Game Theory I

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Outline

- Monday, Thursday 8:30-10:15
  - October 5, 9, 12, 16, 19, 23, 26, 30
  - November 1 (Wed), 2, 6, 9, 13
- Course webpage:
  www.oyama.e.u-tokyo.ac.jp/theory23
- In this course, we study advanced topics on incomplete information games, in particular on information design.

  Half (or 2/3) of the course will consist of lectures by the instructor and the other half (or 1/3) of presentations by students.

Other Information

- Grading
  - Participation
  - Presentation
  - Final project
- Office hours:
  Fridays 14:00-15:30, or by appointment
  10th floor, 1012

Topics

1. Correlated equilibrium/Bayes correlated equilibrium
2. Email game, global games/Full implementation with payoff perturbations
3. Robustness to incomplete information
4. Smallest equilibrium implementation, full implementation
5. Application: Joint design of information and transfers

Incomplete Information Games

- Players: $I = \{1, \dots, |I|\}$
- i's actions: A_i (finite) $(A = A_1 \times \cdots \times A_{|I|})$
- States: Θ (finite)
- $\mu \in \Delta(\Theta)$: Probability distribution over Θ
- i's payoff function: $u_i \colon A \times \Theta \to \mathbb{R}$
- i's types: T_i (countable) $(T = T_1 \times \cdots \times T_{|I|})$
- Common prior: $\pi \in \Delta(T \times \Theta)$
- i's strategy: $\sigma_i \colon T_i \to \Delta(A_i)$.

- Economic/game theoretic models analyze the behavior of solutions (Bayes Nash equilibria, rationalizable strategies, ...) of an incomplete information game.

Implementation via Information Design

- Fix I, $(A_i)_{i \in I}$, Θ, μ, and $(u_i)_{i \in I}$. $\cdots$ "Base game"
- Vary $(T_i)_{i \in I}$ and π. $\cdots$ "Information structure"
- Strategy profile $(\sigma_i)_{i \in I}$ given $\mathcal{T} = ((T_i)_{i \in I}, \pi)$ induces a joint distribution ν over $A \times \Theta$:

$$\nu(a, \theta) = \sum_t \pi(t, \theta) \prod_i \sigma_i(t_i)(a_i).$$

  ν: "outcome"

- What outcomes are induced by an equilibrium of the incomplete information game given by a choice of an information structure?

Implementability

- Once an information structure $\mathcal{T} = ((T_i)_{i \in I}, \pi)$ is chosen, the game has multiple equilibria in general.

- Partial implementation:

  An outcome ν is partially implementable if there exists an information structure $\mathcal{T}$ such that some equilibrium of (the game induced by) $\mathcal{T}$ induces ν.

  PI: Set of partially implementable outcomes

- Full implementation:

  An outcome ν is fully implementable if there exists an information structure $\mathcal{T}$ such that all equilibria of $\mathcal{T}$ induce ν.

  (Equivalent to unique implementation in supermodular games.)

  FI: Set of fully implementable outcomes

- Smallest equilibrium implementation:

  An outcome ν is smallest equilibrium implementable if there exists an information structure $\mathcal{T}$ such that the smallest equilibrium of $\mathcal{T}$ induces ν.

  (Well defined in supermodular games.)

  SI: Set of smallest equilibrium implementable outcomes

Optimal Information Design

- There is an (informed) information designer who designs and provides an information structure to the players.
  - She can commit to the chosen information structure, before the state is realized.
- The information designer chooses an information structure to maximize the expected value of her objective function $V : A \times \Theta \to \mathbb{R}$.

- Best-case information design:

  Assume that, once an information structure $\mathcal{T}$ is given, players will play the best equilibrium for the information designer.

  $\cdots$ Optimization over PI

- Worst-case information design:

  Assume that, once an information structure $\mathcal{T}$ is given, players will play the worst equilibrium for the information designer.

  Assume:

  - The base game is supermodular (where actions are ordered).
  - Designer's objective $V(a, \theta)$ is increasing in a.

  $\cdots$ Optimization over $\overline{SI}$