward-looking migration behavior. First, it is pointed out that, given that a (relatively) higher import barrier in a country tends to work in favor of agglomeration in that country, a peripheral country may have an incentive to increase its import barrier in order to attract firms. This is not possible in standard models, where a core-periphery configuration once realized is never upset under myopic dynamics. Second, due to the possibility of self-fulfilling coordinated migration, the amount of 'agglomeration rents', which is measured by the amount of tax that can be imposed on the firms in the core country without inducing them to move away, is shown to be much smaller under the perfect foresight dynamics than under myopic dynamics.

Baldwin (2001) and Ottaviano (2001) consider the role of expectations in models with two *symmetric* locations under the dynamics due to Krugman (1991b) and Fukao and Benabou (1993), where migrants can move at any instant in time, with moving costs which depend on the speed of migration of the whole society. Baldwin (2001) considers the original core-periphery model by Krugman (1991a), which is not analytically solvable, and conducts simulation analyses, while Ottaviano (2001) proposes an analytically solvable version of the core-periphery model and derives several analytical results. Both conclude, in the case of low trade barriers, that once the economy agglomerates in a single location, self-fulfilling expectations cannot alter the outcome even if the moving cost or the discount rate is small. The equilibrium selection result in the present paper, however, suggests that their conclusion is not robust to exogenous

[2]See also Matsuyama (1992a, 1992b), Matsuyama and Takahashi (1998), and Kaneda (2003). This class of dynamics is extensively studied in game theory since Matsui and Matsuyama (1995). See, e.g., Hofbauer and Sorger (1999, 2002), Oyama (2002), Matsui and Oyama (2006), Oyama et al. (2008), and Oyama and Tercieux (2009).

[3]See also Sandholm (2009) and Ui (2007).

[4]It is worthwhile to note that in general, if there are only two alternatives (locations, in our context) to choose, then a potential function always exists, provided that utilities depend only on own choice and the fraction of individuals choosing each alternative. Any NEG model with two locations that has this property hence admits a potential function.

asymmetries between locations (clearly, if equilibria are all identical, one cannot discriminate among them).[5] It should be noted that the assumption of symmetric locations has typically been made in the NEG literature in order to explore pure economic mechanisms that may generate asymmetric stable spatial outcomes without endowing any location with an exogenously given advantage, and introduction of exogenous asymmetries does not fundamentally affect the main messages of NEG as long as the stability analysis is based on myopic dynamics. In contrast, the model symmetry is no longer innocuous once one allows for forward-looking behavior. This is not a consequence of the particular dynamics we employ in this paper. Indeed, the accompanied paper, Oyama (2009), studies the Krugman-Fukao-Benabou dynamics in a two-country setting and obtains the same stability result as in the present paper: that if the potential function has a unique global maximizer, then it is absorbing and globally accessible also under this class of dynamics for small frictions.

Different from Oyama (2009) as well as Baldwin (2001) and Ottaviano (2001), where, with only two countries, the "dimensionality problem" (Behrens and Thisse, 2007) does not arise in the first place, the present paper aims at investigating the impact of forward-looking expectations in a *multi*-country NEG model. For this purpose, the Poisson formulation of Matsui-Matsuyama employed here has a substantial advantage in terms of analytical tractability over the adjustment cost formulation of Krugman-Fukao-Benabou. As emphasized in Oyama (2009), the Krugman-Fukao-Benabou dynamics requires delicate mathematical treatments even in the two-country case, so that extending the formulation to the setting with more than two countries would be a cumbersome task, whereas it is straightforward in the Matsui-Matsuyama dynamics. These two classes of dynamics also differ in their relevance in economic applications. A similar formulation with explicit adjustment costs had been used by Mussa (1978) in a dynamic version of a standard international trade model, which in effect reduces to a single agent investment problem where the adjustment cost depends only on the agent's own choice variable, and Krugman (1991b) directly translated it into a situation with positive externalities under the alternative assumption that the adjustment cost depends on the other agents' behavior.[6] While the adjustment cost formulation following Krugman (1991b) appears to be more popular in NEG (see, e.g., Baldwin et al., 2003),[7] the Poisson formulation can be found in many other fields where interactions of agents are of importance: our relocation opportunity process is formally equivalent to the birth-death process in the continuous-time overlapping generations model of Yaari (1965) and Blanchard (1985) (see Matsuyama, 1991, 1992a), and can also be viewed as (exogenous) entry-exit of firms which is often modeled with a Poisson process in studies of industry dynamics (e.g., Melitz, 2003; Miao, 2005); equilibrium search models typically introduce frictions by assuming that trading opportunities arrive according to a Poisson process (e.g., Diamond and Fudenberg, 1989; Kiyotaki and Wright, 1991).[8] A difference between the two formulations also lies in the policy implications they may yield when explicit dynamic policy schemes are concerned, as pointed out by Kaneda (2003) in the context of infant industry protection; this issue will be discussed in Section 5.

Methods that utilize potential functions are found in a diverse range of applications, including population genetics (Fisher, 1930; Hofbauer and Sigmund, 1998), network traffic (Beckmann et al., 1956; Rosenthal, 1973), Cournot oligopoly (Slade, 1994; Monderer and Shapley, 1996), environmental economics (Mäler et al., 2003), and mechanism design and implementation (Sandholm, 2002, 2007; Jehiel et al., 2008). Global maximizers of potential functions are known to have nice properties such as the stochastic stability under stochastic evolutionary dynamics with logit choice (Blume, 1993) and the robustness to incomplete information (Ui, 2001), in addition to the stability under perfect foresight dynamics. The present paper adds economic geography to the list of applications where potential methods are of significant use.

The paper is organized as follows. Section 2 describes our static and dynamic setups. Section 3 defines and constructs a potential function for the induced static societal game and states the stability properties under the perfect foresight dynamics. Section 4 characterizes the stable spatial equilibrium for low and high levels of trade barriers. Section 5 discusses trade and tax policy issues. Section 6 concludes.

## 2. Framework

We denote by $\mathbb{R}^n$ the $n$-dimensional real space with a norm $|\cdot|$, by $\Delta$ the $(n-1)$-dimensional simplex (which is a convex and compact subset of $\mathbb{R}^n$), and by $e_i$ $(i = 1, \ldots, n)$ the $i$th vertex of $\Delta$ (i.e., the $i$th unit vector in $\mathbb{R}^n$). By $\mathrm{int}(\Delta)$ and $\partial(\Delta)$ we denote the relative interior and the

[5] If the two locations are completely symmetric, then, in the case of low trade barriers, the potential function is maximized at both vertices of the state space. Any small asymmetry, such as in trade barriers or market size, breaks the tie, leading to a unique (global) maximizer of the potential function. Under myopic dynamics, in contrast, *local* maximizers of a potential function are locally stable, so that small asymmetry does not alter the stability properties.

[6] This assumption makes that model fundamentally different from Mussa's (1978), and is in fact the source of the error in Krugman (1991b) corrected by Fukao and Benabou (1993). See Oyama (2009) for the technical complications that result from this assumption.

[7] Mossay (2006) considers forward-looking rational agents in an adjustment cost model with a continuum of locations on a circle, but where, like in Mussa (1978) and unlike in Krugman (1991b), the adjustment cost each migrant incurs depends only on his own migration speed.

[8] An analogous assumption is also seen in the sticky price literature following Calvo (1983), where opportunities for firms to revise their prices arrive according to independent Poisson processes.

relative boundary of $\Delta$, respectively. For $x \in \Delta$ and $\varepsilon > 0$, we denote $B_\varepsilon(x) = \{y \in \Delta \mid |y - x| < \varepsilon\}$. We regard elements of $\mathbb{R}^n$ as column vectors and let $x'$ denote the transpose of a vector $x \in \mathbb{R}^n$.

*2.1. Environment*

We consider an infinite horizon economy which consists of $n \geq 2$ countries, $1, \ldots, n$, inhabited by a continuum of infinitely lived individuals. Time is continuous, and periods are indexed by $t \in [0, \infty)$. There are two factors of production, skilled and unskilled labor, and two sectors, which produce a differentiated ('modern') good and a homogeneous ('traditional') good,[9] respectively, where goods are assumed not to be durable intertemporarily. There are mass $H + L$ of individuals, $H$ skilled workers and $L$ unskilled workers, each of whom is endowed at each time $t$ with one unit of skilled or unskilled labor, respectively, as well as $\bar{A}$ units of traditional good. Skilled workers are considered as self-employed *entrepreneurs*. They are mobile between countries, and their spatial distribution is endogenously determined. We normalize the total mass of entrepreneurs to be one, i.e., $H \equiv 1$, and denote by $x_i$ the mass of entrepreneurs located in country $i$ $(= 1, \ldots, n)$, so that $x = (x_1, \ldots, x_n)' \in \Delta$. The initial distribution $x^0 \in \Delta$ is exogenously given. Unskilled labor is immobile internationally, and each country $i$ hosts the exogenously fixed mass $L_i$ of unskilled workers, where $L = L_1 + \cdots + L_n$.

Preferences over the modern and the traditional goods are identical across individuals. As in Martin and Rogers (1995) and Pflüger (2004), the instantaneous utility of an individual in country $i$ is given by a logarithmic quasi-linear utility function:

$$U(M_i, A_i) = \mu \log M_i + A_i, \tag{2.1}$$

where $\mu > 0$, and $M_i$ ($A_i$, resp.) is the consumption of the modern (traditional, resp.) good. The modern good is a CES aggregate of varieties:

$$M_i = \left[ \sum_{j=1}^{n} \int_0^{N_j} d_{ji}(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{2.2}$$

where $d_{ji}(z)$ is the consumption in country $i$ of a variety $z$ produced in country $j$, $N_j$ is the mass of varieties produced in country $j$, and $\sigma > 1$ is the elasticity of substitution between any two varieties (Dixit and Stiglitz, 1977). Here, it is assumed that the varieties in each country $j$ are ordered so that they are indexed by $[0, N_j]$. Individuals discount future utility flows exponentially with a common rate of time preference $\theta > 0$.

Firms in the modern sector are monopolistically competitive and employ both skilled and unskilled labor. Entry and exit are free and instantaneous. Each variety $z$ in the economy is produced by a single firm, where its production exhibits increasing returns to scale. As in Ottaviano (2001) and Forslid and Ottaviano (2003), technology is given by

$$F^{\mathrm{M}}(h_i(z), \ell_i(z)) = \begin{cases} 0 & \text{if } h_i(z) < 1 \\ \dfrac{\ell_i(z)}{c} & \text{if } h_i(z) \geq 1, \end{cases}$$

where $c > 0$, and $h_i(z)$ ($\ell_i(z)$, resp.) is the amount of skilled (unskilled, resp.) labor input. That is, in order to produce $Q_i(z)$ units of variety $z$, the firm incurs a fixed input requirement of one unit of skilled labor (i.e., an entrepreneur) and a variable input requirement of $cQ_i(z)$ units of unskilled labor. Note that an entrepreneur and a manufacturing firm thus correspond one to one. The traditional sector is perfectly competitive and employs unskilled labor as the only input under constant returns. Technology is given, without loss of generality, by

$$F^{\mathrm{A}}(\ell_i) = \ell_i,$$

where $\ell_i$ is the amount of unskilled labor input. The skilled and the unskilled labor markets are perfectly competitive.

Goods differ in terms of their spatial mobility. While trade in the traditional good is free, it is costly in the modern good due to trade barriers, which are modeled as iceberg costs. For one unit of a variety to reach country $i$ from country $j \neq i$, $\tau_{ji} > 1$ units must be shipped. We denote

$$\phi_{ji} = \tau_{ji}^{1-\sigma} \in (0, 1).$$

While entrepreneurs are mobile between any two countries, they cannot move at every point in time. As in Matsuyama (1991) and Matsui and Matsuyama (1995), there are frictions in migration decisions. Once an entrepreneur chooses to locate himself and set up a firm in one country, he must commit to the choice for a random time interval, due to a large migration cost. Opportunities to migrate follow a Poisson process with parameter $\lambda > 0$.[10] It is assumed that these processes are independent across individuals and there is no aggregate uncertainty. Thus, during a short time interval $[t, t + dt)$, a fraction $\lambda \cdot dt$ of entrepreneurs are entitled to migrate. We will view the discounted average duration of a commitment,

[9] Our focus will be on the locational distribution of production of the differentiated good. We incorporate the homogeneous good, which will be chosen as the numeraire, in order to ease the analysis. In particular, together with the assumption of zero trade costs in the homogeneous good, it will guarantee equalization of the wage for unskilled labor among countries.

[10] Here we follow the formulation of Matsui and Matsuyama (1995). But this is mathematically equivalent to a continuous-time OLG model à la Blanchard (1985) with irreversibility in migration decisions. That is, one could instead assume as in Matsuyama (1991) that there are a continuum of overlapping agents where each agent is replaced by his successor according to a Poisson process with parameter $\lambda > 0$, and each is entitled to choose a country to locate only upon entry to the economy and then stuck in the country for the rest of his life.

$\delta = \theta/\lambda$, as the *degree of friction* in migration decision. Alternatively, the inverse of the degree of friction, $1/\delta = \lambda/\theta$, which is the expected frequency of the conscious decision made by an entrepreneur per unit of psychological time, may be interpreted as *entrepreneurial alertness*, or simply as *entrepreneurship* (Matsuyama, 1992b).

Finally, we impose the following set of conditions in order to simplify the subsequent analyses.

**Assumption 2.1.** (1) $\bar{A} \geq \mu$.
(2) $\mu < \sigma L_i/(\sigma-1)\left(\sum_j L_j + 1\right)$ for all $i = 1, \ldots, n$.
(3) $\sigma/(\sigma-1) \leq L_i$ for all $i = 1, \ldots, n$.

