Mathematics II
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Homework 5

Due on May 22

1. MWG Exercise 3.I.3.

(There were errors in the lecture slides, corrected in the updated file.)

2. MWG Exercise 3.I.5.

3. Suppose that $u(x)$ is homogeneous of degree one. Show that the Walrasian demand correspondence $x(p, w)$ is homogeneous of degree one in w: i.e., $x(p, \alpha w) = \alpha x(p, w)$ for all $\alpha > 0$ (where $\alpha A = \{\alpha a \mid a \in A\}$).

4. MWG Exercise ~~3.D.5~~ 4.D.5.

5. MWG Exercise ~~3.D.6~~ 4.D.6.

6. [Advanced] Let a complete and transitive $\succsim$ on $X = \mathbb{R}^L_+$ be given. Denote $X^0 = \{x \in X \mid x \succsim 0\}$ (where $0 \in \mathbb{R}^L_+$ is the zero consumption vector).

(1) Suppose that $\succsim$ is continuous. Fix any $\bar{p} \gg 0$, and define the function $u\colon X \to \mathbb{R}$ by

$$u(x) = \min\{\bar{p} \cdot x' \mid x' \succsim x\}.$$

Show that u represents $\succsim$ on X^0.

(2) Suppose in addition that $\succsim$ is locally nonsatiated. Show that u is continuous.

(3) Define the function $\bar{v}\colon \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}$ by

$$\bar{v}(p, w) = \min\{\bar{p} \cdot x' \mid x' \succsim x \text{ for some } x \in x(p, w)\},$$

where $x(p, w)$ is the Walrasian demand correspondence. Show that $\bar{v}$ is the indirect utility function for the utility function u.