Mathematics II
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**Homework 10**

Due on July ~~10~~ 17

**1.** MWG Exercise 10.C.2.

**2.** MWG Exercise 10.D.2.

**3.** MWG Exercise 10.D.3.

**4.** [Advanced] In the class, we proved that Brouwer's Fixed Point Theorem implies the following:

($*$) For any continuous function $z\colon X \to \mathbb{R}^L$, where $X = \mathbb{R}^L_+ \setminus \{0\}$, such that

(a) $z(\alpha p) = z(p)$ for all $p \in X$ and all $\alpha > 0$, and

(b) $p \cdot z(p) = 0$ for all $p \in X$,

there exists $p^* \in X$ such that $z(p^*) \leq 0$.

Here, we want to show that ($*$) in fact implies Brouwer's Fixed Point Theorem. Suppose that ($*$) holds, and prove the following:

($**$) For any continuous function $f\colon \Delta \to \Delta$, where $\Delta = \{p \in \mathbb{R}^L_+ \mid \sum_{\ell=1}^L p_\ell = 1\}$, there exists $p^* \in \Delta$ such that $f(p^*) = p^*$.

Hint. Given $f$, consider the function $z\colon X \to \mathbb{R}^L$ given by

$$z(p) = f\left(\frac{p}{\|p\|_1}\right) - \frac{p \cdot f\left(\frac{p}{\|p\|_1}\right)}{p \cdot p} p,$$

where $\|p\|_1 = \sum_{\ell=1}^L p_\ell$.