Assumption (1) guarantees interior solutions for the utility maximization problem in Subsection 2.2, while (2) is the 'non-full-specialization condition', which guarantees the traditional sector to be always active in all countries (cf. Pflüger, 2004, Footnote 8) and will be used in Subsection 2.2. The condition in (3) is a sufficient condition under which dispersion becomes a unique outcome in the limit as the freeness of trade goes to zero for all countries, which will be used in Subsection 4.2. We note that this condition is stronger than necessary. In the case of two countries (i.e., when $n = 2$), the necessary and sufficient condition, which is called the 'no-black hole condition' in the literature, is that $\sigma/(\sigma-1) \leq L_1 + L_2$ (cf. Pflüger, 2004, Footnote 9).

*2.2. Instantaneous Market Equilibrium*

In this subsection, we solve the static model fixing a distribution of entrepreneurs (and hence firms) $x \in \Delta$ as given, to obtain the instantaneous market equilibrium. Here, following the standard approach in the NEG literature, we rule out any form of intertemporal trade.

We choose the traditional good as the numeraire. Denote by $p_{ji}(z)$ the price of variety $z$ produced in country $j$ and sold in $i$ and by $r_i$ ($w_i^{\mathrm{L}}$, resp.) the wage of entrepreneurs (unskilled workers, resp.) in $i$.

An individual in $i$ maximizes $U(M_i, A_i)$ subject to

$$\sum_{k=1}^{n} \int_0^{N_k} p_{ki}(z) d_{ki}(z) dz + A_i = y_i + \bar{A},$$

where $y_i$ equals $r_i$ (if he is an entrepreneur) or $w_i^{\mathrm{L}}$ (if he is a worker). This yields the demand functions:

$$d_{ki}(z) = \mu p_{ki}(z)^{-\sigma} P_i^{\sigma-1}, \quad M_i = \mu P_i^{-1}, \quad A_i = y_i + \bar{A} - \mu,$$

where $P_i$ is the price index in country $i$:

$$P_i = \left[\sum_{k=1}^{n} \int_0^{N_k} p_{ki}(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}.$$

Since unskilled labor markets are perfectly competitive and technology in the traditional sector exhibits constant returns to scale with unit input coefficient equal to one, we have $w_i^{\mathrm{L}} = 1$ for all $i$ due to our choice of numeraire. Wage equalization holds as long as the traditional good is produced in all countries, which is guaranteed by Assumption 2.1(2).

Skilled labor market clearing implies $N_i = x_i$, so that the number of manufacturing firms (and hence of varieties) in country $i$ is equal to the number of entrepreneurs living in $i$. The total cost of a typical firm located in country $i$ producing variety $z$ (denoted $(i, z)$) is given by $r_i + cQ_i(z)$, where $Q_i(z)$ is the total output of this firm. Firm $(i, z)$ maximizes profit:

$$\Pi_i(z) = p_{ii}(z) D_{ii}(z) + \sum_{k \neq i} p_{ik}(z) D_{ik}(z) - c\left[D_{ii}(z) + \sum_{k \neq i} \tau_{ik} D_{ik}(z)\right] - r_i,$$

where $D_{ik}(z) = (x_k + L_k) d_{ik}(z)$ is the total demand by the residents in location $k$ for the variety produced by firm $(i, z)$. Profit maximization prices are constant markups on marginal costs:

$$p_{ii}(z) = \frac{\sigma}{\sigma-1}c, \quad p_{ik}(z) = \frac{\tau_{ik}\sigma}{\sigma-1}c$$

for all $i$ and $k \neq i$. Thus the price index in country $i$ is given by

$$P_i(x) = \frac{\sigma c}{\sigma-1}\left(N_i + \sum_{k \neq i} \phi_{ki} N_k\right)^{\frac{1}{1-\sigma}} = \frac{\sigma c}{\sigma-1}\left(x_i + \sum_{k \neq i} \phi_{ki} x_k\right)^{\frac{1}{1-\sigma}},$$

where $\phi_{ki} = \tau_{ki}^{1-\sigma} \in (0, 1)$. In matrix notation, $P_i(x)$ is written as

$$P_i(x) = \frac{\sigma c}{\sigma-1}\left[(x'\Phi)_i\right]^{\frac{1}{1-\sigma}}, \tag{2.3}$$

where the matrix $\Phi \in \mathbb{R}^{n \times n}$ is given by

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{pmatrix}$$

with $\phi_{ii} = \tau_{ii}^{1-\sigma} = 1$. We note that $(x'\Phi)_i = x_i + \sum_{k \neq i} \phi_{ki} x_k$ may be viewed as the *effective* total mass of firms in country $i$ where the mass of country-$k$ firms, $x_k$, is "discounted" by $\phi_{ki}$. The individual demands are thus given by

$$d_{ki}(z) = \mu\left(\frac{\sigma c}{\sigma-1}\right)^{-1} \tau_{ki}^{-\sigma}\left[(x'\Phi)_i\right]^{-1}$$

for all $i, k$ and $z$.

Under free entry and exit, and hence zero profit, $\Pi_i(z) = 0$, the reward for an entrepreneur in country $i$, $r_i$, is given by his firm's gross profit:

$$r_i = (p^* - cw^{\mathrm{L}})Q_i(z) = \frac{1}{\sigma}p^* Q_i(z),$$

where $p^* = c\sigma/(\sigma-1)$ is the mill price, $w^{\mathrm{L}} = 1$ is the wage for unskilled workers, and $Q_i(z)$ is the total production of the firm $(i,z)$. Market clearing for the variety $z$ implies

$$\begin{aligned} Q_i(z) &= D_{ii}(z) + \sum_{k\neq i}\tau_{ik}D_{ik}(z) \\ &= \left(\frac{\sigma c}{\sigma-1}\right)^{-1}\frac{\mu(x_i+L_i)}{x_i+\sum_{k\neq i}\phi_{ki}x_k} \\ &\quad + \sum_{k\neq i}\left(\frac{\sigma c}{\sigma-1}\right)^{-1}\phi_{ik}\frac{\mu(x_k+L_k)}{x_k+\sum_{\ell\neq k}\phi_{\ell k}x_\ell} \end{aligned}$$

for all $i$ and $z$. Hence, the equilibrium reward for a country-$i$ agent is computed as

$$r_i(x) = \frac{1}{\sigma}\left[\frac{\mu(x_i+L_i)}{x_i+\sum_{k\neq i}\phi_{ki}x_k} + \sum_{k\neq i}\phi_{ik}\frac{\mu(x_k+L_k)}{x_k+\sum_{\ell\neq k}\phi_{\ell k}x_\ell}\right], \tag{2.4}$$

or, in matrix notation,

$$r(x) = \frac{\mu}{\sigma}\Phi\big[\mathrm{diag}(x'\Phi)\big]^{-1}m(x), \tag{2.5}$$

where $\mathrm{diag}(x'\Phi) \in \mathbb{R}^{n\times n}$ is the diagonal matrix generated by the vector $x'\Phi$, and

$$m(x) = (x_1+L_1,\dots,x_n+L_n)' \in \mathbb{R}^n.$$

The expression (2.4) can be given a straightforward interpretation. The first term in the brackets is the revenue that the firm obtains from the home country $i$, while the other is the sum of those from the foreign countries, which are "discounted" by $\phi_{ik}$'s. In each country $k$, firms compete for the total expenditure on the manufacturing good spent by the residents in $k$, which always equals $\mu m_k(x) = \mu(x_k+L_k)$ due to our assumption of logarithmic quasi-linear preference, and each domestic firm receives $1/(x'\Phi)_k$ of the total expenditure, while each country-$i$ firm receives $\phi_{ik}/(x'\Phi)_k$. Finally, markup pricing implies that each firm's revenue is divided by the entrepreneur and the workers according to the shares $(p^*-cw^{\mathrm{L}})/p^*$ and $cw^{\mathrm{L}}/p^*$, respectively, so that the entrepreneur receives $1/\sigma$ of the revenue.

We now have the instantaneous indirect utility of an entrepreneur located in country $i$, $u_i(x)$, as a function of $x\in\Delta$:

$$\begin{aligned} u_i(x) &= \mu\log(\mu P_i(x)^{-1}) + r_i(x) + \bar{A} - \mu \\ &= \frac{\mu}{\sigma-1}\log(x'\Phi)_i + \frac{\mu}{\sigma}\Big[\Phi\big[\mathrm{diag}(x'\Phi)\big]^{-1}m(x)\Big]_i + C, \end{aligned} \tag{2.6}$$

where

$$C = \mu\log\mu\left(\frac{\sigma c}{\sigma-1}\right)^{-1} + \bar{A} - \mu.$$

### *2.3. Societal Game*

The profile $\mathbf{u} = (u_i)_{i=1}^n$ of $C^1$ functions $u_i\colon \Delta\to\mathbb{R}$ given by (2.6) defines a *societal game*, a special case of nonatomic game (Schmeidler, 1973) in which a continuum of homogeneous players choose among actions $1,\dots,n$, and the payoffs are determined solely by the action distribution $x$ (rather than action profile) as well as one's own action. A distribution $x^*\in\Delta$ is said to be an *equilibrium state* of $\mathbf{u}$ if $x_i^*>0 \Rightarrow u_i(x^*)\geq u_j(x^*)$ for all $j$; $x^*$ is said to be a *strict equilibrium state* of $\mathbf{u}$ if $x_i^*>0\Rightarrow u_i(x^*)>u_j(x^*)$ for all $j\neq i$. Note that $x^*$ is an equilibrium state if and only if $u_i(x^*)=u_j(x^*)$ for all $i,j\in\mathrm{supp}(x^*)$ and $u_i(x^*)\geq u_j(x^*)$ for all $i\in\mathrm{supp}(x^*)$ and $j\notin\mathrm{supp}(x^*)$. Due to the finiteness of actions and the continuity of $u_i$'s, it is a standard exercise to verify the existence of equilibrium state. The proof is formally identical to that of the existence of Nash equilibrium of finite normal form games (Nash, 1950). The best response correspondence of $\mathbf{u}$, $B\colon\Delta\to\Delta$, is defined by

$$B(x) = \{\alpha\in\Delta \mid \alpha_i>0\Rightarrow u_i(x)\geq u_j(x) \text{ for all } j\}, \tag{2.7}$$

where $B(x)$ is the convex hull of $\arg\max_i u_i(x)$ in $\Delta$ (which is obviously nonempty). Since $u_i$'s are continuous, $B$ has a closed graph. It therefore follows from Kakutani's fixed point theorem that $B$ has a fixed point $x^*\in B(x^*)$, which is an equilibrium state of $\mathbf{u}$.

In the rest of this subsection, we make some (coarse) observations about the relationship between the equilibrium states of the societal game $\mathbf{u}$ defined by (2.6) and the parameter values regarding trade openness, $\phi_{ij}$, and market size, $L_i$. We confirm that, as in standard NEG models with symmetric two locations, our model exhibits the general feature that if trade costs are sufficiently small, then agglomeration forces due to scale economies become dominate so that core-periphery states are strict equilibrium states, while if trade costs are sufficiently small, then dispersion forces due to market crowding become dominant so that a dispersion state is a unique equilibrium state. More precisely, we show that if $\phi_{ij}$'s are close to one, then the core-periphery states are all strict equilibrium states provided that $\phi_{ij}$'s and $L_i$'s are not too diverse, while if $\phi_{ij}$'s are close to zero, then there exists a unique equilibrium state, which is a fully dispersed state.

First, for the core-periphery state $e_i$ where all firms are located in country $i$, the utilities are given by

$$u_i(e_i) = \frac{\mu}{\sigma}(1+L) + C,$$

and

$$u_j(e_i) = \frac{\mu}{\sigma - 1}\log\phi_{ij} + \frac{\mu}{\sigma}\left[\phi_{ji}(1+L_i) + \frac{1}{\phi_{ij}}L_j + \sum_{k\neq i,j}\frac{\phi_{jk}}{\phi_{ik}}L_k\right] + C$$

for $j \neq i$. Now let $\hat{\phi} = \max_{i,j}\{L_j/(L_i+1)\}^{1/2}$, and assume that the market sizes are not so different from each other that $\hat{\phi} < 1$. Then, fix $\phi \in (\hat{\phi}, 1)$. Since $1 + L_i + L_j > \phi(1+L_i) + L_j/\phi$, if $\phi_{ij}$'s are in a neighborhood of $\phi$, then for all $i = 1,\dots,n$, $u_i(e_i) > u_j(e_i)$ for all $j \neq i$, that is, $e_i$ is a strict equilibrium state. It is well known that any strict equilibrium state is locally stable under standard myopic evolutionary dynamics such as best response dynamics and replicator dynamics.

Second, consider the limit as all $\phi_{ij}$'s, $i \neq j$, go to zero. The utilities in this limit are given by

$$u_i(x) = \frac{\mu}{\sigma - 1}\log x_i + \frac{\mu}{\sigma}\frac{x_i + L_i}{x_i} + C.$$

Under the Assumption 2.1(3) that $\sigma/(\sigma-1) \leq L_i$, $u_i(x)$ is strictly decreasing in $x_i$ (and independent of $x_j$, $j \neq i$), so that we have

$$(y-x)'(u(y)-u(x)) < 0 \tag{2.8}$$

for all $x, y \in \mathrm{int}(\Delta)$, $x \neq y$. Hence, the limit game as $\phi_{ij} \to 0$ is what is called a 'strictly stable game' by Hofbauer and Sandholm (2009), and thus has a unique equilibrium state, which clearly lies in $\mathrm{int}(\Delta)$. If $\phi_{ij}$'s are positive but sufficiently close to zero, then the game $\mathbf{u}$ continues to be a strictly stable game satisfying the inequality (2.8), and its unique equilibrium lies in $\mathrm{int}(\Delta)$ (i.e., it is a fully dispersed equilibrium). Note that, as shown by Hofbauer and Sandholm (2009), the unique equilibrium of a strictly stable game is locally stable under various classes of myopic evolutionary dynamics including best response dynamics and replicator dynamics.

