Topics in Economic Theory

Winter 2013

Daisuke Oyama

www.oyama.e.u-tokyo.ac.jp/theory13

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Course Description

- This course provides lectures on the theories of
  - dynamical systems, and
  - deterministic approximation of stochastic processes.
- Main part devoted to discussing mathematical techniques.
- Applications: population games,

  games describing “macroeconomic situations”.

Course Outline

1. Theory of (set-valued) dynamical systems (BHS)
2. Application to "sampling best response dynamics" (OST)
3. Theory of stochastic approximations (BW)
4. Forward-looking expectations dynamics (if time permits)

Main Readings

1. Benaïm, M., J. Hofbauer, and S. Sorin (2005). "Stochastic Approximations and Differential Inclusions," *SIAM Journal of Control and Optimization* 44, 328-348.
2. Oyama, O., W. H. Sandholm, and O. Tercieux (2012). "Sampling Best Response Dynamics and Deterministic Equilibrium Selection."
3. Benaïm, M., and J. Weibull (2003). "Deterministic Approximation of Stochastic Evolution in Games," *Econometrica* 71, 873-903.

Other References

- Sandholm, W. H. (2010). *Population Games and Evolutionary Dynamics*, MIT Press, Cambridge.
- Evans, G. W. and S. Honkapohja (2001). *Learning and Expectations in Macroeconomics*, Princeton University Press, Princeton.
- ...

To be posted on the webpage...

Other Course Information

- Webpage:

  www.oyama.e.u-tokyo.ac.jp/theory13

- Grade:

  Term paper or referee reports

- Office hours:

  Friday 14:00-15:30 at 1012

Today: Short introduction to population game dynamics

- Population games
  - Formulation
  - Examples
- Evolutionary dynamics
  - Replicator dynamics
  - Best response dynamics
  - Perturbed best response dynamics
  - Sampling best response dynamics

Population Games

- A continuum of homogeneous and anonymous agents with mass 1 (single-population version)
- $A = \{1, \ldots, n\}$: (common) set of actions
- $\Delta = \{x \in \mathbb{R}^n_+ \mid \sum_{i=1}^n x_i = 1\}$: set of action distributions
  - $x = (x_1, \ldots, x_n) \in \Delta$: action distribution
- $u \colon \Delta \to \mathbb{R}^n$: (common) payoff function (continuous)
  - $u_i(x)$: payoff to action i when the population state is x
- $b(x) = \{i \in A \mid u_i(x) \geq u_j(x)\}$ $(\neq \emptyset)$
  $\cdots$ best response correspondence in pure actions
- $B(x) = \{\alpha \in \Delta \mid \alpha_i > 0 \Rightarrow u_i(x) \geq u_j(x) \text{ for all } j\}$ $(\neq \emptyset)$
  $\cdots$ best response correspondence in mixed actions

Population Games

- $x^* \in \Delta$ is an *equilibrium state* if

$$x_i^* > 0 \Rightarrow u_i(x^*) \geq u_j(x^*) \text{ for all } j.$$

$\iff$ x^* is a fixed point of B, i.e., $x^* \in B(x^*)$.

- Since u_i's are continuous, B has a closed graph, and hence an equilibtium state exists by Kakutani's fixed point theorem (cf. Nash 1950).

- $x^* \in \Delta$ is a *strict equilibrium state* if

$$x_i^* > 0 \Rightarrow u_i(x^*) > u_j(x^*) \text{ for all } j \neq i.$$

A strict equilibrium state must be a monomorphic state (i.e., a vertex of Δ).

Examples

- Traffic networks: $A =$ set of routes
- Spatial economics: $A =$ set of locations (regions, countries...)
  (e.g., Krugman (1991): presence of trade costs)
- Economic development: $A =$ set of sectors
  (e.g., Matsuyama (1991, 1992))

Evolutionary Dynamics

1. Replicator dynamics
2. Best response dynamics
3. Perturbed best response dynamics
4. Sampling best response dynamics