Mathematics II
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Homework 4

Due on May 15

1. MWG Exercise 3.D.5.

2. MWG Exercise 3.E.2.

3. MWG Exercise 3.E.4.

4. MWG Exercise 3.E.6.

5. MWG Exercise 3.E.8.

6. MWG Exercise 3.G.1.

7. MWG Exercise 3.G.2.

8. MWG Exercise 3.G.12.

9. MWG Exercise 3.G.14.

10. Let a list of pairs $(p^1, w^1), \dots, (p^N, w^N)$ be given, where $p^n \in \mathbb{R}^L_{++}$ and $w^n \in \mathbb{R}_{++}$ for each $n = 1, \dots, N$. In addition to the L commodities $1, \dots, L$, there is another commodity labeled 0 whose price is normalized to 1. Suppose that the consumer's consumption when the prices are given by p^n and the wealth level is w^n is $x^n \in \mathbb{R}^L_+$ for the commodities $1, \dots, L$ and $w^n - p^n \cdot x^n$ for commodity 0. We denote $x_0^n = w^n - p^n \cdot x^n$. Assume that $x^m \neq x^n$ for $m \neq n$.

Consider the following conditions:

Condition 1. For any (distinct) $n_1, \dots, n_k \in \{1, \dots, N\}$,

$$p^{n_1} \cdot (x^{n_2} - x^{n_1}) + \cdots + p^{n_{k-1}} \cdot (x^{n_k} - x^{n_{k-1}}) + p^{n_k} \cdot (x^{n_1} - x^{n_k}) > 0.$$

Condition 2. For any (distinct) $n_1, \dots, n_k \in \{1, \dots, N\}$, if $x_0^{n_{i+1}} + p^{n_i} \cdot x^{n_{i+1}} \leq w^{n_i}$ for all $i = 1, \dots k - 1$, then $x_0^{n_1} + p^{n_k} \cdot x^{n_1} > w^{n_k}$.

(1) Show that Condition 1 implies Condition 2.

(2) Show that Condition 1 holds if there exists a function $\phi\colon \mathbb{R}^L_+ \to \mathbb{R}$ such that for any $w \in \mathbb{R}$, $(w - p^n \cdot x^n) + \phi(x^n) > (w - p^n \cdot x^m) + \phi(x^m)$ for all $n = 1, \dots, N$ and all $m \neq n$.

(3) Show that if Condition 1 holds, then there exists a concave function $\phi\colon \mathbb{R}^L_+ \to \mathbb{R}$ such that for any $w \in \mathbb{R}$, $(w - p^n \cdot x^n) + \phi(x^n) > (w - p^n \cdot x^m) + \phi(x^m)$ for all $n = 1, \dots, N$ and all $m \neq n$. (Construct such a function ϕ.)