### *2.4. Equilibrium Migration Dynamics*

A path of the distribution of entrepreneurs is described by a function $x\colon [0,\infty) \to \Delta$, where $x(t) = (x_1(t),\dots,x_n(t))'$ is the distribution at time $t$, with $x_i(t)$ denoting the fraction of the entrepreneurs in country $i$. The assumption of Poisson migration opportunities motivates the following feasibility concept, where $\lambda$ is the Poisson parameter.

**Definition 2.1.** A path $x\colon [0,\infty) \to \Delta$ is said to be *feasible* if it is Lipschitz continuous, and for almost all $t \geq 0$ there exists $\alpha(t) \in \Delta$ such that

$$\dot{x}(t) = \lambda\bigl(\alpha(t) - x(t)\bigr). \tag{2.9}$$

In Equation (2.9), $\alpha(t) = (\alpha_1(t),\dots,\alpha_n(t))' \in \Delta$ denotes the distribution of the entrepreneurs who have an opportunity to migrate during the short time interval $[t, t+dt)$, so that $\alpha_i(t) \times \lambda dt$ is the mass of revising entrepreneurs who choose to locate in region $i$ during this time interval. In terms of the geometry on the state space $\Delta$, along a feasible path $x(\cdot)$ the current distribution $x(t) \in \Delta$ always moves towards the distribution $\alpha(t) \in \Delta$. Formally, a feasible path $x(\cdot)$ satisfies that $\dot{x}_i(t) \geq 0$ whenever $x_i(t) = 0$, and $\sum_{i=1}^n \dot{x}_i(t) = 0$ (and thus $x(\cdot)$ does not leave $\Delta$).

An entrepreneur forms a belief about the future evolution of the distribution and, if given the opportunity to switch locations, migrates to a country that maximizes his expected discounted indirect utility flows. Since the duration of the commitment has an exponential distribution with mean $1/\lambda$, the expected discounted utility of committing to country $i$ at time $t$ with a given anticipated feasible path $x(\cdot)$ is represented by

$$\begin{aligned} V_i(t) &= (\lambda+\theta)\int_0^\infty\int_t^{t+s} e^{-\theta(z-t)}u_i(x(z))\,dz\,\lambda e^{-\lambda s}ds \\ &= (\lambda+\theta)\int_t^\infty e^{-(\lambda+\theta)(s-t)}u_i(x(s))\,ds, \end{aligned} \tag{2.10}$$

where $\theta > 0$ is the common rate of time preference. Note that this value is normalized by the factor $\lambda + \theta$, which is viewed as the *effective* discount rate, and that if $x(\cdot)$ is the constant path at $x^0 \in \Delta$ (i.e., $x(t) = x^0$ for all $t \geq 0$), then we have $V_i(t) = u_i(x^0)$ for all $i = 1,\dots,n$ and all $t \geq 0$.

A perfect foresight path is an equilibrium path of the dynamic model, i.e., a feasible path along which each revising agent optimizes against the future course of migration behavior in the economy.

**Definition 2.2.** A feasible path $x(\cdot)$ is said to be a *perfect foresight path* if for all $i = 1,\dots,n$, and almost all $t \geq 0$,

$$\dot{x}_i(t) > -\lambda x_i(t) \Rightarrow V_i(t) \geq V_j(t) \text{ for all } j = 1,\dots,n. \tag{2.11}$$

Note that $\dot{x}_i(t) > -\lambda x_i(t)$ (i.e., $\alpha_i(t) > 0$ in (2.9)) implies that some positive fraction of entrepreneurs choose to locate in country $i$ during short time interval $[t, t+dt)$. The definition says that such a decision must be an optimal choice against the path $x(\cdot)$ itself.

The continuity of $u_i$'s guarantees the existence of a perfect foresight path.

**Observation 2.1.** *For each initial distribution, there exists a perfect foresight path.*

*Proof.* See Oyama et al. (2008, Subsection 2.3). □

It is immediate to see that the stationary states of the perfect foresight dynamics are precisely the equilibrium states of the societal game.

**Observation 2.2.** *The feasible path $\bar{x}(\cdot)$ such that $\bar{x}(t) = x^*$ for all $t \geq 0$ is a perfect foresight path if and only if $x^*$ is an equilibrium state.*

There may exist, however, another perfect foresight path from an equilibrium state $x^*$ which departs $x^*$ and converges to another equilibrium state. As we will see, when the degree of friction $\delta = \theta/\lambda > 0$ is sufficiently small, this may happen even from a strict equilibrium state. We employ the following stability concepts due to Matsui and Matsuyama (1995).

**Definition 2.3.** (a) $x^* \in \Delta$ is *absorbing* if there exists $\varepsilon > 0$ such that any perfect foresight path from any $x \in B_\varepsilon(x^*)$ converges to $x^*$. $x^*$ is *fragile* if it is not absorbing.

(b) $x^* \in \Delta$ is *accessible* from $x \in \Delta$ if there exists a perfect foresight path from $x$ that converges to $x^*$. $x^*$ is *globally accessible* if $x^*$ is accessible from any $x$.

By definition, if an absorbing state is also globally accessible, then it is a unique absorbing state and any other state is fragile. If $x^*$ is absorbing, then history matters in the following sense: if history picks the initial condition in a neighborhood of $x^*$, then any form of self-fulfilling expectations cannot alter the outcome and the economy necessarily converges to $x^*$. Conversely, if self-fulfilling expectations can drive the economy away from $x^*$, then $x^*$ is fragile. If $x^*$ is globally accessible, then expectations matter in the following sense: whatever initial condition history picks, there exists some form of self-fulfilling expectations that leads the economy to $x^*$. It is straightforward to verify that if the degree of friction is large enough, then any strict equilibrium state, if any, is absorbing, so that self-fulfilling expectations play no role. We are interested in a state that is uniquely absorbing and globally accessible for sufficiently small degrees of friction.

## 3. Potential and Stability

### *3.1. The Symmetric Externality Assumption*

Since we work with an $n$-country setting where the state space $\Delta$ has dimension $n-1$, stability analysis becomes nontrivial when $n \geq 3$. This is true even for myopic evolutionary dynamics: for example, it is well known that even a unique equilibrium may fail to be locally stable even with linear payoff functions when there are more than two alternatives (see, e.g., Hofbauer and Sigmund, 1998).[11,12] Difficulties are naturally accelerated here as we allow for forward-looking behavior. Accordingly, from now on we restrict our attention to the situations in which the following assumption on trade costs $\tau_{ji}$, which we call *Symmetric Externality*, holds (obviously it imposes no restriction when $n = 2$).[13]

**Assumption 3.1** (Symmetric Externality)**.** Trade costs depend only on the destination country: $\tau_{ji} = \tau_i$ for all $j \neq i$.

This assumption covers the following cases:

- two countries with asymmetric trade costs (as in Forslid and Ottaviano, 2003, Section 4[14]);
- $n \geq 3$ countries with symmetric trade costs (as in Tabuchi et al., 2005);
- $n \geq 3$ countries where trade costs arise from import barriers of the destination country and different countries may have different levels of barriers.

Formally, the first two cases are special cases of the third. We refer to $\tau_i$ as the *trade barrier of country* $i$ and to $\phi_i = \tau_i^{1-\sigma}$ as the *trade openness of country* $i$.

Our assumption captures two of the four aspects of trade costs described by Spulber (2007) as the "four Ts": *Transaction costs* that result from doing business at a distance due to differences in customs, business practices, as well as political and legal climates; and *Tariff and non-tariff costs* such as different anti-pollution standards, anti-dumping practices, and the massive regulations that still restrict trade and investment. It is reasonable that transaction and tariff costs are predominantly destination-specific. These are most relevant when policy issues such as trade protection and taxation are concerned, and possible impacts of forward-looking behavior on policy implications will be discussed in Section 5. On the other hand, we are to neglect asymmetries in the other two Ts, *Transportation costs* and *Time costs*, which inherently depend on both origin and destination. Our assumption, although admittedly restrictive, will enable us to obtain clear-cut analytical results, thereby providing a first step towards the understanding of forward-looking expectations and spatial agglomeration in general multi-location situations.

To see why we refer to this assumption as the Symmetric Externality Assumption, recall from Equation (2.6) that the instantaneous utilities depend on the state $x$ through the terms $x'\Phi$ and $m(x)$, where $(x'\Phi)_i = x_i + \sum_{k\neq i} \phi_{ki} x_k$, as noted already, is viewed as the effective total mass of firms in $i$, while $m_i(x) = x_i + L_i$ is the total mass of consumers in $i$. Let the effective mass of firms be written as a function $\Xi_i$ of $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)'$:

$$\Xi_i(x_{-i}) = 1 - \sum_{k\neq i} (1 - \phi_{ki}) x_k.$$

[11] The stability property of a unique equilibrium state may also be different under different evolutionary dynamics depending on their fine details; see, e.g., Hofbauer and Sandholm (2009).

[12] In the NEG literature, the "dimensionality problem" has been left largely untouched (Behrens and Thisse, 2007). Among the few theoretical papers that allow for many locations, Tabuchi et al. (2005) assume that trade costs are the same regardless of the origin and destination, while Behrens et al. (2007) deal with a matrix of trade freeness (under the symmetry assumption that $\phi_{ij} = \phi_{ji}$) but in a 'footloose capital' model where no self-reinforcing agglomeration force is present and thus an equilibrium is always unique.

[13] It is interesting to note that an analogous assumption is introduced by Hofbauer (1985) in a selection-mutation model in population genetics: that mutation rates depend only on the target gene and are independent of the original gene. Hofbauer (1985) shows that under this assumption, his model admits a potential function.

[14] See also Leite et al. (2009).

Consider now a marginal inflow of firms to country $j \neq i$ from other countries. Its impact on the effective mass of firms in $i$ (hence "externality") is represented by $\partial \Xi_i / \partial x_j$. Assumption 3.1 implies that, for all $x_{-i}$,

$$\frac{\partial \Xi_i}{\partial x_j}(x_{-i}) = \frac{\partial \Xi_i}{\partial x_k}(x_{-i})$$

for all $j, k \neq i$ (hence "symmetric"). (The analogous property obviously holds for $m_i$.)

Under Assumption 3.1, for each country $i$, the price index and the reward for entrepreneurs can be written as

$$P_i(x) = \frac{\sigma c}{\sigma - 1} \left\{x_i + \phi_i(1 - x_i)\right\}^{\frac{1}{1-\sigma}}$$

and

$$r_i(x) = \frac{1}{\sigma}\left[\frac{\mu(x_i + L_i)}{x_i + \phi_i(1 - x_i)} + \sum_{k \neq i} \phi_k \frac{\mu(x_k + L_k)}{x_k + \phi_k(1 - x_k)}\right],$$

respectively, so that the instantaneous utility function $u_i$ is now expressed as

$$u_i(x) = S_i(x_i) + R(x), \tag{3.1}$$

where functions $S_i\colon [0,1] \to \mathbb{R}$ and $R\colon \Delta \to \mathbb{R}$ are given by

$$S_i(x_i) = \frac{\mu}{\sigma - 1} \log\left((1 - \phi_i)x_i + \phi_i\right) + \frac{\mu}{\sigma} \frac{(1 - \phi_i)(x_i + L_i)}{(1 - \phi_i)x_i + \phi_i} \tag{3.2}$$

and

$$R(x) = \sum_{k=1}^{n} \frac{\mu}{\sigma} \frac{\phi_k(x_k + L_k)}{(1 - \phi_k)x_k + \phi_k} + C. \tag{3.3}$$

*3.2. Large Population Potential Games*

A potential function of the societal game $\mathbf{u}$ will be defined on

$$\bar{\Delta} = \left\{x \in \mathbb{R}_+ \mid 1 - \eta \leq \textstyle\sum_{i=1}^{n} x_i \leq 1 + \eta\right\},$$

where $\eta$ is a positive constant. While a location distribution of entrepreneurs is always described by a point in $\Delta$, we consider this expanded set $\bar{\Delta}$ only for the partial derivatives of the potential function to be well defined; it is otherwise innocuous.

**Definition 3.1.** A function $v\colon \bar{\Delta} \to \mathbb{R}$ is said to be a *potential function* of $\mathbf{u}$ if it is differentiable, and for all $i, j = 1, \ldots, n$,

$$\frac{\partial v}{\partial x_i}(x) - \frac{\partial v}{\partial x_j}(x) = u_i(x) - u_j(x) \tag{3.4}$$

for all $x \in \Delta$. $\mathbf{u}$ is said to be a *potential game* if it admits a potential function.

This is a natural extension of Monderer and Shapley's (1996) concept of potential to societal games with a continuum of players (see also Sandholm, 2001, 2009; Ui, 2007). To understand the definition, suppose that $u_i(x) - u_j(x) > 0$, and thus an entrepreneur located in country $j$ prefers to migrate to $i$. If $\mathbf{u}$ admits a potential function $v$, then $(\partial v / \partial(e_i - e_j))(x) = (\partial v / \partial x_i)(x) - (\partial v / \partial x_j)(x) > 0$, so that the relocation from $j$ to $i$ by an infinitesimal individual leads to a marginal increase in the *common* function $v$. See Appendix A for characterizations of a potential game.

Observe that the set of states that satisfy the Kuhn-Tucker first-order conditions for the maximization problem,

$$\begin{aligned} &\text{maximize} \quad v(x) \\ &\text{subject to} \quad x \in \Delta, \end{aligned}$$

coincides with the set of equilibrium states of potential game $\mathbf{u}$.[15] In particular, local maximizers of the potential function $v$ are equilibrium states, but not vice versa unless $v$ is single peaked having no saddle point on $\Delta$. As is well known, a potential function works as a Lyapunov function for various classes of myopic evolutionary dynamics, such as replicator dynamics and best response dynamics, and therefore its *local* potential maximizers are all locally stable under those dynamics; see Appendix B. Under the perfect foresight dynamics, in contrast, we will see in Subsection 3.3 that, generically, a *global* potential maximizer is the unique state that is absorbing and globally accessible for any small degree of friction.

The key observation of our analysis is that under Assumption 3.1, the societal game defined by (3.1) admits a potential function.

