4. Aggregate Demand

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Aggregate Demand and Aggregate Wealth

- I consumers ($i = 1, \ldots, I$)
- $\succsim_i$: complete, transitive, continuous, and locally nonsatiated

  Represented by a continuous utility function u_i
- w_i: wealth level
- $\Rightarrow x_i(p, w_i)$: individual Walrasian demand function
- Aggregate demand function: $x(p, w_1, \ldots, w_I) = \sum_i x_i(p, w_i)$
- When can we write it as $x(p, \sum_i w_i)$?
- $\Rightarrow$ Aggregate demand must be independent of the wealth distribution.

Gorman Form

- Suppose that the indirect utility function of each consumer i is written, with a common $b(p)$, as

$$v_i(p, w_i) = a_i(p) + b(p)w_i.$$

- By Roy's identity, for each commodity ℓ,

$$x_{\ell i}(p, w_i) = -\frac{\frac{\partial a_i}{\partial p_\ell}(p) + \frac{\partial b}{\partial p_\ell}(p)w_i}{b(p)}.$$

- Thus,

$$\sum_i x_{\ell i}(p, w_i) = -\frac{\sum_i \frac{\partial a_i}{\partial p_\ell}(p) + \frac{\partial b}{\partial p_\ell}(p)\sum_i w_i}{b(p)}.$$

$\cdots$ Depends only on $\sum_i w_i$, not on $(w_1, \ldots, w_I)$.

- This aggregate demand function is derived from the "representative consumer" with indirect utility function

$$v(p, w) = \sum_i a_i(p) + b(p)w.$$

- It is known that the Gorman form is the only case where $\sum_i x_i(p, w_i)$ is written as a function of $\sum_i w_i$.

Example: Quasi-Linear Preference Relations

- Suppose that the utility function of each i is given by

$$u_i(x) = x_1 + \phi_i(x_2, \ldots, x_L).$$

- $x_{\ell i}(p, w_i)$ does not depend on w_i for $\ell \neq 1$, and

$$x_{1i}(p, w_i) = \frac{1}{p_1} w_i - \frac{1}{p_1} \sum_{\ell \neq 1} p_\ell x_{\ell i}(p).$$

- Thus i's indirect utility function is written in the form

$$v_i(p, w_i) = \frac{1}{p_1} w_i + a_i(p).$$

Example: Identical Homethetic Preference Relations

- Suppose that all consumers have an identical homothetic preference relation, which is represented by a homogeneous utility function u.

- Then

$$v(p, w_i) = w_i v(p, 1).$$

- Let $b(p) = v(p, 1)$ $(= u(x(p, 1))$ and $a_i(p) = 0$.

Representative Consumer

- Fix a wealth distribution rule $(w_1(p,w), \ldots, w_I(p,w))$:
  - $\sum_i w_i(p,w) = w$
  - $w_i(p,w)$: continuous, homogeneous of degree one
- Aggregate demand function:

$$x(p,w) = \sum_i x_i(p, w_i(p,w))$$

continuous, homogeneous of degree one, Walras' law

Definition 4.1

A positive representative consumer exists if there exists a complete and transitive $\succsim$ on $\mathbb{R}^L_+$ that rationalizes $x(p,w)$.

- Social welfare function $W\colon \mathbb{R}^I \to \mathbb{R}$

  strictly increasing, continuous

- $W(u_1, \ldots, u_I)$: "Social utility index" for a profile $(u_1, \ldots, u_I)$ of individual utility levels

Definition 4.2

The positive representative consumer is a normative representative consumer relative to W if for all $p \gg 0$ and $w > 0$, $(w_1(p, w), \ldots, w_I(p, w))$ is a solution to

$$\begin{aligned} \max_{w_1, \ldots, w_I} \quad & W(v_1(p, w_1), \ldots, v_I(p, w_I)) \\ \text{s. t.} \quad & \sum_i w_i \leq w. \end{aligned}$$

Proposition 4.1

Suppose that the positive representative consumer is a normative representative consumer relative to W.
Define $u\colon \mathbb{R}^L_+ \to \mathbb{R}$ by

$$u(x) = \max_{x_1,\dots,x_I} \{W(u_1(x_1),\dots,u_I(x_I)) \mid \textstyle\sum_i x_i \le x\}.$$

Then u rationalizes $x(p,w)$.

Proof

- Fix (p, w).
- Let

$$\begin{aligned}\bar{v}(p,w) &= \max_{w_1,\ldots,w_I} \{W(v_1(p,w_1),\ldots,v_I(p,w_I)) \mid \textstyle\sum_i w_i \leq w\} \\ &= W(v_1(p,w_1(p,w)),\ldots,v_I(p,w_I(p,w))).\end{aligned}$$

- We want to show:
  1. $p \cdot x \leq w \Rightarrow u(x) \leq \bar{v}(p,w)$,
  2. $u(x(p,w)) \geq \bar{v}(p,w)$.

1.

- Suppose that $p \cdot x \leq w$.
- Take any $(x_i)_{i \in I}$ such that $\sum_i x_i \leq x$.
- Let $w_i = p \cdot x_i$.

  Then $\sum_i w_i = p \cdot \sum_i x_i \leq p \cdot x \leq w$.
- Then we have

$$
\begin{aligned}
&W(u_1(x_1), \dots, u_I(x_I)) \\
&\qquad \leq W(v_1(p, w_1), \dots, v_I(p, w_I)) \qquad (W \text{ increasing}) \\
&\qquad \leq \bar{v}(p, w) \qquad (\text{by definition of } \bar{v}).
\end{aligned}
$$

- Therefore, we have $u(x) \leq \bar{v}(p, w)$ (by definition of u).

2.

- By $\sum_i x_i(p, w_i(p, w)) = x(p, w)$, we have

$$\begin{aligned} u(x(p,w)) &\geq W(u_1(x_1(p,w_1(p,w))),\dots,u_I(x_I(p,w_I(p,w)))) \\ &\quad \text{(by definition of } u\text{)} \\ &= W(v_1(p,w_1(p,w)),\dots,v_I(p,w_I(p,w))) \\ &= \bar{v}(p,w) \quad \text{(by normative representative consumer).} \end{aligned}$$

Gorman Form

- Suppose that the indirect utility function of each consumer i is written, with a common $b(p)$, as

$$v_i(p, w_i) = a_i(p) + b(p)w_i.$$

- In this case, there is a positive representative consumer, with indirect utility $v(p, w) = \sum_i a_i(p) + b(p)w$.
- Consider the "utilitarian" social welfare function $\overline{W}(u_1, \ldots, u_I) = \sum_i u_i$.

- Then for any $(w_1, \dots, w_I)$ such that $\sum_i w_i \leq w$,

$$\begin{aligned}\overline{W}(v_1(p, w_1), \dots, v_I(p, w_I)) &= \sum_i a_i(p) + b(p) \sum_i w_i \\ &\leq \sum_i a_i(p) + b(p) w.\end{aligned}$$

- Thus, for any distribution rule $w_i(p, w_i)$, the positive representative consumer is a normative representative consumer with respect to $\overline{W}$.