Mathematics II
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Homework 7

Due on June 26

1. MWG Exercise 5.B.2.

2. MWG Exercise 5.B.3.

3. MWG Exercise 5.C.1.

4. Prove properties 2, 5, and 6 in Proposition 6.3 in the lecture slides.

5. Find an example of Y that is nonempty, closed, and convex and satisfies free disposal and such that

$$Y \neq \{y \in \mathbb{R}^L \mid p \cdot y \leq \pi(p) \text{ for all } p \gg 0\}.$$

6. MWG Exercise 5.C.9.

7. MWG Exercise 5.C.10.

8. MWG Exercise 5.C.12.

9. MWG Exercise 5.D.1.

10. MWG Exercise 5.D.2.

11. [Advanced] Prove property 3 in Proposition 6.12 in the lecture slides.

Fix any $q > f(0)$, and write $V = \{z \in \mathbb{R}^{L-1} \mid z \geq 0,\ f(z) \geq q\}$. Suppose that $\bar{z} \notin V$.

(1) Let $A = \{\bar{z}\} - V$. Prove that A is convex and closed and satisfies $A - \mathbb{R}^{L-1}_{++} \subset A$.

(2) Prove that there exist $w^1 \geq 0$ and α such that $w \cdot \bar{z} < \alpha \leq w \cdot z$ for all $z \in V$.

(3) Fix any $w^0 \gg 0$; then $w^0 \cdot z > 0$ for all $z \in V$. Let $w = w^1 + \varepsilon w^0 \gg 0$ for sufficiently small $\varepsilon > 0$ (how small?), and conclude the proof.