**Proposition 3.1.** *Let* $\mathbf{u}$ *be given by* (3.1). *Then,* $\mathbf{u}$ *admits a potential function* $v$ *defined by*

$$v(x) = \frac{\mu}{\sigma} \sum_{i=1}^{n} \left[\frac{1}{\sigma - 1} \frac{1}{1 - \phi_i} \{(1 - \phi_i)x_i + \phi_i\} + x_i + L_i\right] \times \log\left((1 - \phi_i)x_i + \phi_i\right). \tag{3.5}$$

This function is obtained by

$$v(x) = \sum_{i=1}^{n} \int_0^{x_i} S_i(x_i')\, dx_i' + D$$

with some constant $D$, where $S_i$ is defined in (3.2). We thus have

$$\frac{\partial v}{\partial x_i}(x) - \frac{\partial v}{\partial x_j}(x) = S_i(x_i) - S_j(x_j) = u_i(x) - u_j(x)$$

for all $i, j$.

[15] Formally, this is reminiscent of Negishi's (1960) characterization of competitive equilibria of general equilibrium models without externalities in terms of solutions to social welfare maximizing problems. Note, however, that in the presence of externalities, potential functions generally do not allow any such normative interpretation.

*3.3. Global Accessibility and Absorption*

Hofbauer and Sorger (1999) show for games with linear payoff functions that a unique potential maximizer is globally accessible for sufficiently small degrees of friction $\delta = \theta/\lambda$, and is always absorbing independently of the degree of friction $\delta$. As shown in Appendix C, their results hold also in our framework with nonlinear payoff functions.

**Theorem 3.2.** *Let* $\mathbf{u}$ *be given by* (3.1)*. Assume that* $x^*$ *is the unique global maximizer of the potential function* $v$ *given by* (3.5) *over* $\Delta$*. Then,*

(1) *there exists* $\bar{\delta} > 0$ *such that* $x^*$ *is globally accessible for all* $\delta \in (0, \bar{\delta}]$,

(2) $x^*$ *is absorbing for all* $\delta > 0$.

*Proof.* See Appendix C. □

In particular, the potential maximizer $x^*$ is a unique state that is absorbing and globally accessible whenever the friction $\delta$ is sufficiently small.

The general idea of potential function methods is to reduce the analysis of equilibria themselves, which are fixed points of a certain function/correspondence, to that of solutions to a single optimization problem that involves the potential function, which in many cases are much easier to manipulate than fixed points. Here we consider, for a given initial condition $x^0 \in \Delta$, the following maximization problem:

$$\text{maximize} \quad J(x(\cdot)) = (\lambda + \theta) \int_0^\infty e^{-\theta t} v(x(t))\, dt \tag{3.6a}$$

$$\text{subject to} \quad \dot{x}(t) \in \lambda\big(\alpha(t) - x(t)\big),\ \alpha(t) \in \Delta, \quad x(0) = x^0. \tag{3.6b}$$

It is shown that any solution to this problem is a perfect foresight path (Lemma C.2), and it must visit neighborhoods of the potential maximizer $x^*$ when $\delta = \theta/\lambda$ is sufficiently small (Lemma C.3). In light of the former result, the functional $J$ can be seen as the dynamic extension of the static potential function $v$. The latter is analogous to the so-called "visit lemma" in turnpike theory (see, e.g., Scheinkman, 1976) and is understood as follows. When $\delta$ is small, the far future values of $v(x(t))$ are important (when $\theta$ is small) and/or the adjustment of $x(\cdot)$ is very fast (when $\lambda$ is large). In any case, the values of $v(x(t))$ for small $t$ do not have much impact on the value of $J$. Therefore, for any small neighborhood of the maximizer $x^*$ of $v$, if $x(\cdot)$ does not visit this neighborhood, then we have $J(x(\cdot)) < J(y(\cdot))$ for some feasible path $y(\cdot)$ that converges to $x^*$, and hence $x(\cdot)$ does not maximize $J$, provided that $\delta$ is sufficiently small. These lemmata together with the absorption result prove the global accessibility of $x^*$.

To prove the absorption of $x^*$, we consider the function $H^* \colon \bar{\Delta} \times \mathbb{R}^n \to \mathbb{R}$ given by

$$H^*(x, V) = (\lambda + \theta) v(x) + \lambda\big(\bar{V} - V' x\big),$$

which is the maximized Hamiltonian of the optimization problem (3.6), where $\bar{V} = \max_i V_i$. It is shown that $H^*$ can be used as a Lyapunov function. Note that because of forward-looking expectations, the static potential function $v$ does not suffice by itself, and thus we need to incorporate future utilities through $V$. First, if $x(\cdot)$ is a perfect foresight path and $V(\cdot)$ is given by (2.10), then we have $(d/dt)H^*(x(t), V(t)) \geq 0$, and therefore $H^*(x(t), V(t))$ is nondecreasing in $t$ (Lemma C.5). Second, if $\hat{x}$ is an accumulation point of $x(\cdot)$, then it must hold that $v(\hat{x}) \geq v(x(0))$ and that $\hat{x}$ is a critical point of $v$ (Lemma C.6). Hence, if we take a neighborhood of the potential maximizer $x^*$ such that $v(x) > v(\hat{x})$ for all $x$ in the neighborhood and for all other critical points $\hat{x} \neq x^*$, then any perfect foresight path from this neighborhood converges to $x^*$, which implies the absorption of $x^*$.

## 4. Agglomeration and Dispersion

Once we have the stability results in the previous section in hand, our task is to characterize the shape of the potential function $v$ for various values of trade barriers $\tau_i$, or equivalently, of trade openness $\phi_i$. In particular, we are interested in conditions under which $v$ becomes convex or concave on the state space $\Delta$. A necessary and sufficient condition for $v$ to be convex (concave, resp.) on $\Delta$ is that, for all $x \in \Delta$, its Hessian matrix $D^2 v(x)$ at $x$ is positive (negative, resp.) semidefinite with respect to the tangent space of $\Delta$, $T(\Delta) = \{z \in \mathbb{R}^n \mid \sum_{i=1}^n z_i = 0\}$, i.e., it satisfies $z' D^2 v(x) z \geq 0$ ($\leq 0$, resp.) for all $z \in T(\Delta)$. This condition, however, will be quite complex, and thus we choose to consider a simpler, sufficient condition that $(\partial^2 v/\partial x_i^2)(x) \geq 0$ ($\leq 0$, resp.) for all $i$ and all $x \in \Delta$, where

$$\begin{aligned}
&\frac{\partial^2 v}{\partial x_i^2}(x) \\
&= \frac{\mu}{\sigma - 1} \frac{1 - \phi_i}{(1 - \phi_i) x_i + \phi_i} - \frac{\mu}{\sigma} \frac{(1 - \phi_i)\{L_i - (L_i + 1)\phi_i\}}{\{(1 - \phi_i) x_i + \phi_i\}^2} \\
&= \frac{\mu (1 - \phi_i)^2}{\{(1 - \phi_i) x_i + \phi_i\}^2} \\
&\quad \times \left[ \frac{1}{\sigma - 1} \left( x_i + \frac{\phi_i}{1 - \phi_i} \right) - \frac{1}{\sigma} \left( L_i - \frac{\phi_i}{1 - \phi_i} \right) \right]
\end{aligned} \tag{4.1}$$

for all $i$, and $(\partial^2 v/\partial x_i \partial x_j)(x) = 0$ for all $i \neq j$.

Recall that $e_i$ denotes the $i$th vertex of the simplex $\Delta$, the core-periphery configuration in which all the entrepreneurs (and hence manufacturing firms) are located in country $i$.

*4.1. Low Trade Barriers and Agglomeration*

In this subsection, we consider the case in which countries have high trade openness (or equivalently, low trade barriers), i.e., $\phi_i$'s are sufficiently close to one. For each $i$, define

$$\bar{\phi}_i = \frac{(\sigma - 1) L_i}{(\sigma - 1) L_i + 2\sigma - 1}. \tag{4.2}$$

If $\phi_i > \bar{\phi}_i$ for all $i$, then $(\partial^2 v/\partial x_i^2)(x) > 0$ for all $i$ and all $x \in \Delta$ and hence $v$ is strictly convex, so that the global maximum is attained at a vertex of $\Delta$. We have $v(e_i) > v(e_j)$ if and only if

$$\sum_{k\neq i}\left(\frac{1}{\sigma-1}\frac{\phi_k}{1-\phi_k}+L_k\right)\log\phi_k > \sum_{k\neq j}\left(\frac{1}{\sigma-1}\frac{\phi_k}{1-\phi_k}+L_k\right)\log\phi_k,$$

or

$$\left(\frac{1}{\sigma-1}\frac{\phi_i}{1-\phi_i}+L_i\right)\log\phi_i < \left(\frac{1}{\sigma-1}\frac{\phi_j}{1-\phi_j}+L_j\right)\log\phi_j. \quad (4.3)$$

In the case where countries have identical market size (i.e., $L_i = L_j$ for all $i, j$), $\{e_{i^*}\} = \arg\max_{x\in\Delta} v(x)$ if and only if $\phi_{i^*} < \phi_j$ for all $j \neq i^*$, while in the case where countries have identical trade openness (i.e., $\phi_i = \phi_j$ for all $i, j$), $\{e_{i^*}\} = \arg\max_{x\in\Delta} v(x)$ if and only if $L_{i^*} > L_j$ for all $j \neq i^*$.

**Proposition 4.1.** *Assume that countries have high trade openness so that $\phi_i > \bar{\phi}_i$ for all $i$. Then, the potential function is strictly convex and hence admits a global maximum at a core-periphery configuration. Furthermore,*

(1) *if $\phi_{i^*} < \phi_j$ for all $j \neq i^*$ and $L_i = L_j$ for all $i, j$, then $e_{i^*}$ is absorbing and globally accessible for sufficiently small $\delta > 0$,*
(2) *if $L_{i^*} > L_j$ for all $j \neq i^*$ and $\phi_i = \phi_j$ for all $i, j$, then $e_{i^*}$ is absorbing and globally accessible for sufficiently small $\delta > 0$.*

*That is, if countries are symmetric with respect to market sizes (trade openness, resp.) and country $i^*$ has the lowest trade openness (the largest market size), then the core-periphery configuration with full agglomeration in country $i^*$ is absorbing and globally accessible for sufficiently small friction.*

Note that properties of the potential function are, of course, those of the underlying static model. Thus, what shape the potential function are exactly the agglomeration and dispersion forces identified in the standard NEG models. These forces can be observed by considering the second partial derivatives $\partial^2 v/\partial x_i^2$, where the first partial derivative with respect to $x_i$ is given by

$$\frac{\partial v}{\partial x_i}(x) = \frac{\mu}{\sigma-1}\log\left(x_i+\phi_i(1-x_i)\right) + \frac{\mu}{\sigma}\frac{(1-\phi_i)(x_i+L_i)}{x_i+\phi_i(1-x_i)}, \quad (4.4)$$

which is equal to $S_i(x_i)$ in (3.2). Consider a small migration of entrepreneurs into country $i$. On the one hand, this increases the local expenditures on the manufacturing good, which has a positive effect on demand per firm (market size effect). It is captured in the numerator of the second term in (4.4), which is increasing in $x_i$. On the other hand, this also increases the "effective" number of firms competing in $i$, $x_i + \phi_i(1 - x_i)$, which has a positive effect on consumer surplus through the decrease in the price index (cost-of-living effect) and a negative effect on demand per firm (local competition effect). The former is captured in the first term in (4.4), which is increasing in $x_i$, while the latter in the denominator in the second term, where the increase in $x_i$ negatively affects $\partial v/\partial x_i$. The first two effects generate a positive feedback fostering agglomeration of entrepreneurs, which leads to convexity of the potential function $v$, while the third effect encourages dispersion, working in favor of concavity of $v$. As seen in (4.1), the agglomeration forces dominate the dispersion force and thus $v$ becomes convex if $\phi_i$'s are sufficiently close to one (i.e., trade barriers are sufficiently low), and vice versa if $\phi_i$'s are sufficiently close to zero (i.e., trade barriers are sufficiently high).

When $v$ is convex, it is *locally* maximized at vertices of the simplex $\Delta$, or core-periphery configurations, which are necessarily equilibrium states of the static model. Standard exercise in the NEG literature is to consider stability of equilibrium states under *myopic* dynamics. As is well known, *local* maximizers of a potential function are all locally stable under various classes of myopic evolutionary dynamics. Under the perfect foresight dynamics, by contrast, all the equilibrium states but the *global* maximizer of the potential are destabilized when the degree of friction is sufficiently small: the global potential maximizer is globally accessible and absorbing, while all the other equilibrium states are fragile, for sufficiently small frictions. Thus, allowing for forward-looking expectations helps to select a unique equilibrium outcome through the global stability consideration under the perfect foresight dynamics. The condition (4.3) says that the global potential maximizer is the core-periphery configuration with the core in the country that is most (relatively) protected or has a largest market size. Located in such a country, firms can have better access to consumers due to cost reduction in terms of trade barriers (see also Forslid and Ottaviano, 2003, Section 4).

### *4.2. High Trade Barriers and Dispersion*

In this subsection, we consider the case in which countries have low trade openness (or equivalently, high trade barriers), i.e., $\phi_i$'s are sufficiently close to zero. In this case, as long as $L_i > \sigma/(\sigma-1)$ holds (Assumption 2.1(3)), we have that $(\partial^2 v/\partial x_i^2)(x) < 0$ for all $i$ and all $x \in \Delta$, and hence $v$ is strictly concave, and that on the boundary of $\Delta$, $|\nabla v| \to \infty$ as $\phi_i$'s tend to zero. Therefore, $v$ has a unique maximizer in the interior and no other critical point. More precisely, a sufficient condition for this is obtained as follows. Let

$$\phi_i^0 = \frac{(\sigma-1)L_i - \sigma}{(\sigma-1)(L_i+1)}, \tag{4.5}$$

and $\phi_i^1$ be the solution to the equation in $\phi$:

$$\frac{\sigma}{\sigma-1}\log\phi + \frac{L_i}{\phi} = L+1, \tag{4.6}$$

where $L = L_1 + \cdots + L_n$. Verify that, under Assumption 2.1(3), the left hand side of (4.6) is decreasing in $\phi$. If $\phi_i < \phi_i^0$, then $(\partial^2 v/\partial x_i^2)(x) < 0$ for all $x \in \Delta$, while if $\phi_i < \phi_i^1$, then $v$ does not attain its global maximum on the face of $\Delta$ where $x_i = 0$. Let

$$\underline{\phi}_i = \min\left\{\phi_i^0, \phi_i^1\right\}. \tag{4.7}$$

By Lemma C.6 in Appendix C, we thus have a stronger stability result.

**Proposition 4.2.** *Assume that countries have low trade openness so that $\phi_i < \underline{\phi}_i$ for all $i$. Then, the potential function is strictly concave and steep on $\partial(\Delta)$ and hence admits a global maximum at some fully dispersed configuration $x^* \in \operatorname{int}(\Delta)$, and any perfect foresight path converges to $x^*$ for any $\delta > 0$.*

*Proof.* See Appendix D. □

When trade barriers are sufficiently high (i.e., $\phi_i$'s are sufficiently close to zero), firms sell mainly in the local markets, so that the local competition effect becomes dominant, discouraging spatial clustering of firms. Accordingly, the potential function $v$ becomes sufficiently concave and is maximized at a unique dispersed configuration $x^* \in \operatorname{int}(\Delta)$. The configuration $x^*$ is a unique equilibrium state and attracts any perfect foresight path regardless of the initial condition as well as the degree of friction. Thus, in this case of large trade barriers, expectations as well as history play no role.

### *4.3. Two-Country Case*

In this subsection, we illustrate our results in the simple case of two countries (i.e., $n = 2$). Let $f\colon [0,1] \to \mathbb{R}$ be defined by

$$f(x_1) = u_1(x_1, 1-x_1) - u_2(x_1, 1-x_1). \tag{4.8}$$

That is, $f(x_1)$ is the increment in indirect utility from locating in country 1 instead of in 2 when fraction $x_1 \in [0,1]$ of entrepreneurs are located in 1. In this case where the state space is one dimensional, as the potential function it is natural to consider the function $F\colon [0,1] \to \mathbb{R}$ defined by

$$F(x_1) = \int_0^{x_1} f(x_1')\, dx_1', \tag{4.9}$$

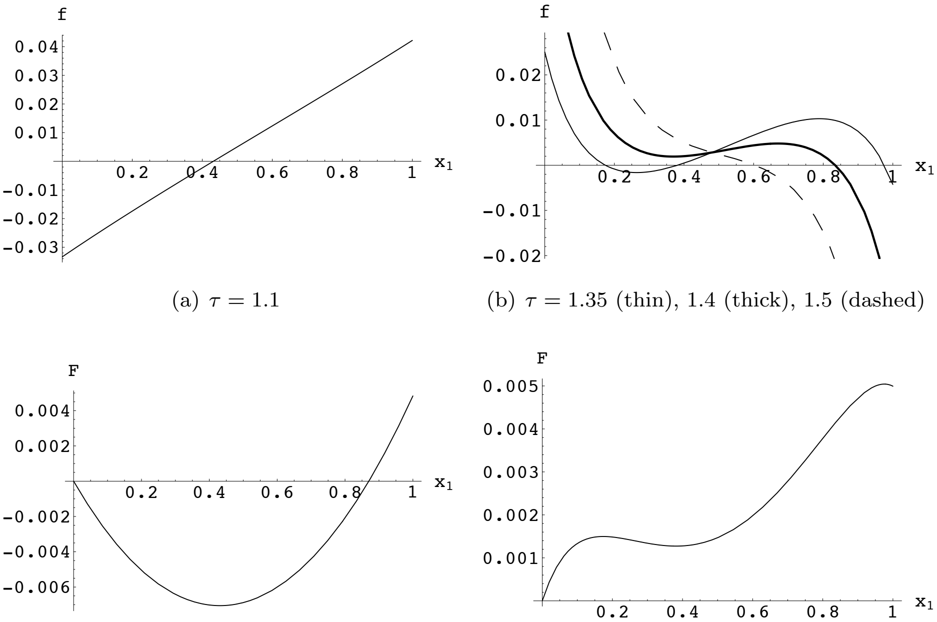
which is equal to $v(x_1, 1-x_1) + D'$ with some constant $D'$.

Figure 1 depicts the utility difference function $f$ and the corresponding potential function $F$ for different values of trade barriers $\tau_1$ and $\tau_2$ with parameter values $L_1 = L_2 = 1$, $\sigma = 6$, and $\mu = 0.3$. Here we set $\tau_1 = 0.9^{1/(1-\sigma)}\tau$ and $\tau_2 = \tau$ so that $\phi_1 = 0.9\phi_2$ and thus $\phi_1 < \phi_2$. The utility difference $f$ is depicted in Figure 1(a) for $\tau = 1.1$ and in Figure 1(b) for $\tau = 1.35$ (thin curve), $\tau = 1.4$ (thick curve), and $\tau = 1.5$ (dashed curve), while the potential function $F$ is depicted in Figure 1(c) for $\tau = 1.1$ and in Figure 1(d) for $\tau = 1.35$. Compare Figure 1 in Pflüger (2004, p.570), who considers symmetric countries.

As seen in the previous subsections, when trade barriers $\tau_1$ and $\tau_2$ are low (i.e., $\phi_1$ and $\phi_2$ are close to one), the utility difference function $f$ is upward sloping (as in Figure 1(a)) and thus the potential function $F$ becomes convex (as in Figure 1(c)). In this case, while both core-periphery configurations, $x_1 = 0$ and $x_1 = 1$, are strict equilibrium states and in fact *local* maximizers of $F$, they generically have different stability properties under the perfect foresight dynamics. If, as in Figure 1(c), $F(1) > F(0)$ so that $x_1 = 1$ is the *global* maximizer of $F$, then $x_1 = 1$ is globally accessible and absorbing, while $x_1 = 0$ is fragile, when the degree of friction is small. That is, even if all the entrepreneurs are located in country 2 (i.e., the initial state is at $x_1 = 0$), the expectations can become self-fulfilling for small friction that all the entrepreneurs will eventually move to country 1, and once a large fraction of entrepreneurs have been located in country 1 (i.e., the system reaches a small neighborhood of $x_1 = 1$), no self-fulfilling expectation can upset the outcome and the system must converge to the state $x_1 = 1$.

In contrast, when trade barriers are high (i.e., $\phi_1$ and $\phi_2$ are close to zero), $f$ is downward sloping, so that $F$ becomes concave and attains the global maximum at some dispersed configuration $x_1^* \in (0,1)$. In this case, $x_1^*$ is a unique equilibrium and attracts all the perfect foresight paths regardless of the degree of friction. For some intermediate values of trade barriers, there are three equilibrium states; see the thin curve in Figure 1(b) for $\tau = 1.35$. While the left and the right equilibria are locally stable under myopic dynamics, the right equilibrium is the global maximizer of the potential $F$ as shown in Figure 1(d) and hence is globally accessible and absorbing under the perfect foresight dynamics for small frictions.

Figure 2 is the bifurcation diagram which depicts the loci of the equilibrium states for different values of $\tau$. The bold curve (uppermost) is the locus of the global maximizer of the potential, the thin curve (lowermost) is that of the local, but not global, maximizer, and the dotted curve (middle) is that of the other equilibrium state which minimizes the potential and is unstable even under myopic dynamics. Compare Figure 2 in Pflüger (2004, p.570), which exhibits a 'pitchfork bifurcation' with symmetric locations. Our figure above suggests, however, that when locations are asymmetric, the pitchfork bifurcation disappears and




(a) $\tau = 1.1$

(b) $\tau = 1.35$ (thin), 1.4 (thick), 1.5 (dashed)

(c) $\tau = 1.1$

(d) $\tau = 1.35$

Figure 1: Utility difference $f$ and potential $F$

the model exhibits a 'saddle-node bifurcation' instead.[16]

Let us take again the case where trade barriers are low so that the utility difference $f$ is upward sloping, and see how the strict equilibrium states have distinct stability properties under the perfect foresight dynamics. Assume that the potential $F$ is such that $F(1) > F(0)$ $(= 0)$ as in Figure 1(c) where the strict equilibrium state $x_1 = 1$ is the global potential maximizer. To see that the other strict equilibrium state $x_1 = 0$ is fragile when the friction $\delta = \theta/\lambda$ is close to zero, consider the feasible path $x(\cdot)$ given by $x_1(t) = 1 - e^{-\lambda t}$, along which all entrepreneurs are anticipated to move from country 2 to country 1 at their first relocation opportunities. Given this path $x(\cdot)$, the difference in discounted utilities over the expected duration of a commitment for an entrepreneur making a decision at time 0 is computed as

$$\begin{aligned} \Delta V(0) &= V_1(0) - V_2(0) \\ &= (\lambda+\theta) \int_0^\infty e^{-(\lambda+\theta)t} f\big(1 - e^{-\lambda t}\big)\, dt \\ &= (1+\delta) \int_0^1 (1-x_1)^\delta f(x_1)\, dx_1 \\ &\to F(1) > 0 \quad \text{as } \delta \to 0, \end{aligned} \tag{4.10}$$

where the last inequality follows from the assumption that $F(1) > F(0) = 0$. Thus, this entrepreneur has an incentive to move to country 1, provided that the future is sufficiently important (i.e., $\delta$ is sufficiently small), as the future utility gain is large enough to compensate the current loss. Since the utility difference $f(x_1)$ is increasing in $x_1$, this suffices to guarantee that the path $x(\cdot)$ is a perfect foresight path, which in turn implies that, for small $\delta$, the state $x_1 = 0$ is fragile and in fact $x_1 = 1$ is globally accessible.

To verify that $x_1 = 1$ is absorbing, consider the feasible path $x'(\cdot)$ given by $x'_1(t) = e^{-\lambda t}$, along which all entrepreneurs are anticipated to move from 1 to 2. Against this path $x'(\cdot)$, the expected discounted utility difference is given by

$$\begin{aligned} \Delta V(0) &= (\lambda+\theta) \int_0^\infty e^{-(\lambda+\theta)t} f\big(e^{-\lambda t}\big)\, dt \\ &= (1+\delta) \int_0^1 (x_1)^\delta f(x_1)\, dx_1 \\ &\to F(1) > 0 \quad \text{as } \delta \to 0, \end{aligned} \tag{4.11}$$

where again the inequality follows from the assumption that $F(1) > F(0) = 0$. That is, even when the friction $\delta$ is small, the future gain relative to the current loss is not large enough, so that entrepreneurs choose to stay in county 1 postponing migration to country 2 for a next or later relocation opportunity. Hence, the escaping path $x'(\cdot)$ does not become a perfect foresight path. Since this argument remains valid when the initial state is in a neighborhood of $x_1 = 1$, we may conclude that $x_1 = 1$ is absorbing (for any $\delta > 0$).

[16] In fact, this is a general phenomenon. See Berliant and Kung (2009), who show that pitchfork or tomahawk bifurcations appearing in NEG models with symmetric locations are nongeneric patterns which result from the model symmetry.

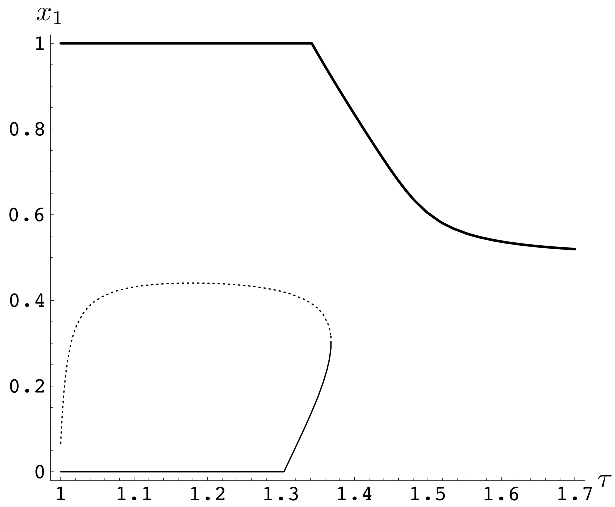



Figure 2: Bifurcation diagram

## 5. Policy Issues

In this section, we briefly argue how new insights on policy issues are gained when we allow for forward-looking behavior of mobile production factors. In what follows, we assume that the countries have the same size and consider the case in which trade barriers are already low so that $\phi_i > \bar{\phi}_i$ for all $i$, where $\bar{\phi}_i$ is defined in (4.2); we know that when they are large, there is no substantial change from the myopic case. Our purpose here is to point out that allowing for forward-looking expectations may alter results in existing NEG models with myopic agents, where all the core-periphery outcomes are locally stable under myopic dynamics in the case of low trade barriers.

### *5.1. Trade Policy*

To make the discussion as simple as possible, we assume that each government $i$ can exercise trade protection by raising trade barrier $\tau_i$ (i.e., lowering trade openness $\phi_i$).[17] By Proposition 4.1, if unilateral protection by country $i^*$ is large enough that $\phi_{i^*} < \phi_j$ for all $j \neq i^*$ (but still $\phi_{i^*} > \bar{\phi}_{i^*}$), then the potential is maximized at $e_{i^*}$ (the core-periphery configuration with full agglomeration in $i^*$), so that $e_{i^*}$ is globally accessible for small frictions. Thus, by trade protection it is possible for country $i^*$ to induce relocation of firms toward country $i^*$ provided that the degree of friction is sufficiently small. This is due to the 'price-lowering effect' of protection; see, e.g., Baldwin et al. (2003, Chapter 12). It is not possible in standard NEG models with myopic migrants, where each agglomeration state is locally stable under myopic dynamics.

Notice, however, that largest trade protection does not automatically imply agglomeration of firms in country $i^*$, since global accessibility of $e_{i^*}$ only says that from each distribution $x \in \Delta$ there *exists* a perfect foresight path convergent to $e_{i^*}$ and does not exclude existence of other perfect foresight paths. In order to attract firms to country $i^*$, it is necessary to encourage the firms to coordinate on the expectation that leads to $i^*$.

Two remarks are in order. First, in our model, countries are identical in terms of production technology, and therefore, comparative advantage is absent. The above result might be modified if we incorporate effects of comparative advantage. Second, given the benefit from unilateral trade protection, countries may well be involved in tariff competition; see, e.g., Mai et al. (2008).

### *5.2. Taxation*

Several papers argue in models with myopic migrants that there are 'agglomeration rents' that can be taxed without inducing firms to move out (see Baldwin et al., 2003, Chapter 15 and the references therein). This is understood in terms of potential. Assume for simplicity that the government of country $i$ collects lump sum taxes $T_i$ from entrepreneurs located in $i$ (as in Borck and Pflüger, 2006 in the context of tax competition).[18] Then, the instantaneous indirect utility of entrepreneurs in $i$ becomes $u_i(x) = S_i(x_i) - T_i + R(x)$, $x \in \Delta$, where $S_i(x_i)$ and $R(x)$ are as in (3.2) and (3.3), respectively. The potential function is now given by

$$\tilde{v}(x) = v(x) - \sum_{i=1}^{n} T_i x_i,$$

[17] Given our specification of quasi-linear utilities, it is straightforward to model tariffs explicitly, with an assumption that revenue from tariffs is distributed lump-sum to local immobile workers.

[18] More generally, the tax instrument may be varied depending on time $t$ and current state $x$ (this applies also for tariff policy). See e.g., Kaneda (2003), who, in a different context of infant industry protection but with the same class of perfect foresight dynamics, studies protection schemes with the duration and rate of subsidy as policy variables.

where $v(x)$ is as in (3.5). Therefore, taxation in country $i$ implies lowering the potential at $e_i$, where $\tilde{v}(e_i) = v(e_i) - T_i$. Under myopic dynamics, each local maximizer of the potential is locally stable, so that $e_i$ is still stable unless tax is so large that $e_i$ fails to *locally* maximize the potential $\tilde{v}$. Under the perfect foresight dynamics, however, agglomeration rents are much smaller. When $e_i$ is not a global maximizer of the potential, and hence is fragile when the friction is small, taxation may trigger the firms to coordinate on expectations that lead them to move out (the same caveat as in the previous subsection applies here). Even if $e_i$ is a global potential maximizer, which is absorbing, tax must be small enough that $e_i$ still *globally* maximizes the potential $\tilde{v}$ in order not to trigger relocation of the firms. Re-examination of agglomeration rents under forward-looking expectations seems an interesting future topic.

### *5.3. Speed of Adjustment*

The discussions in the preceding subsections were essentially static, reducing to those relying on the shape of the potential function, since only the stable long-run distributions of industry were concerned. However, once one is interested also in the speed of adjustment of firms (e.g., for political reasons), different formulations in dynamics may yield different policy implications depending on how the adjustment process is modeled. This is indeed the case, as pointed out by Kaneda (2003) in a different context, for the two particular classes of dynamics, the dynamics of Matsuyama (1991) and Matsui and Matsuyama (1995) (MM-dynamics, in short), which the present paper follows, and that of Krugman (1991b) and Fukao and Benabou (1993) (KFB-dynamics, in short), studied by Baldwin (2001), Ottaviano (2001), and Oyama (2009) in two-location settings. In this subsection, we illustrate the difference between them in dynamic policy implications.

Consider as in Subsection 5.1 the unilateral protection by country $i^*$ and suppose that the agglomeration in $i^*$, $e_{i^*}$, is a global maximizer of the potential function (an analogous argument holds for taxation, when $i^*$ lowers its tax rate while the other countries set higher rates so that $e_{i^*}$ is a potential maximizer). Assume also that there are only two countries (in order to apply the results of Oyama, 2009) and that the rate of time preference of agents is sufficiently small. Then, under the KFB-, as well as the MM-, dynamics, there are self-fulfilling expectations that *eventually* lead the firms to agglomerate in country $i^*$, and once the basin of attraction of $e_{i^*}$ is reached, no self-fulfilling expectation can upset the outcome.

Now, we ask whether the government can control the *speed* of agglomeration by changing the rate of protection (i.e., tariff rate). It is here that the two classes of dynamics yield different answers. In the KFB-dynamics, migration is perfectly reversible in that any agent can migrate at any point in time, by incurring moving costs which are assumed to be increasing in the number of moving agents. Therefore, the speed of aggregate adjustment is a priori unbounded, and in equilibrium it is determined by the non-arbitrage condition (i.e., that all the agents be always indifferent between staying and moving) and thus is linked with individuals' utilities through this condition. This implies that a protection scheme with a higher rate, by affecting the utilities of agents, can speed up the migration process. By contrast, this is not the case under the independent Poisson assumption in the MM-dynamics, where migration is irreversible, constrained by exogenous factors, so that for each short time interval, only a given fraction of agents have the opportunity to migrate, and therefore the speed of adjustment is bounded and independent of agents' utilities. Thus, in this case the government can control only the direction, but not the speed, of relocation of industry.

The relevance of the policy implications derived by the two formulations depends on the extent to which migration is an irreversible activity. At least when international migration is concerned, it seems quite reasonable to assume that it is irreversible to a large extent. Clearly, however, a closer look is called for at the hidden components behind the reduced-form moving cost function or the exogenous factors that restrict migration.

## 6. Concluding Remarks

In this paper, we have studied the impact of forward-looking expectations in a multi-country NEG model. We considered a version of Krugman's (1991a) static core-periphery model, allowing for any finite number of countries as well as exogenous asymmetries among countries with respect to their trade barriers and market sizes, and embedded it into the class of perfect foresight dynamics due to Matsuyama (1991) and Matsui and Matsuyama (1995). The dynamics may possess multiple stationary states, which is indeed the case when barriers to trade are low enough that the agglomeration externalities arising from scale economies dominate the market competition effect due to trade barriers. We were nevertheless able to conduct global analysis of this dynamics, by invoking the techniques due to Hofbauer and Sorger (1999) which utilize the concept of potential function.

We demonstrated that in the case of low trade barriers, incorporating forward-looking migration decisions alters the predictions on the long-run spatial configuration of industry obtained in standard NEG models which assume myopic migrants: while core-periphery equilibrium configurations are all locally stable under myopic dynamics, generically a unique configuration among them becomes absorbing and globally accessible and the others become fragile under the perfect foresight dynamics, whenever the degree of friction is sufficiently small and hence the future is sufficiently important in migration decisions. In particular, if the countries have the same market size (trade barrier, resp.), then the core-periphery configuration with the core in the country with the highest barrier (largest market size, resp.) is the unique such globally stable state. When trade barriers are high enough, in contrast, expectations as well as history play no decisive role: there is a unique equilibrium state, which is a dispersed configuration and to which any perfect foresight path converges regardless of the degree of friction and the initial condition.

We then pointed out, for the case of low trade barriers, that the insights regarding policy issues gained in existing models with myopic agents may be modified when one allows for forward-looking expectations. In particular, we argued that, due to the possibility of self-fulfilling coordinated relocation away from a country, the amount of 'agglomeration rents', which can be taxed without inducing firms to move out from the country, is much smaller under the perfect foresight dynamics than under myopic dynamics. Our discussion on policy implications is, however, obviously quite limited and preliminary. Detailed formal analysis is left for future research.

While we worked with a 'footloose entrepreneur' model in which firms which produce final goods move internationally with their entrepreneurs, in some situations it may be more reasonable to consider mobile capital (as in 'footloose capital' models) or inter-sectorally mobile labor allowing for input-output linkages among firms (as in 'vertical linkage' models); see Baldwin et al. (2003, Chapter 15) and the references therein. It would be interesting to see how our potential methods apply to these models (possibly starting with two countries).

Finally, the approach in this paper has been to examine the stability of spatial configurations under perfect foresight.[19] Thus our model is silent about the issue of expectation formation. As we noted, this can be of relevance when one considers policy implications based on our stability results. It would therefore be desirable to construct a model that explicitly addresses this issue. The work by Vega-Redondo (1997) may be a possible starting point, who studies, in a closed economy, how the government can direct the expectations of the population towards a preferred outcome by exercising tax/subsidy schemes. It would be an interesting future research topic to introduce such policy instruments in our multi-country model, where the governments would naturally behave strategically.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## Appendix A. Potential Functions

In this section, we state useful characterizations of large population potential games based on Hofbauer (1985), Hofbauer and Sigmund (1998, Section 19.5), and Sandholm (2001, 2009).

Let $\Delta_{-n} = \{y \in \mathbb{R}^{n-1} \mid y_i \geq 0,\ \sum_{i=1}^{n-1} y_i \leq 1\}$. Each point $x_{-n} = (x_1, \ldots, x_{n-1})' \in \Delta_{-n}$ is identified with the point $x = (x_1, \ldots, x_{n-1}, x_n)' \in \Delta$ by $x_n = 1 - \sum_{i=1}^{n-1} x_i$. Since only the payoff *differences* are relevant in optimal choice, it is convenient to consider $g_i \colon \Delta_{-n} \to \mathbb{R}$, $i = 1, \ldots, n-1$, defined by

$$g_i(x_{-n}) = u_i(x_{-n}, x_n) - u_n(x_{-n}, x_n). \tag{A.1}$$

Accordingly, given a potential function $v \colon \Delta \to \mathbb{R}$, define the function $G \colon \Delta_{-n} \to \mathbb{R}$ by

$$G(x_{-n}) = v(x_{-n}, x_n). \tag{A.2}$$

(In the case of $n = 2$, the functions $g_1$ and $G$ correspond to $f$ and $F$ in Subsection 4.3, respectively.) If $v$ is a potential function of $\mathbf{u}$ (recall Definition 3.1), then for all $x_{-n} \in \Delta_{-n}$,

$$\begin{aligned}\frac{\partial G}{\partial x_i}(x_{-n}) &= \frac{\partial v}{\partial x_i}(x) - \frac{\partial v}{\partial x_n}(x) \\ &= u_i(x) - u_n(x) = g_i(x_{-n}),\end{aligned}$$

or

$$\nabla G(x_{-n}) = g(x_{-n}). \tag{A.3}$$

This is where the term *potential* comes from. That is, the function $G$ is the integral of the payoff difference vector field $g$. As is well known, a necessary and sufficient condition for $g$ to be integrable (i.e., its line integral to be

[19]For approaches to relaxation of the perfect foresight assumption in large population situations, see, e.g., Matsui and Oyama (2006) and Antoci et al. (2008).

independent of the path of integration) and hence admit a potential is that

$$\frac{\partial g_i}{\partial x_j}(x_{-n}) = \frac{\partial g_j}{\partial x_i}(x_{-n}) \tag{A.4}$$

for all $i, j$ and all $x_{-n} \in \operatorname{int}(\Delta_{-n})$. Clearly, when $n = 2$, so that the state space is one-dimensional, $g$ is always integrable. If $u$ is defined on a neighborhood of $\Delta$, the *integrability condition* (A.4) is written in terms of the original payoff function as

$$\frac{\partial u_i}{\partial x_j}(x) + \frac{\partial u_j}{\partial x_k}(x) + \frac{\partial u_k}{\partial x_i}(x) = \frac{\partial u_i}{\partial x_k}(x) + \frac{\partial u_k}{\partial x_j}(x) + \frac{\partial u_j}{\partial x_i}(x)$$

for all $i, j, k$ and all $x \in \operatorname{int}(\Delta)$, which is trivially satisfied if $n = 2$. This condition can be rewritten as

$$\frac{\partial(u_i - u_k)}{\partial(e_j - e_k)}(x) = \frac{\partial(u_j - u_k)}{\partial(e_i - e_k)}(x),$$

which exhibits *symmetric externalities* on utility differences. That is, for any $i, j, k$, the effect of a marginal relocation of agents in $k$ to $j$ on the incentive of migration from $k$ to $i$ is equal to the effect of a marginal relocation of agents in $k$ to $i$ on the incentive of migration from $k$ to $j$.

## Appendix B. Myopic Evolutionary Dynamics in Potential Games

In this section, we review known facts in evolutionary game theory on stability of local maximizers of a potential function under myopic evolutionary dynamics.

Given a societal game $\mathbf{u}$, let $\Psi$ be a (set-valued) dynamical system, which maps a state $x \in \Delta$ to a set of Lipschitz paths on $\Delta$ that start at $x$. This general formulation accommodates possible multiplicity of solution paths as in the best response dynamics (and also in the perfect foresight dynamics). Note that, since it stays in $\Delta$, any $x(\cdot) \in \Psi(x)$ must satisfy, for almost all $t \geq 0$,

$$\textstyle\sum_{i=1}^n \dot{x}_i(t) = 0, \tag{B.1a}$$

$$\dot{x}_i(t) \geq 0 \text{ whenever } i \notin \operatorname{supp}(x(t)), \tag{B.1b}$$

where $\operatorname{supp}(x) = \{i \mid x_i > 0\}$.

A dynamical system $\Psi$ is said to be a *myopic adjustment dynamics* (Swinkels, 1993) if every path $x(\cdot) \in \Psi(x)$ satisfies

$$\dot{x}(t)' u(x(t)) \geq 0 \tag{B.2}$$

for almost all $t \geq 0$; it is said to be a *strict* myopic adjustment dynamics if (B.2) holds with equality only if $x(t)$ is an equilibrium state of $\mathbf{u}$.[20] (Note that the converse of this follows from (B.2): since $\sum_i \dot{x}_i(t) u_i(x(t)) = \sum_i \dot{x}_i(t)[u_i(x(t)) - \bar{u}(x(t))]$ by (B.1a), where $\bar{u}(x(t)) = x(t)'u(x(t))$, if $x(t)$ is an equilibrium state of $\mathbf{u}$, then $\dot{x}(t)'u(x(t)) = \sum_{i \notin \operatorname{supp}(x(t))} \dot{x}_i(t)[u_i(x(t)) - \bar{u}(x(t))] \leq 0$, where the inequality follows from (B.1b); hence, $\dot{x}(t)'u(x(t)) = 0$ holds under (B.2).)

By (B.1a), the condition (B.2) can be written in terms of the payoff differences $g_i$ defined in (A.1) as

$$\dot{x}_{-n}(t)' g(x_{-n}(t)) \geq 0. \tag{B.3}$$

When $n = 2$, this reads

$$\dot{x}_1(t)\big(u_1(x(t)) - u_2(x(t))\big) \geq 0.$$

That is, the condition postulates that agents tend to switch from a lower-payoff alternative to a higher-payoff one.

Examples of myopic adjustment dynamics include the *best response dynamics* and the *replicator dynamics*.

*Example* B.1. The *best response dynamics* (Gilboa and Matsui, 1991; Matsui, 1992; Hofbauer, 1995), the representative of *innovative* dynamics, is defined by the differential inclusion

$$\dot{x}(t) \in B(x(t)) - x(t), \tag{B.4}$$

where $B$ is the best response correspondence as defined in (2.7). This dynamics may be viewed as the limit of the perfect foresight dynamics as the discount rate $\theta$ goes to $\infty$. Note that (B.4) can be written as

$$\dot{x}_i(t) > -x_i(t) \Rightarrow u_i(x(t)) \geq u_j(x(t)) \text{ for all } j = 1, \dots, n.$$

The best response dynamics is generated by the following decision rule at the individual level. Migration opportunities are assumed to arrive according to independent Poisson processes with parameter $\lambda$ so that for each time interval $[t, t + dt)$, a fraction $\lambda \cdot dt$ of agents receive the opportunity to migrate. Given an opportunity, each agent chooses a location that maximizes the current payoff given the state $x(t)$. Therefore, if $\alpha(t) \in \Delta$ denotes the choice distribution of the agents who receive the opportunity in $[t, t + dt)$, we have $\alpha(t) \in B(x(t))$. Thus,

$$x(t + dt) = (1 - \lambda dt)\, x(t) + \lambda dt\, \alpha(t), \quad \alpha(t) \in B(x(t)).$$

By normalizing the unit of time so that $\lambda = 1$, we obtain (B.4).

It is easy to verify that the best response dynamics is a strict myopic adjustment dynamics. Since $\dot{x}(t) + x(t) \in B(x(t))$ by (B.4), we have

$$(\dot{x}(t) + x(t))' u(x(t)) \geq x(t)' u(x(t)),$$

or $\dot{x}(t)' u(x(t)) \geq 0$, where the equality holds if and only if $x(t)$ is an equilibrium state.

*Example* B.2. The *replicator dynamics* (Taylor and Jonker, 1978), the representative of *imitative* dynamics, is defined by the differential equation

$$\dot{x}_i(t) = x_i(t)\big(u_i(x(t)) - \bar{u}(x(t))\big) \tag{B.5}$$

[20] For standard (i.e., point-valued) dynamical systems, this strict myopic adjustment condition is equivalent to 'positive correlation' in the sense of Sandholm (2001) under the additional condition of 'noncomplacency' (or 'Nash stationarity').

for $i = 1, \ldots, n$, where $\bar{u}(x(t)) = x(t)'u(x(t))$.

The replicator dynamics is generated by the *proportional imitation rule* (Schlag, 1998): Migration opportunities are assumed to arrive according to independent Poisson processes with parameter $\lambda$ so that for each time interval $[t, t+dt)$, a fraction $\lambda \cdot dt$ of agents receive the opportunity to migrate. Given an opportunity, each agent randomly samples an agent, a candidate to imitate, from the whole population and observes this agent's payoff, where the probability that a location-$j$ agent is selected is $x_j$. If the sampled payoff is higher than his own, the agent migrates to location $j$ from his current location $i$ with a probability proportional to the payoff difference $u_j(x(t)) - u_i(x(t))$; otherwise, he stays at location $i$. Thus,

$$
\begin{aligned}
&x_i(t+dt) \\
&= x_i(t) + \sum_{j\neq i} \lambda dt\, x_j(t) \times x_i(t) \frac{[u_i(x(t)) - u_j(x(t))]_+}{K} \\
&\qquad - \lambda dt\, x_i(t) \times \sum_{j\neq i} x_j(t) \frac{[u_j(x(t)) - u_i(x(t))]_+}{K},
\end{aligned}
$$

where $[a]_+ = \max\{a, 0\}$, and $K$ is a constant appropriately chosen for the imitation probabilities to be well defined. By normalizing the unit of time so that $\lambda = K$, we obtain (B.5).

It is easy to verify that the replicator dynamics is a myopic adjustment dynamics.[21] By (B.1a) and (B.5), we have

$$
\begin{aligned}
\dot{x}(t)'u(x(t)) &= \sum_{i=1}^{n} \dot{x}_i(t)\big[u_i(x(t)) - \bar{u}(x(t))\big] \\
&= \sum_{i=1}^{n} x_i(t)\big[u_i(x(t)) - \bar{u}(x(t))\big]^2 \geq 0,
\end{aligned}
$$

where the equality holds if and only if $u_i(x(t)) = \bar{u}(x(t))$ for all $i \in \operatorname{supp}(x(t))$, which holds in particular when $x(t)$ is an equilibrium state.

Note however that the replicator dynamics satisfies the strict myopic adjustment condition only in the interior of the state space $\Delta$. It violates the condition on the boundary of $\Delta$ because of its imitative nature: unused actions are never chosen.

As shown by Sandholm (2001),[22] if the game admits a potential function $v$, then it works as a Lyapunov function of strict myopic adjustment dynamics. This is easily verified by using the functions $g_i$ and $G$ in (A.1) and (A.2):

$$
\begin{aligned}
\frac{d}{dt} v(x(t)) &= \nabla v(x(t))'\dot{x}(t) \\
&= \nabla G(x_{-n}(t))'\dot{x}_{-n}(t) \\
&= g(x_{-n})'\dot{x}_{-n}(t) \geq 0,
\end{aligned}
$$

where the second equality follows from (B.1a), the third equality from (A.3), and the inequality from (B.3).

Thus we have:

**Theorem B.1.** *Assume that equilibrium states of* $\mathbf{u}$ *are all isolated. If* $\mathbf{u}$ *admits a potential function, then every trajectory of every strict myopic adjustment dynamics converges to an equilibrium state, and every local maximizer of the potential function is asymptotically stable.*

Two corollaries follow from this theorem. First, strict equilibrium states of a potential game are asymptotically stable under any strict myopic adjustment dynamics. Second, if the potential function is strictly concave on $\Delta$, then its unique maximizer is globally asymptotically stable under any strict myopic adjustment dynamics.

For the replicator dynamics, which satisfies the strict myopic adjustment condition only in $\operatorname{int}(\Delta)$, a slightly weaker result holds, that every trajectory starting in $\operatorname{int}(\Delta)$ converges to a local maximizer of the potential function (while every local maximizer is asymptotically stable).

Outside the class of potential games, the best response and the replicator dynamics may exhibit different asymptotic behavior; see, e.g., Hofbauer and Sigmund (1998, Section 8.4).

When there are only two locations (i.e., $n = 2$), a potential function always exists and is given by $F(x_1) = \int_0^{x_1} f(x_1')\, dx_1'$, where $f(x_1) = u_1(x_1, 1-x_1) - u_2(x_1, 1-x_1)$. Let $(x_1^*, x_2^*)$ be an interior equilibrium state, $x_1^* \in (0,1)$, which satisfies $f(x_1^*) = 0$. Under any strict myopic adjustment dynamics, if $F''(x_1^*) = f'(x_1^*) < 0$, then $(x_1^*, x_2^*)$ locally maximizes the potential and hence is asymptotically stable, while if $F''(x_1^*) = f'(x_1^*) > 0$, then $(x_1^*, x_2^*)$ locally minimizes the potential and hence is unstable.

## Appendix C. Perfect Foresight Dynamics in Potential Games

In this section, we derive the stability results for the perfect foresight dynamics in potential games, established by Hofbauer and Sorger (1999, Theorems 3 and 4) for linear payoff functions. We provide proofs adapted to our nonlinear framework.

Consider a potential game $\mathbf{u}$ with a potential function $v$. The stability results are proved for nonlinear payoff functions under the following regularity condition. We call $x^c \in \Delta$ a *critical point* of $v$ on $\Delta$ if

$$
\frac{\partial v}{\partial x_i}(x^c) = \frac{\partial v}{\partial x_j}(x^c)
$$

for all $i, j \in \operatorname{supp}(x^c)$. If $x^c$ is a critical point of $v$, then we call $v(x^c)$ a *critical value* of $v$.

**Assumption C.1.** $v$ has finitely many different critical values.

Our societal game $\mathbf{u}$ defined in (3.1) satisfies this regularity assumption, since its potential function $v$ in (3.5) is

[21] More generally, the positive definite dynamics (Hopkins, 1999) satisfy the myopic adjustment condition.

[22] See also Hofbauer (2000) for the case of the best response dynamics.

real analytic (and not identically zero), so that it admits only finitely many critical values on any compact set.[23]

Subsection C.1 proves the global accessibility result, that a unique global potential maximizer is globally accessible whenever the degree of friction $\delta = \theta/\lambda$ is sufficiently small, while Subsection C.2 proves the absorption result, that a unique global potential maximizer is always absorbing independently of $\delta$. (While we maintain $\lambda$ as given, one may, as in Hofbauer and Sorger (1999), simplify the notations by normalizing the unit of time so that $\lambda = 1$.)

### C.1. Global Accessibility

**Theorem C.1.** *Suppose that* $\mathbf{u}$ *admits a potential function* $v$ *and satisfies Assumption C.1. Assume that* $x^*$ *is the unique global maximizer of* $v$ *over* $\Delta$. *Then, there exists* $\bar\delta > 0$ *such that* $x^*$ *is globally accessible for all* $\delta \in (0, \bar\delta]$.

For a fixed initial condition $x^0 \in \Delta$, let $X$ be the set of feasible paths from $x^0$, which is compact with respect to the topology of uniform convergence on compact intervals. Consider the optimal control problem:

$$\text{maximize} \quad J(x(\cdot)) = (\lambda+\theta)\int_0^\infty e^{-\theta t} v(x(t))\,dt \tag{C.1a}$$

$$\text{subject to} \quad x(\cdot) \in X. \tag{C.1b}$$

The continuity of $v$ implies the continuity of $J$, which in turn guarantees the existence of an optimal solution to the problem.

The following lemmata correspond to Theorem 2 and Lemma 1 in Hofbauer and Sorger (1999), respectively.

**Lemma C.2.** *Every optimal solution to the problem* (C.1) *is a perfect foresight path from* $x^0$.

*Proof.* Let $H\colon \bar\Delta \times \Delta \times \mathbb{R}^n \to \mathbb{R}$ be the current value Hamiltonian:

$$H(x, \alpha, q) = (\lambda+\theta)v(x) + q'\lambda(\alpha - x). \tag{C.2}$$

The necessary conditions for optimality are as follows: If $x(\cdot) \in X$ is an optimal solution of (C.1) and $\alpha(\cdot)$ the corresponding control path (i.e., $\dot x(t) = \lambda(\alpha(t) - x(t))$, $\alpha(t) \in \Delta$), then there exists an absolutely continuous adjoint function $q\colon [0,\infty) \to \mathbb{R}^n$ such that

$$\alpha(t) \in \arg\max\nolimits_{\alpha\in\Delta} H(x(t), \alpha, q(t)), \tag{C.3}$$

$$\dot q_i(t) = \theta q_i(t) - \frac{\partial H}{\partial x_i}(x(t), \alpha(t), q(t)), \tag{C.4}$$

$$\lim_{t\to\infty} e^{-\theta t} q_i(t) = 0, \tag{C.5}$$

where (C.3) is the maximum principle, (C.4) is the adjoint equation, and (C.5) is the transversality condition. Conditions (C.3) and (C.4) are written as

$$\alpha_i(t) > 0 \Rightarrow q_i(t) \ge q_j(t) \text{ for all } j = 1, \ldots, n, \tag{C.3$'$}$$

$$\dot q_i(t) = (\lambda+\theta)\left(q_i(t) - \frac{\partial v}{\partial x_i}(x(t))\right). \tag{C.4$'$}$$

Under (C.5), it follows from (C.4′) that

$$q_i(t) = (\lambda+\theta)\int_t^\infty e^{-(\lambda+\theta)(s-t)} \frac{\partial v}{\partial x_i}(x(s))\,ds.$$

Thus, by the definition of potential function, we have $q_i(t) \gtreqless q_j(t)$ if and only if $V_i(t) \gtreqless V_j(t)$, where $V_i$ is defined as in (2.10). It therefore follows form (C.3′) that $x(\cdot)$ is a perfect foresight path. $\square$

**Lemma C.3.** *For any* $\varepsilon > 0$, *there exists* $\bar\delta = \bar\delta(\varepsilon) > 0$ *such that for all* $\delta \in (0, \bar\delta]$ *and for all* $x^0 \in \Delta$, *if* $x(\cdot)$ *is an optimal solution to the problem* (C.1), *then there exists* $t \ge 0$ *such that* $|x(t) - x^*| < \varepsilon$.

*Proof.*[24] Assume the contrary: i.e., there exists $\varepsilon > 0$ such that for all $\bar\delta > 0$, there exists an optimal solution $x(\cdot)$ for some $\lambda$ and $\theta$ with $\delta = \theta/\lambda \in (0, \bar\delta]$ and some $x^0 \in \Delta$ such that $|x(t) - x^*| \ge \varepsilon$ for all $t \ge 0$. Given such an $\varepsilon > 0$, let $c = c(\varepsilon) > 0$ be such that

$$c = v(x^*) - \max\bigl\{v(x) \bigm| |x - x^*| \ge \varepsilon\bigr\};$$

$T = T(\varepsilon) \in [0, \infty)$ be such that

$$v\bigl(e^{-t}x^0 + (1 - e^{-t})x^*\bigr) \ge v(x^*) - c/2$$

for all $t \ge T$; and $\bar\delta = \bar\delta(\varepsilon) > 0$ be such that

$$(1 - e^{-\bar\delta T})2M < e^{-\bar\delta T}c/2,$$

where $M > 0$ is a constant such that $|v(x)| \le M$ for all $x \in \Delta$. Given such a $\bar\delta > 0$, let $x(\cdot)$ be an optimal solution with $\delta \in (0, \bar\delta]$ and $x^0 \in \Delta$ such that $|x(t) - x^*| \ge \varepsilon$ for all $t \ge 0$, as assumed.

Let $y(\cdot) \in X$ be the feasible path from $x^0$ such that $y(t) = e^{-\lambda t}x^0 + (1 - e^{-\lambda t})x^*$ (which converges to $x^*$). Then,

$$\begin{aligned}
&J(x(\cdot)) - J(y(\cdot)) \\
&= (\lambda+\theta)\int_0^{T/\lambda} e^{-\theta t}\bigl(v(x(t)) - v(y(t))\bigr)\,dt \\
&\quad + (\lambda+\theta)\int_{T/\lambda}^\infty e^{-\theta t}\bigl(v(x(t)) - v(y(t))\bigr)\,dt \\
&\le (\lambda+\theta)\int_0^{T/\lambda} e^{-\theta t}2M\,dt + (\lambda+\theta)\int_{T/\lambda}^\infty e^{-\theta t}(-c/2)\,dt \\
&= (1 + 1/\delta)\left\{(1 - e^{-\delta T})2M - e^{-\delta T}c/2\right\} \\
&\le (1 + 1/\delta)\left\{(1 - e^{-\bar\delta T})2M - e^{-\bar\delta T}c/2\right\} < 0,
\end{aligned}$$

which contradicts the optimality of $x(\cdot)$. $\square$

[23]This follows from the Lojasiewicz gradient inequality (see, e.g., Absil and Kurdyka, 2006, Lemma 4 and the references therein). I am grateful to Josef Hofbauer for pointing me to this fact.

[24]The proof provided here fixes a small gap in the proof of Hofbauer and Sorger (1999, Lemma 1).

Now let us conclude the proof of Theorem C.1. Let $x^*$ be the unique maximizer of $v$ over $\Delta$. By Theorem C.4 to be proved below, there exists $\varepsilon > 0$ such that any perfect foresight path starting in $B_\varepsilon(x^*)$ converges to $x^*$, where $\varepsilon$ can be taken independently of $\delta$. Given this value of $\varepsilon$, let $\bar{\delta} = \bar{\delta}(\varepsilon)$ as in Lemma C.3, and assume $\delta \leq \bar{\delta}$. Fix any $x^0 \in \Delta$, and let $x(\cdot)$ be a solution to the optimization problem (C.1), which is a perfect foresight path from $x^0$ by Lemma C.2. By Lemma C.3, $x(\cdot)$ must visit $B_\varepsilon(x^*)$. Since $x^*$ is absorbing by Theorem C.4, it follows that $\lim_{t\to\infty} x(t) = x^*$ by the choice of $\varepsilon$. Initial condition $x^0$ taken arbitrarily, this proves the global accessibility of $x^*$ for $\delta \leq \bar{\delta}$.

*C.2. Absorption*

**Theorem C.4.** *Suppose that* **u** *admits a potential function $v$ and satisfies Assumption C.1. Assume that $x^*$ is the unique global maximizer of $v$ over $\Delta$. Then, $x^*$ is absorbing for all $\delta > 0$.*

For $x^0 \in \Delta$, consider the following system:

$$\dot{x}(t) \in \lambda\big(\mathcal{B}(t) - x(t)\big), \qquad x(0) = x^0 \tag{C.6a}$$

$$\dot{V}(t) = (\lambda+\theta)\big(V(t) - u(x(t))\big), \tag{C.6b}$$

where

$$\mathcal{B}(t) = \big\{\alpha \in \Delta \;\big|\; \alpha_i > 0 \Rightarrow V_i(t) \geq V_j(t) \text{ for all } j\big\},$$

which is the convex hull of $\arg\max_i V_i(t)$ in $\Delta$. Observe that $x(\cdot)$ is a perfect foresight path from $x^0$ if and only if it is a solution to (C.6) for which $V(\cdot)$ stays bounded. Then, define $H^* \colon \bar{\Delta} \times \mathbb{R}^n \to \mathbb{R}$ by

$$H^*(x,V) = (\lambda+\theta)v(x) + \lambda\big(\bar{V} - V'x\big), \tag{C.7}$$

where $\bar{V} = \max_i V_i$. This function is the maximized Hamiltonian of the optimization problem (C.1): i.e., $H^*(x,V) = \max_{\alpha\in\Delta} H(x,\alpha,V)$, where $H$ is defined in (C.2). Note that $H^*(x,V) \geq (\lambda+\theta)v(x)$ for all $x \in \Delta$ and $V \in \mathbb{R}^n$.

The following lemmata correspond to Lemmata 3 and 4 in Hofbauer and Sorger (1999), respectively.

**Lemma C.5.** *Let $(x(\cdot), V(\cdot))$ be a solution to (C.6). Then, the function $t \mapsto H^*(x(t), V(t))$ is Lipschitz continuous, satisfies*

$$\frac{d}{dt}H^*(x(t),V(t)) = \theta\lambda\big(\bar{V}(t) - V(t)'x(t)\big) \geq 0 \tag{C.8}$$

*for almost all $t \geq 0$, and therefore is nondecreasing.*

*Proof.* Let $(x(\cdot), V(\cdot))$ be a solution to (C.6). It is easy to verify that the function $t \mapsto H^*(x(t), V(t))$ is Lipschitz continuous.

By Danskin's envelope theorem,[25] we have, for almost all $t \geq 0$, $(d/dt)\bar{V}(t) = \alpha'\dot{V}(t)$ for all $\alpha \in \mathcal{B}(t)$, and in particular, $(d/dt)\bar{V}(t) = (\dot{x}(t)/\lambda + x(t))'\dot{V}(t)$ by (C.6a). Using this, we obtain, for almost all $t \geq 0$,

$$\begin{aligned}
&\frac{d}{dt}H^*(x(t),V(t)) \\
&= (\lambda+\theta)\nabla v(x(t))'\dot{x}(t) \\
&\qquad + \lambda\big(\dot{x}(t)/\lambda + x(t)\big)'\dot{V}(t) - \lambda\big(\dot{V}(t)'x(t) + V(t)'\dot{x}(t)\big) \\
&= (\lambda+\theta)\nabla v(x(t))'\dot{x}(t) + \dot{x}(t)'\dot{V}(t) - \lambda V(t)'\dot{x}(t) \\
&= (\lambda+\theta)\nabla v(x(t))'\dot{x}(t) \\
&\qquad + \dot{x}(t)'(\lambda+\theta)\big(V(t) - u(x(t))\big) - \lambda V(t)'\dot{x}(t) \\
&= \theta V(t)'\dot{x}(t) + (\lambda+\theta)\big(\nabla v(x(t)) - u(x)\big)'\dot{x}(t) \\
&= \theta\lambda\big[V(t)'\big(\dot{x}(t)/\lambda + x(t)\big) - V(t)'x(t)\big] \\
&= \theta\lambda\big[\bar{V}(t) - V(t)'x(t)\big] \geq 0,
\end{aligned}$$

where the third equality follows from (C.6b), the fifth from the definition of potential function as well as $\sum_i \dot{x}_i(t) = 0$, and the last from (C.6a). ☐

**Lemma C.6.** *Let $x(\cdot)$ be a perfect foresight path from $x^0$. If $\hat{x}$ is an accumulation point of $x(\cdot)$, then*
*(1) $v(\hat{x}) \geq v(x^0)$,*
*(2) $\hat{x}$ is a critical point of $v$ on $\Delta$.*

*Proof.* Let $V(\cdot)$ be associated with the perfect foresight path $x(\cdot)$. Let $\{t_k\}$ be a sequence such that $\lim_{k\to\infty} t_k = \infty$ and $\lim_{k\to\infty} x(t_k) = \hat{x}$. Let, without loss of generality, $\hat{V} = \lim_{k\to\infty} V(t_k)$. Define $(x^*(\cdot), V^*(\cdot))$ by $t \mapsto \lim_{k\to\infty}(x(t+t_k), V(t+t_k))$, which satisfies (C.6) with $(x^*(0), V^*(0)) = (\hat{x}, \hat{V})$.

Using Lemma C.5, one can prove by contradiction that $H^*(x^*(t), V^*(t))$ is constant, which implies that $(d/dt)H^*(x^*(t), V^*(t)) = 0$, and therefore

$$\bar{V}^*(t) = V^*(t)'x^*(t) \tag{C.9}$$

for almost all $t \geq 0$ by (C.8). From this equation as well as Lemma C.5, we obtain that

$$\begin{aligned}
(\lambda+\theta)v(x^0) &\leq H^*(x(0),V(0)) \\
&\leq H^*(x(t),V(t)) \\
&\leq H^*(x^*(t),V^*(t)) = (\lambda+\theta)v(\hat{x}),
\end{aligned}$$

which proves (1).

Equation (C.9) also implies that, for almost all $t \geq 0$, $V_i^*(t) = V_j^*(t)$ and $\dot{V}_i^*(t) = \dot{V}_j^*(t)$ for all $i,j \in \operatorname{supp}(x^*(t))$, where the latter follows from Danskin's envelope theorem. It therefore follows from (C.6b) that, for almost all $t \geq 0$, $u_i(x^*(t)) = u_j(x^*(t))$, or equivalently $(\partial v/\partial x_i)(x^*(t)) = (\partial v/\partial x_j)(x^*(t))$, for all $i,j \in \operatorname{supp}(x^*(t))$. But by continuity, this must hold for all $t \geq 0$, which means that $x^*(t)$ is a critical point of $v$ for all $t \geq 0$. In particular, $\hat{x} = x^*(0)$ is a critical point, which proves (2). ☐

Now let us conclude the proof of Theorem C.4. Let $x^*$ be the unique maximizer of $v$ over $\Delta$. It is a critical point of $v$. By Assumption C.1, there exists $\varepsilon > 0$ such that

[25] See, e.g., Hofbauer and Sandholm (2009, Theorem A.4). The following argument is due to Josef Hofbauer (personal communication), who informed me that use of this theorem significantly simplifies the original proof of Hofbauer and Sorger (1999, Lemma 3).

$v(x) > v(x^c)$ for all $x \in B_\varepsilon(x^*)$ and all critical points $x^c \neq x^*$. Lemma C.6 thus implies that any perfect foresight path $x(\cdot)$ from any $x^0 \in B_\varepsilon(x^*)$ satisfies $\lim_{t\to\infty} x(t) = x^*$, which proves the absorption of $x^*$.

## Appendix D. Proof of Proposition 4.2

We only verify that if $\phi_i < \phi_i^1$, then $v$ does not attain its global maximum on the face of $\Delta$ where $x_i = 0$. Fix any $x \in \partial(\Delta)$ such that $x_i = 0$. We denote $\mathrm{supp}(x) = \{j \mid x_j > 0\}$. For $s \in [0,1]$, let $\kappa(s) = v((1-s)x + se_i) - v(x)$. It suffices to show that if $\phi_i < \phi_i^1$, then $\kappa'(0) > 0$. Notice that

$$\begin{aligned}\kappa'(0) &= \frac{\partial v}{\partial x_i}(x) - \sum_{j\in\mathrm{supp}(x)} x_j \frac{\partial v}{\partial x_j}(x) \\ &= S_i(0;\phi_i) - \sum_{j\in\mathrm{supp}(x)} x_j S_j(x_j;\phi_j),\end{aligned}$$

where $S_j(x_j;\phi_j)$ is as in (3.2); i.e.,

$$\begin{aligned}S_j(x_j;\phi_j) = \frac{\mu}{\sigma-1}\log\left((1-\phi_j)x_j + \phi_j\right) \\ + \frac{\mu}{\sigma}\frac{(1-\phi_j)(x_j+L_j)}{(1-\phi_j)x_j+\phi_j}.\end{aligned}$$

Since, under the assumption that $L_i > \sigma/(\sigma-1)$, $S_j(x_j;\phi_j)$ is decreasing in $\phi_j$, we have

$$S_j(x_j;\phi_j) \leq S_j(x_j;0) = \frac{\mu}{\sigma-1}\log x_j + \frac{\mu}{\sigma}\frac{x_j+L_j}{x_j}$$

for $j \in \mathrm{supp}(x)$. It follows that

$$\begin{aligned}&\sum_{j\in\mathrm{supp}(x)} x_j S_j(x_j;\phi_j) \\ &\leq \sum_{j\in\mathrm{supp}(x)} x_j S_j(x_j;0) \\ &= \frac{\mu}{\sigma-1}\sum_{j\in\mathrm{supp}(x)} x_j\log x_j + \frac{\mu}{\sigma}\left(1 + \sum_{j\in\mathrm{supp}(x)} L_j\right) \\ &\leq \frac{\mu}{\sigma}\left(1+\sum_{j\neq i} L_j\right).\end{aligned}$$

On the other hand,

$$S_i(0;\phi_i) = \frac{\mu}{\sigma-1}\log\phi_i + \frac{\mu}{\sigma}\frac{L_i}{\phi_i} - \frac{\mu}{\sigma}L_i.$$

Hence, we have $\kappa'(0) > 0$ if

$$\frac{\mu}{\sigma-1}\log\phi_i + \frac{\mu}{\sigma}\frac{L_i}{\phi_i} - \frac{\mu}{\sigma}L_i > \frac{\mu}{\sigma}\left(1+\sum_{j\neq i}L_j\right),$$

or

$$\frac{\sigma}{\sigma-1}\log\phi_i + \frac{L_i}{\phi_i} > L + 1,$$

which is true if $\phi_i < \phi_i^1$. □

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